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ON THE DIFFUSIVE WAVES IN HEAT CONDUCTING SOLIDS

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ABSTRACT: The objective of the present paper is study the generalized thermoelastic diffusive waves in heat conducting solids. The governing equations for heat conducting generalized thermodiffusion materials are solved symbolically. It is shown that the characteristic equation, three waves namely, elasto-diffusive (ED), mass-diffusion (MD-mode) and thermo-diffusive (TD-mode), can propagate in such solids in addition to transverse waves. The transverse waves remain unaffected due to temperature change and mass diffusion effects and get decoupled from rest of the fields and travel without attenuation and dispersion. It is also shown that how easily one can study that the generalized thermoelastic diffusive waves are significantly affected by the interacting parameters using this computational technique. Finally, the results are discussed graphically.
KEYWORDS: Thermodifusion, thermoelastic, elasto-diffusive, mass-diffusion, thermo-diffusive

INTRODUCTION

Thermodiffusion in elastic solids is due to coupling of the temperature, mass diffusion and of strain and heat and mass exchange with the environment during this process. These days, oil companies are interested in the process of thermodiffusion for more efficient extraction of oil from oil deposits. Diffusion can be defined as the random walk of an ensemble of particles from regions of high concentration to regions of lower concentration. In most of the applications, the concentration is calculated using what is known as Fick's law. This is a simple law that does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of the temperature on this interaction.

In the recent years increasing attention is directed towards the generalized theory of thermoelasticity, which was found to give more realistic results than the coupled or uncoupled theories of thermoelasticity, especially when short time effects or step temperature gradients are considered. The theory of generalized thermoelasticity with one relaxation time was first introduced by Lord and Shulman [5], who obtained a wave-type heat equation by postulating a new law of heat conduction instead of the classical Fourier's law. A review of various representative theories in the range of generalized thermoelasticity has been presented Chanderashekhariah [3]. Nowacki [8] developed the theory of thermoelastic diffusion by using a coupled thermoelasticity. Dudziak and Kowalski [4] and Olesiak and Yryev [7], respectively, discussed the theory of thermodiffusion and coupled quasi-stationary problems of thermal diffusion in an elastic layer. Verma [13-15] and Verma Hasbe [16-17] studied problems in generalized thermoelasticity with thermal relaxation. Many authors [1,2, 6, 7, 9-12] considered problems in the theory of generalized thermoelastic diffusion. In this article the generalized thermoelastic diffusive waves in heat conducting solids is study using the symbolic processing. The governing equations for heat conducting generalized thermodiffusion materials are solved symbolically. It is shown that the characteristic equation, three waves namely, elasto-diffusive (ED), mass-diffusion (MD-mode) and thermo-diffusive (TD-mode), can propagate in such solids in addition to transverse waves. It is shown that transverse waves get decoupled from rest of the fields and travel without attenuation and dispersion stay unaffected with temperature change and mass diffusion. It is also exhibited that how easily one can study that the generalized thermoelastic diffusive waves are significantly affected by the interacting parameters using this computational technique. Finally, the results are discussed graphically.

GOVERNING EQUATIONS AND SOLUTION

The governing equations for an isotropic, homogeneous elastic solid with generalized thermoelastic diffusion at constant temperature T0 in the absence of body forces given by Sherief et al. [6] are:

The equation of motion:

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \beta_1 T_{,i} - \beta_2 C_{,i} = \rho \ddot{u}_i \tag{1}$$

The equation of heat conduction:

$$\rho c_E(\dot{T}+\tau_0\ddot{T})+\beta_1 T_0(\dot{e}+\tau_0\ddot{e})+aT_0(\dot{C}+\tau_0\ddot{C})=K\ddot{T}_{,ii} \tag{2}$$

The equation of mass diffusion:

$$D\beta_2 e_{,ii}+DaT_{,ii}+\dot{C}+\tau_0\ddot{C}-DbC_{,ii}=o \tag{3}$$

The constitutive equations:

$$\begin{aligned}\sigma_{ij}&=2\mu e_{ij}+\delta_{ij}(\lambda e_{kk}-\beta_1 T-\beta_2 C),\\ \rho T_0 S&=\rho c_E T+\beta_1 T_0 e_{kk}+aT_0 C,\\ P&=-\beta_2 e_{kk}+bC-aT,\end{aligned} \tag{4}$$

where material constants and are given by

$$\beta_1=(3\lambda+2\mu)\alpha_t \text{ and } \beta_2=(3\lambda+2\mu)\alpha_c \tag{5}$$

α_t coefficient of linear thermal expansion, α_c coefficient of linear diffusion expansion, λ,μ are Lame's constants, T0 is the temperature of the medium in its natural state assumed to be such that $T-T_0$, T is the absolute temperature, σ_{ij} are the components of the stress tensor, u_i are the components of the displacement vector, ρ is the density assumed independent of time, e_{ij} are the components of the strain tensor, S is the entropy per unit mass, P is the chemical potential per unit mass, C is the mass concentration, c_E is the specific heat at constant strain, K is the coefficient of thermal conductivity, D is thermodiffusion constant. τ_0 is the thermal relaxation which will ensure that the heat conduction equation, satisfied by the temperature T will predict infinite speeds of heat propagation. τ is the diffusion relaxation time, which will ensure that the equation, satisfied by the concentration C will also predict finite speeds of propagation of matter from one medium to the other. The constants 'a' and 'b' are the measures of thermodiffusion effects and diffusive effects, respectively. The superposed dots (.) denotes derivative with respect to time and comma (,) denotes the spatial derivative.

For two-dimensional motion in x-z plane, the eq. (1)-(3) is written as

$$(\lambda+2\mu)u_{1,11}+(\lambda+\mu)u_{3,13}+\mu u_{1,33}-\beta_1 T_{,1}-\beta_2 \mathrm{C}_{,1}=\rho\ddot{u}_1 \tag{6}$$

$$\mu u_{3,11}+(\lambda+2\mu)u_{3,33}+(\lambda+\mu)u_{1,13}-\beta_1 T_{,3}-\beta_2 \mathrm{C}_{,3}=\rho\ddot{u}_3 \tag{7}$$

$$K\left(\Theta_{,11}+\Theta_{,33}\right)=\rho c_E\dot{\Theta}+\beta_1 T_0\tau_m\dot{e}+aT_0\tau_m\dot{C} \tag{8}$$

$$D\beta_2\left(e_{,11}+e_{,33}\right)+Da\left(\Theta_{,11}+\Theta_{,33}\right)-Db\left(C_{,11}+C_{,33}\right)+\tau_n\dot{C}=0 \tag{9}$$

where $\tau_m=\tau_0+i\dfrac{\partial}{\partial t}$, $\tau_n=\tau+i\dfrac{\partial}{\partial t}u_3$.

The displacement components u_1 and u_3 may be written in terms of potential functions ϕ and ψ as

$$u_1=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z}, \qquad u_3=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x} \tag{10}$$

Using (10) into equations (6)-(9), we obtain

$$\mu\nabla^2\psi=\rho\ddot{\psi} \tag{11}$$

$$(\lambda+2\mu)\nabla^2\phi-\beta_1 T-\beta_2\mathrm{C}=\rho\ddot{\phi} \tag{12}$$

$$K\left(\Theta_{,11}+\Theta_{,33}\right)=\rho c_E\dot{\Theta}+\beta_1 T_0\tau_m\frac{\partial}{\partial t}\nabla^2\phi+aT_0\tau_m\dot{C} \tag{13}$$

$$K\left(\Theta_{,11}+\Theta_{,33}\right)=\rho c_E\dot{\Theta}+\beta_1 T_0\tau_m\frac{\partial}{\partial t}\nabla^2\phi+aT_0\tau_m\dot{C} \qquad K\nabla^2\Theta=\rho c_E\dot{\Theta}+\beta_1 T_0\tau_m\frac{\partial}{\partial t}\nabla^2\phi+aT_0\tau_m\dot{C}$$

$$D\beta_2\nabla^2\phi+Da\left(\Theta_{,11}+\Theta_{,33}\right)-Db\left(C_{,11}+C_{,33}\right)+\tau_n\dot{C}=0 \tag{14}$$

$$D\beta_2\nabla^2\phi+Da\nabla^2\Theta-Db\nabla^2 C+\tau_n\dot{C}=0$$

Equation (11) is decoupled, whereas the equations (12)-(14) are coupled in ϕ, Θ and C. From equations (12)-(14), we see that three longitudinal waves namely, elastodiffusive (ED), mass diffusion (MD-mode) and thermodiffusive (TD-mode), can propagate in such solids in addition to transverse waves and are affected due to the presence of thermal and mass diffusion waves, while the transverse waves unaffected

And get decoupled from rest of the fields and hence remain unaffected due to temperature change and mass diffusion effects. The solution of equation (11) corresponds to the propagation of transverse wave with velocity $v_4 = \sqrt{\mu/\rho}$.

ANALYSIS

Solutions of the equations (12)-(14) are now sought in the form of the harmonic traveling wave

$$(\phi, \Theta, C) = (P, Q, R)e^{ik(xd_1 + zd_2 - vt)} \tag{15}$$

in which v is the phase speed, k is the wave number, (d_1, d_2) denotes the projection of the wave normal onto the $x - z$ plane. The homogeneous system of equations in P, Q and R, obtained by inserting (15) using symbolically explorer version into equations (12)-(14),

$$\begin{aligned} & k^2(\lambda + 2\mu - \rho v^2)P + \beta_1 Q + \beta_2 R = 0 \\ & -(\tau_0 + i\omega^{-1})v^2\beta_1 T_0 k^3 P + \left[(i + vk\tau_0)\rho v C_e - kK\right]Q + (i + vk\tau_0)avT_0 R = 0 \\ & -\beta_1 Dk^3 P + kDaQ + \left[(\tau_0 + i\omega^{-1})kv^2 - kDb\right]R = 0 \end{aligned} \tag{16}$$

The system of equations (16) has a non-trivial solution if the determinant of the coefficients vanishes. This leads to the cubic equation in v^2.

$$A_0 v^6 + A_1 v^4 + A_2 v^2 + A_3 = 0 \tag{17}$$

where

$$\begin{aligned} A_0 &= (\tau_0 + i\omega^{-1})(i + \tau_1\omega)C_e\rho^2 \\ A_1 &= C_e(i + \tau_1\omega)(\lambda + 2\mu)\rho(\tau_0 + i\omega^{-1}) + (\tau_0 + i\omega^{-1})(i + \tau_1\omega)T_0\beta_1^2 \\ &\quad + \rho\omega(\tau_0 + i\omega^{-1})C_e Db\rho + T_0 a^2(\tau_0 + i\omega^{-1})D + \tau_1 K \\ A_2 &= \left[-D\omega(T_0 a^2 + C_e b\rho)(\tau_0 + i\omega^{-1}) - K\tau_1\omega - iK\right](\lambda + 2\mu) \\ &\quad + D\omega(\tau_0 + i\omega^{-1})(\beta_1 b + 2a\beta_2)\beta_1 T_0 - D\omega\left[\beta_2^2\rho(\tau_0 + i\omega^{-1})C_e + bK\rho\right] \\ A_3 &= D\omega K\left[b(\lambda + 2\mu) - \beta_2^2\right] \end{aligned} \tag{18}$$

Particular Cases

In the absence of mass concentration $a = \beta_2 = 0$, and thermo-mechanical coupling $\beta_1 = 0$, the matrix (17) reduces to (19), corresponding to three elastic waves propagating in any fixed direction.

$$\left[(\lambda + 2\mu) - \rho v^2\right]\left(-(i + \tau_1\omega)v^2 + \omega Db\right)\left(v^2 C_e \tau_3 \rho - K\right) = 0 \tag{19}$$

where $\tau 3 = \tau 0 + i\omega^{-1}$.

In the absence of mass concentration $a = \beta_2 = 0$, and thermo-mechanical coupling $\beta_1 \neq 0$,

$$-\tau_3 C_e \rho^2 v^4 + \left[\tau_3 C_e \rho(\lambda + 2\mu) + K\rho + \tau_3 T_0 \beta_1^2\right]v^2 - (\lambda + 2\mu)K = 0 \tag{20a}$$

$$v^2 = \omega Db(i + \tau_1\omega)^{-1} \tag{20b}$$

The secular equation (17) decoupled, (20a) in this case reduce to quadratic in v^2 having in general, complex roots. Equation (20b) corresponds to the mass diffusion wave.

NUMERICAL RESULTS

For computational work, the following material constants at T0 = 27±C are considered for an elastic solid with generalized thermodiffusion

$\lambda = 5.775 \times 10^{11} dyne/cm^2$,

$\mu = 2.646 \times 10^{11} dyne/cm^2$, $\quad K = 0.492 cal/cms^0C$,

$\rho = 2.7 gm/cm^3$, $\quad \tau_0 = 0.05s, \tau_1 = 0.04s, \alpha_t = 0.05, \alpha_c = 0.06$

$C_e = 2.361 cal/gm^0C$, $\quad \omega = 2s^{-1}, a = 0.005, b = 0.05, D = 0.5$

The cubic equation (17) is solved by the computer program of Cardan method to obtain the numerical values of velocities of three dilatational waves viz.,P, MD and T waves, the variations of velocities.

CONCLUSIONS

Generalized thermoelastic diffusive waves in heat conducting solids are investigated using computational techniques and symbolically. It is exhibited that how easily one can study that the generalized thermoelastic diffusive waves are significantly affected by the interacting parameters using this computational technique. Finally, the results are discussed graphically.

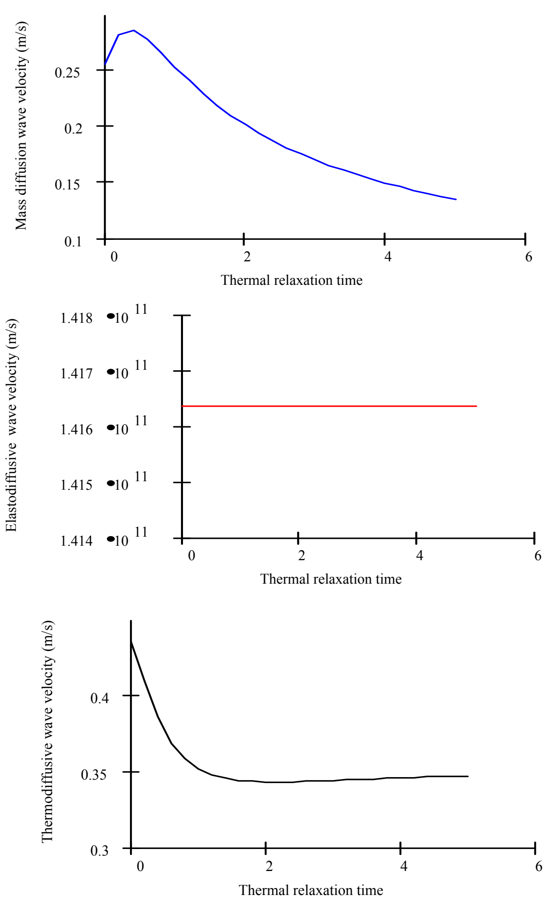

Figure 1. Variations of velocities of Massdiffusive, Elastodiffusive, thermodiffusive and waves with thermal relaxation times

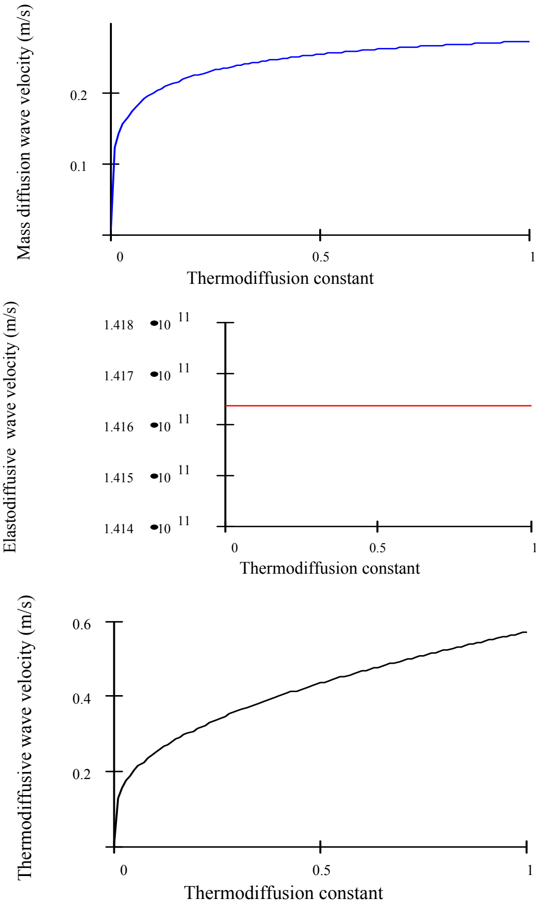

Figure 2. Variations of velocities of Massdiffusive, Elastodiffusive, thermodiffusive and waves with thermodiffusion constant D

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