ANNALS OF FACULTY ENGINEERING HUNEDOARA
– International Journal of Engineering
Tome XI (Year 2013) – FASCICULE 2 (ISSN 1584 – 2665)

1. Octavian FIERARU, 2. Ticuşor CIOBOTARU,
3. Valentin VÎNTURIŞ, 4. Dănuț GROSU, 5. Ioan BĂRBUŞ

THE STUDY OF THE POWER FLOW IN THE PLANETARY GEARBOX

1-5. MILITARY TECHNICAL ACADEMY, BUCUREȘTI, ROMANIA

ABSTRACT: The paper focuses on the study of the power flow in the planetary gearbox of a tracked vehicle and presents an algorithm that analyzes and determines the distribution of moments, angular velocities and power factors. The planetary gearbox used for the analysis operates in a transitory movement system and is part of a transmission from a Romanian main battle tank production. The analysis itself is elaborated for the second level of the planetary gearbox. The way of analysis is based on the network pattern where the planetary gearbox is seen as an energetical network that converts and sends the power flow to the driving mechanism of the tracked vehicle. Using the network pattern, the planetary gearbox is graphically represented as a general nodal scheme that is made of nodes and sides. The characteristics of nodal schemes point out the physical processes which are involved in the performance of the planetary gearbox. The paper ends in the representation of the circulation of the power flow to the second level of the planetary gearbox, and in a comparison between the analytical data resulted from the analysis of planetary gearbox performing both in the dynamic and in the stationary regimes.
KEYWORDS: planetary gearbox, power flow, power factor, load and kinematic factors, nodal scheme

INTRODUCTION

The planetary gearboxes are complex mechanical structures, both from a constructive and from a functional point of view. In the specialized literature [4], the planetary gearboxes are represented by some principles schemes or kinematic schemes which express in an adapted method the way of representation and organization without emphasizing the physical processes which take part in the performance.

The kinematic scheme of the studied planetary gearbox and the number of teeth from the gear wheel which are part of the simple planetary mechanisms are shown in Figure 1.

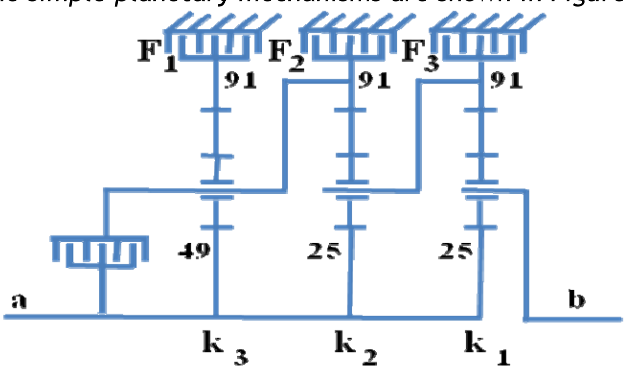

Figure 1 – The kinematic scheme of the planetary gearbox

If we interpret the kinematic scheme of the planetary gearbox shown in figure 1 we are able to observe that there are three simple planetary mechanisms and four friction elements – three brakes and a clutch. The simple planetary mechanisms are symmetric, of type E-I, and having external and internal engagements.

Another way of representing the gearboxes is by using the nodal schemes where the planetary gearbox is seen as an energetic network formed by nodes and sides. The nodes are active elements where the transformations of the load and/or kinematic factors take place. The sides have more of a passive role because they are connections between the nodes. The main characteristic of the sides is that they allow the energy transmission between the nodes without internal losses of power. Through the agency of the nodal schemes there are obviously and simply emphasized the physical processes which appear in the performance of the gearbox and the circulating track of the power flow.

In the accelerated movement system the kinematic factors are not constant and that thing presupposes that the transmission and the conversion of the energy vary in time. Such situations are characterized by the fact that the energy accumulates in the guise of inertial power flows in the energetic network.

The inertial flows don't express the proper energy of the component from the planetary gearbox which is implied in the rotary motion, but how its energy varies inside the energetic network[2] *according to the relation:*

$$A_j = -\frac{dE_c}{dt} \rightarrow A_j = -\frac{d\left(\frac{m \cdot r^2 \cdot \omega_j^2}{2}\right)}{dt}\Bigg|_{I = m \cdot r^2} \rightarrow A_j = -I_j \cdot \frac{d\omega_j}{dt} \cdot \omega_j, \qquad (1)$$

where I, ω, m, r, E_c *are the moment of inertia, the angular speed, the mass, the range and the kinetic energy of the element which is implied in the rotary motion.*

Figure 2 - The traditional representation of the divergent power flow

The power factor of the inertial flow A_j *is characterized by the load factor* $M_j = -I_j \cdot \frac{d\omega_j}{dt}$ *and by the kinematic factor* $C = \omega$.

The traditional way of representing the accumulating or divergent inertial flow is the one in figure 2.

THE MAKING OF THE GENERAL NODAL SCHEME

Keeping in mind the conditions imposed by the transitory system of elements as a part of the planetary gearbox, and the kinematic scheme from figure 1, it can be represented the nodal scheme of the analyzed planetary gearbox.

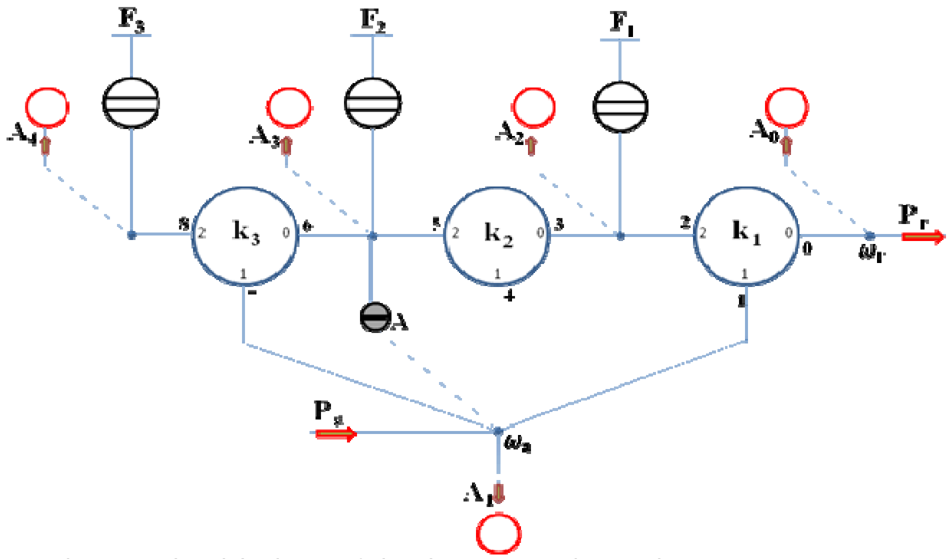

Figure 3 - The general nodal scheme of the planetary gearbox in the transitory movement system

On the general nodal scheme are attached, using traditional symbols, the inertial power flows A_0, A_1, A_2, A_3, A_4 *of the elements which are included in the rotary movement, the brakes* F_1, F_2 *and* F_3 *which are meant to block some elements from the gearbox in order to connect the levels, the clutch* A *which sets the direct connection. The planetary mechanism* k_1, k_2 *and* k_3 *are traditionally represented on the nodal scheme through global nodes. The connection between the simple planetary mechanisms, the clutch, the brakes and the inertial power flows is being made through the sides and the branched nodes* n_0, n_1, n_2, n_3 *and* n_4.

THE MAKING OF THE NODAL SCHEME FOR THE SECOND LEVEL OF THE PLANETARY GEARBOX

The making of the nodal scheme for the second level has as a starting point the general nodal scheme of the planetary gearbox. According to the cyclorama which was presented within the paper [1]*, the second level of the gearbox is acquired by connecting the brake* F_2*. We need to analyze all the three planetary mechanisms in order to establish the circulating way of the flow power and the participating elements in the transmission and conversion of the power factor. Each simple planetary* *mechanism is made of three external elements, the central wheel, the crown and the port satellites board. On the general nodal scheme these are symbolically marked with numbers from zero to eight.*

The inertial power flows are present only at the external elements which are found in the rotary movement. That's why, because of the connection of the brake F_2, on the nodal scheme of the second level will appear only the accumulating inertial flows A_0, A_1, A_2 and A_4. The friction elements A, F_1 and F_3 won't be represented on the scheme because they are passive.

Through the branched node n_1, the nodal scheme emphasizes the fact that the power flow is directed towards the central wheels of the simple planetary mechanisms. According to the characteristics of the branched nodes, the central wheels come into contact with the same kinetic factor, but with different load factors. From these constraints we have established that the central wheel of the simple planetary mechanisms do take part in the transmission of the power flow.

It is already known the fact that the second level of the gearbox is acquired by connecting the brake F_2. By closely examining the planetary mechanism K_2, we are able to see that, by blocking the crown, the power flow is being transmitted from the central wheel to the post satellites board, symbolically marked with "3". The crown of the planetary mechanism k_1 is in adherence with the board of the planetary mechanism k_2 and takes part in the transmission of the power flow. We can say that the planetary mechanism k_1 does also take part in the transmission of the power flow if we keep in mind the movement law of the central wheel.

In the equability law of the load factors applied on the branched node n_4 it is shown that the external element „8" has a load factor. At the same time it is characterized also by a kinematic factor other than zero which means that it takes part in the transmission of the power flow. The post satellites board is connected with the crown of the planetary mechanism k_2 which is blocked. The analysis of the branched node n_1 showed that the central wheel of the mechanism has both a load factor and a kinematic factor. As all the elements that are part of the mechanism are characterized by a movement law and by load factors, the planetary mechanism k_3 takes part in the transmission of the power flow also.

The nodal scheme for the second level of the planetary gearbox is represented in figure 4 by completing the general nodal scheme with the data acquired from the analysis.

Considering the things that we have already shown we can conclude that in the accelerated movement system all the planetary mechanisms and some of the inertial power flows take part in the transmission and conversion of the power flow.

THE ANALYSIS OF THE TRANSMISSION AND CONVERSION METHOD OF THE POWER FLOW IN THE SECOND LEVEL OF THE PLANETARY GEARBOX

The analysis is being made through the analytical method which resembles the analysis of the uniform movement system utilized in the specialized literature [1,4] and its purpose is to determine the path of the power flow in the second level of the planetary gearbox. The approach is different because of the inertial power flows that appear in the energetic network as results of the power used for accelerating the elements which take part in the rotary movement.

The starting point in the course of the operations included in the analytical method is represented by the directional power flow found at the admission and outcome of the gearbox. At the admission of the gearbox the power flow is considered to be convergent and that thing means that the sign of its power factor is conventionally chosen positive. At the outcome of the gearbox the power flow is divergent. Based on this theory, the power factor found at the outcome of the gearbox has a sign conventionally negative.

In the transitory system the kinematic factors which characterize the branched nodes and the global ones are not being affected by the inertial power accumulations or by the internal power losses which appear in the energetic network. That is why the complete kinematic and dynamic rapports of transmission are similar to the ones in the stationary system both as concerning the value and the analytical structure.

The angular rates of the elements that are part of the gearbox are not being affected by the internal power losses, but they vary in time. We can observe on the general nodal scheme that the angular rates of the solar wheels which are part of the planetary mechanisms have the same value as the one of the admission arbor from the planetary gearbox. The equality concept of the angular rates also appears at some other elements of the gearbox, for example at the port satellites board and at the crown that can be found in the adjacent planetary mechanisms.

The nodal scheme of the second level represented in figure 4 emphasizes the simple planetary mechanisms which take part in the transmission of the power flow but does not show its path. For showing how the power flow circulates it is necessary to determine the analytical expression and the sign of the power factor which loads every external element of the planetary gearbox.

The operation algorithm presupposes the determination of the angular rates, of the complete rapports of transmission which characterize the stationary and dynamic systems, and of the torsion moments which load the external elements of the planetary mechanisms. The internal losses of power *appear because of frictions from the engagements, the bearings and the frictions between the gear wheels and the grease oil.*

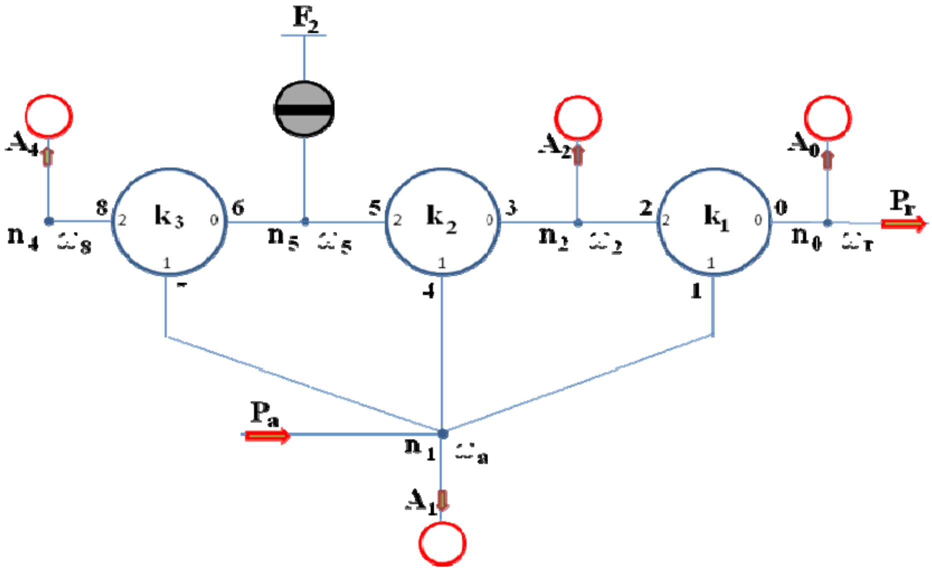

Figure 4 - The nodal scheme for the second level of the planetary gearbox. The transitory movement system

The angular rates and the complete rapports of transmission i_{cvII} and $\tilde{i}_{cvII}$ will be the result of solving the equations system which is formed from Willis's equations, the restrictions imposed for the achievement of the second level of planetary gearbox and the definition of the efficiency [1] (eq. 2).

$$\left.\begin{array}{l}\omega_1 + k_1\cdot\omega_2 - (1+k_1)\cdot\omega_r = 0\\ \omega_4 + k_2\cdot\omega_5 - (1+k_2)\cdot\omega_3 = 0\\ \omega_7 + k_3\cdot\omega_8 - (1+k_3)\cdot\omega_6 = 0\\ \omega_1 = \omega_4 = \omega_7 = \omega_a\\ \omega_2 = \omega_3\\ \omega_5 = \omega_6\\ \omega_8 = 0\\ (\eta_0)_{k_1} = \dfrac{\tilde{i}_{12}^{0}}{-k_1}\\ (\eta_0)_{k_2} = \dfrac{\tilde{i}_{45}^{3}}{-k_2}\\ (\eta_0)_{k_3} = \dfrac{\tilde{i}_{78}^{6}}{-k_3}\end{array}\right| \Rightarrow \left|\begin{array}{l}\omega_1 = \omega_4 = \omega_7 = \omega_a\\ \omega_2 = \omega_3 = \dfrac{1+k_2+k_3}{(k_2+1)(k_3+1)}\omega_a\\ \omega_5 = \omega_6 = \dfrac{1}{(k_3+1)}\omega_a\\ \omega_r = \dfrac{(k_1+1)(k_2+1)(k_3+1) - k_1\cdot k_2\cdot k_3}{(k_1+1)(k_2+1)(k_3+1)}\omega_a\\ \omega_8 = 0\\ i_{cvII} = \dfrac{(k_1+1)(k_2+1)(k_3+1)}{(k_1+1)(k_2+1)(k_3+1) - k_1\cdot k_2\cdot k_3}\\ \tilde{i}_{cvII} = \dfrac{(\eta_0\cdot k_1+1)(\eta_0\cdot k_2+1)(\eta_0\cdot k_3+1)}{(\eta_0\cdot k_1+1)(\eta_0\cdot k_2+1)(\eta_0\cdot k_3+1) - \eta_0^{3}\cdot k_1\cdot k_2\cdot k_3}\end{array}\right. \tag{2}$$

where η_0 is the efficiency of the planetary mechanisms k_1, k_2 and k_3, and $\tilde{i}_{12}^{0}, \tilde{i}_{45}^{3}$ and $\tilde{i}_{78}^{6}$ are the rapports of transmission of the dynamic moments which load the simple planetary mechanisms.

Considering the blocking restriction of the port satellites board which converts the planetary mechanism in a serial transmission we can observe that the efficiency of the planetary mechanisms $(\eta_0)_{k_1}$, $(\eta_0)_{k_2}$ and $(\eta_0)_{k_3}$ have the same value [1]. For simplifying the operation relations the following notation is being introduced:

$$(\eta_0)_{k_1} = (\eta_0)_{k_2} = (\eta_0)_{k_3} = \eta_0 \tag{3}$$

The mathematical expressions of the angular rates which characterize the dynamic system are analytically identical with the ones of the stationary system, both of them being dependent of ω_a. If we analyze them from a physical point of view we can observe that they are different because of the ascendant variation of the angular rate at the admission of the planetary gearbox. The complete rapports of transmission for both movement systems are identical because they are not dependent of the angular rates, but of the constants and efficiencies of the planetary mechanisms k_1, k_2 and k_3.

The load factors which charge the elements of the planetary gearbox are shown through applying the equability laws of the load factors and the distribution relations of the moments [1,2] on the general and branched nodes specific for the second level (eq. 4).

A system of 11 equations containing 11 unknown elements resulted from the distribution and equability relations - $\tilde{M}_r, \tilde{M}_0, \tilde{M}_1, \tilde{M}_2, \tilde{M}_3, \tilde{M}_4, \tilde{M}_5, \tilde{M}_6, \tilde{M}_7, \tilde{M}_8$ and $\tilde{M}_{F2}$.

$$\begin{array}{ll} \tilde{M}_0 + M_{j0} + \tilde{M}_r = 0 & \\ \tilde{M}_0 = -\left(1 + k_1\eta_0\right)\tilde{M}_1 & \tilde{M}_8 + M_{j4} = 0 \\ \tilde{M}_0 \cdot k_1\eta_0 = -\left(1 + k_1\eta_0\right)\tilde{M}_2 & \tilde{M}_6 + \tilde{M}_{F2} + \tilde{M}_5 = 0 \\ \tilde{M}_3 + M_{j2} + \tilde{M}_2 = 0 & \tilde{M}_7 \cdot k_3\eta_0 = \tilde{M}_8 \\ \tilde{M}_3 = -\left(1 + k_2\eta_0\right)\tilde{M}_4 & \tilde{M}_6 \cdot k_3\eta_0 = -\left(1 + k_3\eta_0\right)\tilde{M}_8 \\ \tilde{M}_3 \cdot k_2\eta_0 = -\left(1 + k_2\eta_0\right)\tilde{M}_5 & \tilde{M}_a + M_{j1} + \tilde{M}_1 + \tilde{M}_4 + \tilde{M}_7 = 0 \end{array} \tag{4}$$

The load factors of the inertial power flows are being defined as constants and are shown in the relations:

$$\begin{array}{ll} M_{j0} = I_0 \cdot \dfrac{d\omega_r}{dt} & M_{j2} = I_2 \cdot \dfrac{d\omega_2}{dt} \\ M_{j1} = I_1 \cdot \dfrac{d\omega_a}{dt} & M_{j4} = I_4 \cdot \dfrac{d\omega_4}{dt} \end{array}, \tag{5}$$

where I_0 is the inertia moment of the planetary mechanism board k_1 and of the outcome arbor of the gearbox, I_1 - the inertia moment of the admission arbor and of the central wheels, I_2 - the inertia moment of the planetary mechanism board k_2, of the planetary mechanism crown k_1 and of the F1 brake discs and I_4- the inertia moment of the planetary mechanism crown k_3 and of the F3 brake discs.

So as to diminish the operation errors, the solutions for the equations system (6) are shown with the aid of the MathCad soft, and the following results are acquired:

$$\tilde{M}_r = \tilde{M}_a \cdot i_{cvII} \cdot \eta_{II} - i_{cvII} \cdot \eta_{II} \cdot \varepsilon_a \left[\frac{1}{i_{cv}^2 \cdot \eta_{II}} I_0 + I_1 + \frac{1}{(k_2 \cdot \eta_0 + 1)(k_2 + 1)} I_2 + \frac{1}{k_3^2 \cdot \eta_0} I_4 \right] \tag{6}$$

$$\tilde{M}_0 = -\left\{ \tilde{M}_a \cdot i_{cvII} \cdot \eta_{II} - i_{cvII} \cdot \eta_{II} \cdot \varepsilon_a \left[I_1 + \frac{1}{(k_2+1)(k_2 \cdot \eta_0 + 1)} I_2 + \frac{1}{k_3^2 \cdot \eta_0} I_4 \right] \right\}$$

$$\tilde{M}_1 = \frac{\tilde{M}_a \cdot i_{cvII} \cdot \eta_{II}}{(k_1 \cdot \eta_0 + 1)} - i_{cvII} \cdot \eta_{II} \cdot \varepsilon_a \left[\frac{1}{(k_1 \cdot \eta_0 + 1)} I_1 + \frac{1}{(k_2+1)(k_2 \cdot \eta_0 + 1)(k_1 \cdot \eta_0 + 1)} I_2 + \frac{1}{k_3^2 \cdot \eta_0 (k_1 \cdot \eta_0 + 1)} I_4 \right]$$

$$\tilde{M}_2 = \frac{\tilde{M}_a \cdot i_{cvII} \cdot \eta_{II} \cdot k_1 \cdot \eta_0}{(k_1 \cdot \eta_0 + 1)} - i_{cvII} \cdot \eta_{II} \cdot \varepsilon_a \left[\frac{k_1 \cdot \eta_0}{(k_1 \cdot \eta_0 + 1)} I_1 + \frac{k_1 \cdot \eta_0}{(k_2+1)(k_2 \cdot \eta_0 + 1)(k_1 \cdot \eta_0 + 1)} I_2 + \frac{k_1 \cdot \eta_0}{k_3^2 \cdot \eta_0 (k_1 \cdot \eta_0 + 1)} I_4 \right]$$

$$\tilde{M}_3 = -\left\{ \frac{\tilde{M}_a \cdot i_{cvII} \cdot \eta_{II} \cdot k_1 \cdot \eta_0}{(k_1 \cdot \eta_0 + 1)} - i_{cvII} \cdot \eta_{II} \cdot \varepsilon_a \left[\frac{k_1 \cdot \eta_0}{(k_1 \cdot \eta_0 + 1)} I_1 - \frac{1}{(k_2+1)(k_1 \cdot \eta_0 + 1)} I_2 + \frac{k_1 \cdot \eta_0}{k_3^2 \cdot \eta_0 (k_1 \cdot \eta_0 + 1)} I_4 \right] \right\}$$

$$\tilde{M}_4 = \frac{\tilde{M}_a \cdot i_{cvII} \cdot \eta_{II} \cdot k_1 \cdot \eta_0}{(k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} - i_{cvII} \cdot \eta_{II} \cdot \varepsilon_a \left[\frac{k_1 \cdot \eta_0}{(k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} I_1 - \frac{1}{(k_2+1)(k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} I_2 + \frac{k_1 \cdot \eta_0}{k_3^2 \cdot \eta_0 (k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} I_4 \right]$$

$$\tilde{M}_5 = \frac{\tilde{M}_a \cdot i_{cvII} \cdot \eta_{II} \cdot k_1 \cdot k_2 \cdot \eta_0^2}{(k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} - i_{cvII} \cdot \eta_{II} \cdot \varepsilon_a \left[\frac{k_1 \cdot k_2 \cdot \eta_0^2}{(k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} I_1 - \frac{k_2 \cdot \eta_0}{(k_2+1)(k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} I_2 + \frac{k_1 \cdot k_2 \cdot \eta_0^2}{k_3^2 \cdot \eta_0 (k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} I_4 \right]$$

$$\tilde{M}_{F2} = \frac{\tilde{M}_a \cdot i_{cvII} \cdot \eta_{II} \cdot k_1 \cdot k_2 \cdot \eta_0^2}{(k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} - i_{cvII} \cdot \eta_{II} \cdot \varepsilon_a \left[\frac{k_1 \cdot k_2 \cdot \eta_0^2}{(k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} I_1 - \frac{k_2 \cdot \eta_0}{(k_2+1)(k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} I_2 + \frac{(k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)(k_3 \cdot \eta_0 + 1) - k_1 \cdot k_2 \cdot k_3 \cdot \eta_0^3}{k_3^2 \cdot \eta_0 (k_1 \cdot \eta_0 + 1)(k_2 \cdot \eta_0 + 1)} I_4 \right]$$

$$\tilde{M}_6 = -\frac{(k_3 \cdot \eta_0 + 1)}{k_3^2 \cdot \eta_0} \varepsilon_a \cdot I_4 \quad \tilde{M}_7 = \frac{1}{k_3^2 \cdot \eta_0} \varepsilon_a \cdot I_4 \quad \tilde{M}_8 = \frac{1}{k_3} \varepsilon_a \cdot I_4$$

The analytical expressions of the power factors and the circulation path of the power flow will be determined according to the kinematic and load factors. By replacing the moments and the angular rates in the relation of defining the power [1], the following analytical expressions are acquired:

$$\tilde{P}_r = \tilde{P}_a \cdot \eta_{II} - \omega_a \cdot \varepsilon_a \cdot \eta_{II} \left[\frac{1}{i_{cv}^2 \cdot \eta_{II}} I_0 + I_1 + \frac{1}{(k_2 \cdot \eta_0 + 1)(k_2 + 1)} I_2 + \frac{1}{k_3^2 \cdot \eta_0} I_4 \right] \tag{7}$$

$$\tilde{P}_0 = -\left\{ \tilde{P}_a \cdot \eta_{II} - \omega_a \cdot \varepsilon_a \cdot \eta_{II} \left[I_1 + \frac{1}{(k_2+1)(k_2 \cdot \eta_0 + 1)} I_2 + \frac{1}{k_3^2 \cdot \eta_0} I_4 \right] \right\}$$

$$\tilde{P}_1=\frac{\tilde{P}_a\cdot i_{cvII}\cdot\eta_{II}}{\left(k_1\cdot\eta_0+1\right)}-i_{cvII}\cdot\eta_{II}\cdot\omega_a\cdot\varepsilon_a\left[\frac{1}{\left(k_1\cdot\eta_0+1\right)}I_1+\frac{1}{\left(k_2+1\right)\left(k_2\cdot\eta_0+1\right)\left(k_1\cdot\eta_0+1\right)}I_2+\frac{1}{k_3^2\cdot\eta_0\left(k_1\cdot\eta_0+1\right)}I_4\right]$$

$$\tilde{P}_2=\frac{\tilde{P}_a\cdot k_1\cdot\eta_0\cdot i_{cvII}\cdot\eta_{II}}{\left(k_2+1\right)\left(k_1\cdot\eta_0+1\right)}-i_{cvII}\cdot\eta_{II}\cdot\omega_a\cdot\varepsilon_a\left[\frac{k_1\cdot\eta_0}{\left(k_2+1\right)\left(k_1\cdot\eta_0+1\right)}I_1+\frac{k_1\cdot\eta_0}{\left(k_2+1\right)^2\left(k_2\cdot\eta_0+1\right)\left(k_1\cdot\eta_0+1\right)}I_2+\frac{k_1\cdot\eta_0}{\left(k_2+1\right)k_3^2\cdot\eta_0\left(k_1\cdot\eta_0+1\right)}I_4\right]$$

$$\tilde{P}_3=-\left\{\frac{\tilde{P}_a\cdot k_1\cdot\eta_0\cdot i_{cvII}\cdot\eta_{II}}{\left(k_2+1\right)\left(k_1\cdot\eta_0+1\right)}-i_{cvII}\cdot\eta_{II}\cdot\omega_a\cdot\varepsilon_a\left[\frac{k_1\cdot\eta_0}{\left(k_2+1\right)\left(k_1\cdot\eta_0+1\right)}I_1-\frac{1}{\left(k_2+1\right)^2\left(k_1\cdot\eta_0+1\right)}I_2+\frac{k_1\cdot\eta_0}{\left(k_2+1\right)k_3^2\cdot\eta_0\left(k_1\cdot\eta_0+1\right)}I_4\right]\right\}$$

$$\tilde{P}_4=\frac{\tilde{P}_a\cdot k_1\cdot\eta_0\cdot i_{cvII}\cdot\eta_{II}}{\left(k_1\cdot\eta_0+1\right)\left(k_2\cdot\eta_0+1\right)}-i_{cvII}\cdot\eta_{II}\cdot\omega_a\cdot\varepsilon_a\left[\frac{k_1\cdot\eta_0}{\left(k_1\cdot\eta_0+1\right)\left(k_2\cdot\eta_0+1\right)}I_1-\frac{1}{\left(k_2+1\right)\left(k_1\cdot\eta_0+1\right)\left(k_2\cdot\eta_0+1\right)}I_2+\frac{k_1\cdot\eta_0}{k_3^2\cdot\eta_0\left(k_1\cdot\eta_0+1\right)\left(k_2\cdot\eta_0+1\right)}I_4\right]$$

$$\tilde{P}_5=\tilde{P}_6=0\quad \tilde{P}_7=\frac{1}{k_3^2\cdot\eta_0}\varepsilon_a\cdot\omega_a\cdot I_4\quad \tilde{P}_8=-\frac{1}{k_3^2}\varepsilon_a\cdot\omega_a\cdot I_4$$

According to the acquired results from the operation algorithm, the circulation path of the power flow in the second level of the planetary gearbox is being represented in figure 5.

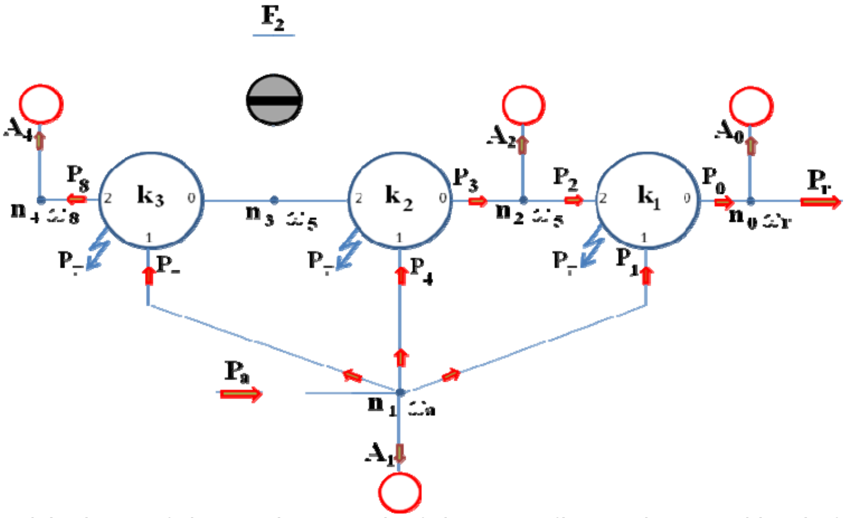

Figure 5 - The nodal scheme of the circulation path of the power flow in the second level of the planetary gearbox. The transitory movement system

In order to check the operation algorithm used for finding the power flow we have to apply the equability laws of the load factors belonging to the global nodes. Considering the load factors of the convergent and divergent flows in the nodes and replacing their analytical expressions, we can observe that the equability relations are checked by obtaining the zero value.

$$\begin{aligned}
&\tilde{M}_a+\tilde{M}_{1n}+\tilde{M}_{4n}+\tilde{M}_{7n}+M_{j1}\xrightarrow{replacement}0\\
&\tilde{M}_0+\tilde{M}_1+\tilde{M}_2\xrightarrow{ireplacement}0\\
&\tilde{M}_3+\tilde{M}_4+\tilde{M}_5\xrightarrow{replacement}0\\
&\tilde{M}_6+\tilde{M}_7+\tilde{M}_8\xrightarrow{replacement}0
\end{aligned}\tag{8}$$

The power factor from the outcome of the planetary gearbox can be determined also from the conversion of the planetary gearbox in the global node from figure 6.

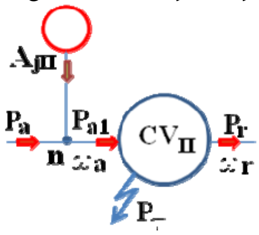

Figure 6 - The nodal scheme of the second floor of the planetary gearbox. The transitory movement system

The directional flow of the admission in the gearbox is convergent. The power flow of the outcome of the gearbox is determined from the calculation relation of the efficiency and from the equability of the load factors.

The analytical expression of the efficiency in the second level of the planetary gearbox is expressed by the structure [1]:

$$\eta_{II} = -\frac{\tilde{P}_r}{\tilde{P}_{a1}} \rightarrow \eta_{II} \cdot \tilde{P}_{a1} = -\tilde{P}_r \tag{9}$$

Through applying the conservation of energy law on the branched node and considering the sign and the expression of the power factors, the relation of defining the efficiency becomes:

$$\eta_{II} \cdot \left(\tilde{P}_a - A_{jII}\right) = -\tilde{P}_r \rightarrow \eta_{II} \cdot \omega_a \cdot \left(\tilde{M}_a - I_{jII} \cdot \varepsilon_a\right) = -\tilde{M}_r \cdot \omega_r \tag{10}$$

By dividing the relation of defining the efficiency with the angular rate of the outcome arbor of the gearbox - „ω_r”, it will result the analytical expression of the torsion moment sent to the admission element of the addition planetary mechanism:

$$\tilde{M}_r = \tilde{M}_a \cdot \eta_{II} \cdot i_{cvII} - I_{jII} \cdot \varepsilon_a \cdot \eta_{II} \cdot i_{cvII} \tag{11}$$

Therefore, by applying the two operation methods of the torsion moment of distribution which loads the outcome arbor of the planetary gearbox, we have the following analytical expressions:

$$\begin{aligned} \tilde{M}_r &= \tilde{M}_a \cdot i_{cvII} \cdot \eta_{II} - i_{cvII} \cdot \eta_{II} \cdot \varepsilon_a \left[\frac{1}{i_{cv}^2 \cdot \eta_{II}} I_0 + I_1 + \frac{1}{\left(k_2 \cdot \eta_0 + 1\right)\left(k_2 + 1\right)} I_2 + \frac{1}{k_3^2 \cdot \eta_0} I_4\right] \\ \tilde{M}_r &= \tilde{M}_a \cdot \eta_{II} \cdot i_{cvII} - I_j \cdot \varepsilon_a \cdot \eta_{II} \cdot i_{cvII} \end{aligned} \tag{12}$$

We get the expression of the equivalent inertia moment marked by I_{jII} if we compare the two operation relations:

$$I_{jII} = \frac{1}{i_{cv}^2 \cdot \eta_{II}} I_0 + I_1 + \frac{1}{\left(k_2 \cdot \eta_0 + 1\right)\left(k_2 + 1\right)} I_2 + \frac{1}{k_3^2 \cdot \eta_0} I_4 \tag{13}$$

CONCLUSIONS

The nodal scheme shown in figure 4 presents the external elements of the second level of the planetary gearbox which take part in the transmission and the conversion of the power flow. These are characterized both by the kinematic factors, and the load factors. By comparison [1], we observe that the nodal schemes that characterize the dynamic system and the stationary system are different. In the dynamic system both the planetary mechanisms k_1 and k_2 and the planetary mechanism k_3 take part in the transmission of the power flow. The presence on the nodal scheme of the planetary mechanism k_3 is due to the inertial power flow which affects the crown of the planetary mechanism that has the load factor M_{j4}.

We can observe in the nodal scheme shown in figure 5 that the power flow which is sent by the planetary gearbox is affected by the inertial power flows and by the dissipative power flows. The internal losses of power are active both in the stationary and in the transitory systems.

If we analyze the circulation path of the power flow, we can observe that part of the power factor present at the admission of the planetary gearbox is distributed towards the inertial accumulating power flow A_1 and towards the central wheels of the planetary mechanisms. The presence of the power flow is due to the fact that part of the energy is consumed in accelerating the admission arbor, the central wheels of the planetary mechanisms and the conducting elements of the A clutch.

The power flow received by the global node k_3 is totally used for accelerating the crown of the planetary mechanism k_3 and the F_3 brake discs. The power lost by the energetic network is converted into the accumulating inertial flow A_4 and into the dissipative flow P_τ.

The power factor of the convergent flow P_4 is sent through the planetary mechanism k_2 towards the crown of the planetary mechanism k_1. We can see in the nodal scheme that the power factor of the divergent directional flow – P_3 is affected by the internal losses of power which appear in the planetary mechanism k_2 because of the frictions from the engagements, the bearings and the frictions between the gear wheels and the grease oil.

The directional flow is sent towards the crown of the planetary mechanism k_1 with a diminished power factor because of the consumption of energy necessary for accelerating the crown of the planetary mechanism k_1, the F_1 brake discs and the planetary mechanism board k_2. The energy losses are marked on the nodal scheme with A_2.

The global node k_1 acts like a accumulator because of the circulating path of the power flow and it has the convergent power on two sides and the divergent power on the third side. The power losses from the planetary mechanism k_1 are represented through the dissipative flow P_τ and through a part of the accumulating inertial flows A_1, A_2 and A_0. Moreover, the inertial flow A_0 accumulates the adequate energy for accelerating the outcome arbor of the planetary gearbox.

The power flow from the outcome of the planetary gearbox is divergent and is directed towards the admission element of the addition planetary mechanism. We can observe that the analytical expressions of the power factors are rational functions which are dependent of the partial efficiencies, of the planetary mechanisms constants, of the dynamic rapport of transmission, of the *inertia moment that characterize the masses found in the rotary movement, and of the angular rate and acceleration of the admission arbor belonging to the planetary gearbox.*

The presence in the operation expression of the angular rates and acceleration show the fact that the power factor vary according to these. In comparison with the stationary system [1] where the load factor was established only according to the constants and to the partial efficiencies of the planetary mechanisms, in the dynamic system shown in the operation relations we can see that these vary according to the angular acceleration of the admission element. That is due to the presence of the inertial power flows in the energetic network.

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