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THE USE OF DIGITAL HYPERBOLIC FILTER AS TOOL TO DENOISING RESISTIVITY DATA MAP OF MOROCCAN PHOSPHATE ANOMALOUS "DISTURBANCES"

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ABSTRACT: Low- and band-pass filters can be designed by a combination of hyperbolic tangent functions in the frequency domain using the scaling and shifting theorems of the Fourier Transforms. The corresponding filter function in the time domain can be derived analytically from the frequency domain expression. The smoothness parameters control the slopes at the cut off regions and permit the construction of a relatively short filter while reducing the oscillations of the filter response in the time domain. Different smoothness parameters can be chosen for the low and high cut off frequencies in the band-pass filter design. Following the proposed scheme can easily derive the other type of the filters.
KEYWORDS: filter, hyperbolic, resistiviy, geoelectric, Moroccan, phosphate

INTRODUCTION

Several methods are currently used to optimize edges and contours of geophysical data maps. A resistivity map was expected to allow the electrical resistivity signal to be imaged in 2D in Moroccan resistivity survey in the mining domain. Anomalous zones of phosphate deposit "disturbances" correspond to resistivity anomalies. We propose a new method for white noise reduction of Moroccan phosphate disturbances map of resistivity data based on the use of digital hyperbolic tangent functions filter approach. The effectiveness of our approach for successfully reducing noise and smoothing the resistivity data has been used much success in the analysis of stationary geophysical data. The resistivity data base used was a compilation of 51 traverses at a spacing of 20 m. There were 101 stations at 5 m distance for every traverse, which makes 5151 stations all together in the resistivity survey which represented by map of the phosphate anomalous zones or phosphate "disturbance. The application deals with analyzing resistivity data map using smoothing approach using digital hyperbolic tangent functions filter to denoise anomalous zones map of phosphate deposit disturbances resistivity signal. The digital hyperbolic tangent functions filter approach method is an efficient tool in the interpretation of geophysical potential field data particulary suitable in denoising, filtering and analyzing resistivity data singularities. The digital hyperbolic tangent functions filter smoothing approach applied to Moroccan resistivity data of phosphate "disturbances" was found to be consistently useful.

THE MATHEMATICAL CONTEXT

We combined two hyperbolic tangent functions in the frequency domain to construct an analytical expression, which gives the filter coefficients for the numerical evaluation of the Hankel type integrals as follows (Johansen et al., 1979)):

$$P(\nu)=\frac{1}{2}\tanh\left[\frac{\pi}{a}\left(\nu+\frac{1}{2}\right)\right]-\frac{1}{2}\tanh\left[\frac{\pi}{a}\left(\nu-\frac{1}{2}\right)\right] \quad (1)$$

where ν and a denote the frequency and the 'smoothness' parameter, respectively. The smoothness parameter a is a small constant, which takes values less than unity. $P(\nu)$ is referred to as "$P-function$" and it is analytical in whole space from $-\infty$ to $+\infty$ and has no discontinuity. The shape of the "$P-function$" resembles a box-car function and consequently the corresponding time domain function resembles a "$\sin c-function$". The inverse Fourier transform of Equation 1 yields (Johansen et al., 1979)

$$P(t)=\frac{a.\sin(\pi t)}{\sinh(\pi a t)} \quad (2)$$

$P(t)$ is referred as "$\sin sh$" function. The above Fourier transform pair is usually also used for the same purposes, namely as a truncation function in the frequency domain and as a interpolating function in the time domain (Christensen, 1990; Sorensen et al., 1994)). This paper presents a potential use of the hyperbolic tangent functions low and band-pass filters applied to denoise and smooth resistivity data responses of Moroccan phosphate "perturbations" (Bakkali, 2005, 2006; Bakkali et al., 2006, 2007).

The low-pass filter

An expression for a low-pass filter can be developed by making use of the following Fourier transform pair given (Bracewell, 1965).

$$\frac{i}{\sinh(\pi t)} \leftrightarrow \tanh(\pi \nu) \tag{3}$$

where $i^2 = -1$. By applying the scaling property of the Fourier transform one finds

$$h(t) = \frac{i a \nu_L}{\sinh(2\pi \nu_L t)} \leftrightarrow H(\nu) = \frac{1}{2}\tanh\left[\frac{\pi \nu}{2 a \nu_L}\right] \tag{4}$$

where ν_L is a constant and it will be used as the cutoff frequency of a low-pass filter in the subsequent development. Figure 1 shows a combination of two hyperbolic tangent functions giving the desired low pass filter expression

$$H_L(\nu) = A\left[H(\nu + \nu_L) - H(\nu - \nu_L)\right] \tag{5}$$

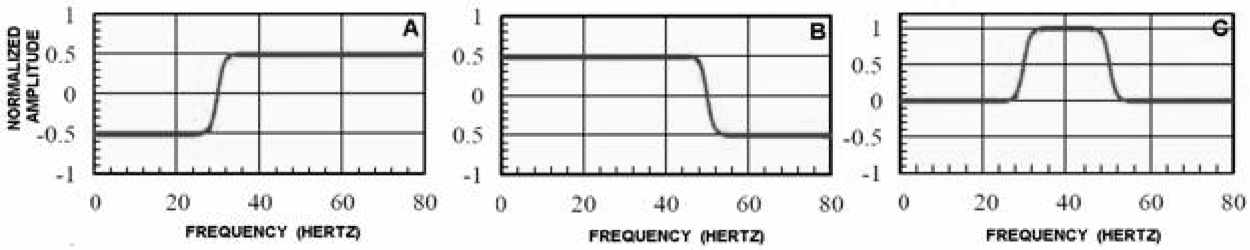

Figure 1: A low-pass hyperbolic tangent filter in the frequency domain.

The combination of the functions $H(\nu + \nu_L)$[A] and $-H(\nu - \nu_L)$ [B] derived from the Equation 4 gives the desired low-pass filter $H(\nu)$[C], where A is the gain of the filter. The amplitudes of the input are not modified if A is chosen being equal to the sampling time. Substituting Equation 4 into Equation 5 yields the final expression

$$H_L(\nu) = \frac{A}{2}\left[\tanh\left(\frac{\pi(\nu + \nu_L)}{2 a \nu_L}\right) - \tanh\left(\frac{\pi(\nu - \nu_L)}{2 a \nu_L}\right)\right] \tag{6}$$

The filter coefficients can be computed from the time domain equivalence of the equation (6). An expression for this purpose can be easily derived from Equation 4 and Equation 5 by using the shift theorem for Fourier transform (Bracewell, 1965)

$$h_L(t) = h(t)\left(e^{-2\pi i \nu_L t} - e^{2\pi i \nu_L t}\right) \tag{7}$$

which becomes

$$h_L(t) = 2 A a \nu_L \frac{\sin(2\pi \nu_L t)}{\sinh(2\pi a \nu_L t)} \tag{8}$$

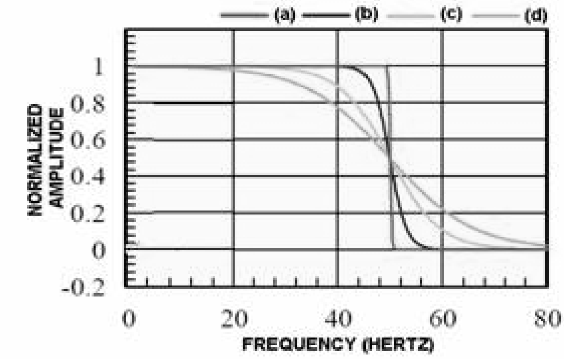

Figure 2: Low-pass hyperbolic tangent filters (smoothness parameters (a) = 0.01, (c) = 0.05, (c) = 0.3, (d) = 0.5.).

Equations (6) and (8) serve the calculation of the filter transfer function in the frequency domain and the filter response in the time domain, respectively. The smoothness parameter controls the slope of the filter transfer function around the cut off frequency. If this parameter takes very small values, then $H_L(\nu)$ resembles an ideal filter represented by a box-car function (Figure 2).

Consequently, time-domain response of the filter approaches to a "$\sin c - function$" since $\sinh(x) \cong x$ for small arguments. If the smoothness parameter takes relatively high values, then the slope of the filter function in the frequency domain decreases.

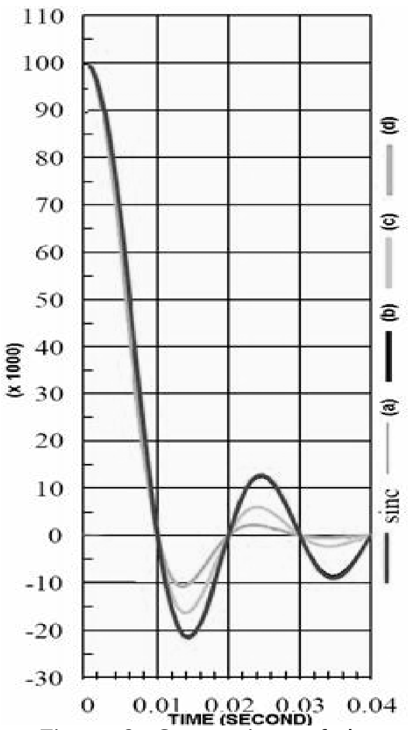

Figure 3: Comparison of the $\sin c$ function and time domain responses of low-pass hyperbolic tangent filters given in Figure 2.

This will significantly reduce the oscillations of the filter response function in the time-domain and permits the construction of a relatively short filter in length (Figure 3). A special care is needed in the selection of the smoothness parameter because a high value may lead unsuccessful extraction of unwanted frequency components from the input signal. However, the easy controlled degree of attenuation in the transition band is always possible.

The smoothness parameters control the oscillations of the $\sinh$ responses.

The band-pass filter

A band-pass filter can be obtained by subtracting one low-pass filter from another each having different cut off frequencies. Thus, the transfer function of a band-pass filter can be obtained by combining four tangent hyperbolic functions as done by the following Equation 9:

$$H_B(\nu)=\frac{A}{2}\left[\tanh\left(\frac{\pi(\nu+\nu_H)}{2a_2(\nu_H-\nu_L)}\right)-\tanh\left(\frac{\pi(\nu-\nu_H)}{2a_2(\nu_H-\nu_L)}\right)\right]$$
$$-\frac{A}{2}\left[\tanh\left(\frac{\pi(\nu+\nu_L)}{2a_1(\nu_H-\nu_L)}\right)+\tanh\left(\frac{\pi(\nu-\nu_L)}{2a_1(\nu_H-\nu_L)}\right)\right]$$

where ν_L and ν_H are the low and high cut off frequencies and a_1 and a_2 are smoothness parameters. The different numerical values of the smoothness parameters (a) and (b) permit the independent adjustment of the slopes in the transition bands (Figure 4).

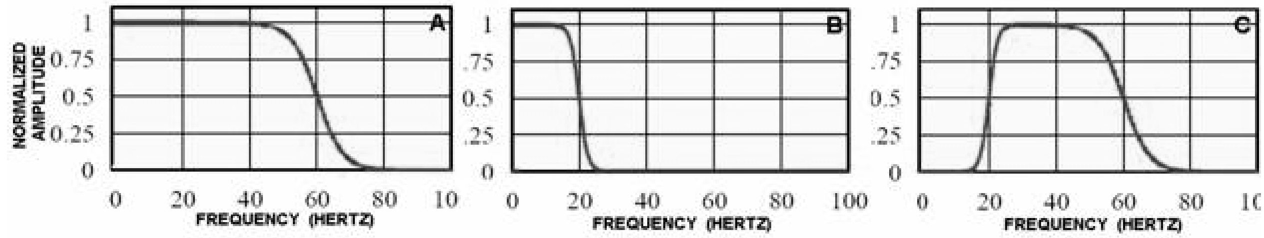

Figure 4: A band-pass hyperbolic tangent filter

A band-pass hyperbolic tangent filter in the frequency domain by the summation of the low-pass filters shown in (A) and (B) having the cut off frequency 60 and 20 Hz and the smoothness parameters 0.3 and 0.01, respectively. The resulted band-pass filter has different slopes at the low and high cut off regions.

The inverse Fourier Transform of Equation 9 yields the response function in the time domain

$$h_B(t)=2A(\nu_H-\nu_L)\left[a_2\frac{\sin(2\pi\nu_H t)}{\sinh(2\pi a_2 t(\nu_H-\nu_L))}-a_1\frac{\sin(2\pi\nu_L t)}{\sinh(2\pi a_1 t(\nu_H-\nu_L))}\right] \quad (10)$$

The sample values of Equation 10 gives the desired filter coefficients (Figure 5). Other properties of the band pass filter are the same as that of the low-pass filter. The main properties of the suggested filter can be summarised as follows:

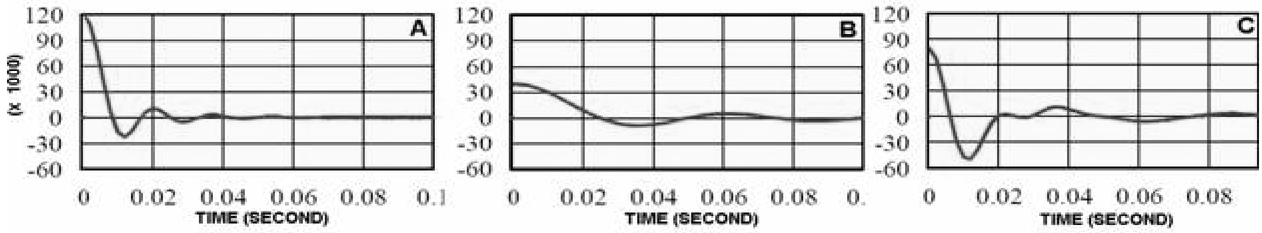

Figure 5: Time domain response of a band-pass filter.

(A), (B) and (C) correspond the time domain representations of the filter transfer functions give in Figure 4 [(A), (B), (C)].

The transfer function of the hyperbolic tangent filters is analytical in whole space from - □□to □□ and has no discontinuity. The smoothness parameters control the slopes at the cut off regions and *different smoothness parameters can be chosen for the low and high cut-off frequencies in the band-pass filter design. The corresponding filter function in the time domain can be derived analytically from the frequency domain expression and permit the construction of a relatively short filter while reducing the oscillations of the filter response in the time domain. The easy control of the slope of the transfer function and the suppression of the ripples of the response function are the main advantages of the suggested filters. The decision about the value of the smoothness parameter can be made interactively depending on the given problem.*

APPLICATIONS

Resistivity data collected in the survey are often contaminated with noise and artifacts coming from various sources. The presence of noise in data resistivity distorts the characteristics of the geophysical signal, resulting in poor quality of any subsequent processing. Consequently, the first step in any processing of such geophysical data is the "cleaning up" of the noise in a way that preserves the signal sharp variations. The smoothing approach using digital hyperbolic tangent functions filter has become a powerful signal and image processing tool which has found applications in many scientific areas. This method is a widely used technique that is applicable to smoothing geophysical data more effective than one of the last methods used recently (Bakkali, 2007).

The present application deals with analyzing resistivity data map using smoothing approach using digital hyperbolic tangent functions filter to denoise anomalous zones map of phosphate deposit disturbances resistivity signal (figure 6).

The resistivity data base is a compilation of 51 traverses (figure 7) at a spacing of 20 m. There were 101 stations at 5 m distance for every traverse, which makes 5151 stations all together in the resistivity survey which represented by map of the phosphate anomalous zones or phosphate "disturbance" map (Bakkali, 2005, 2006, 2007) (figure 8). We calculated the output digital hyperbolic tangent functions filter for each resistivity traverse (Figure 9).

Figure 6: Example of "disturbance" affecting the Moroccan phosphate strates

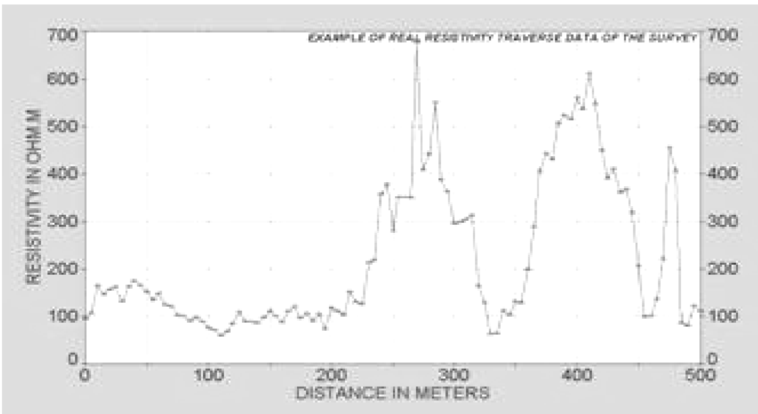

Figure 7: Example of real resistivity traverse data of the survey

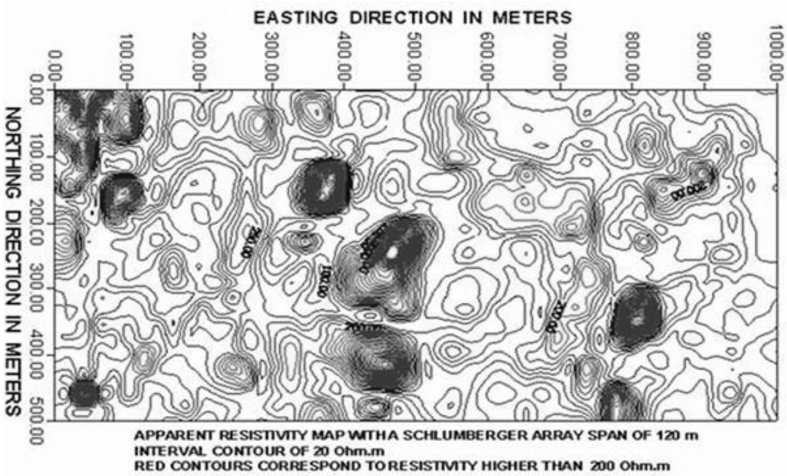

Figure 8: A map of the Moroccan disturbed phosphate zones

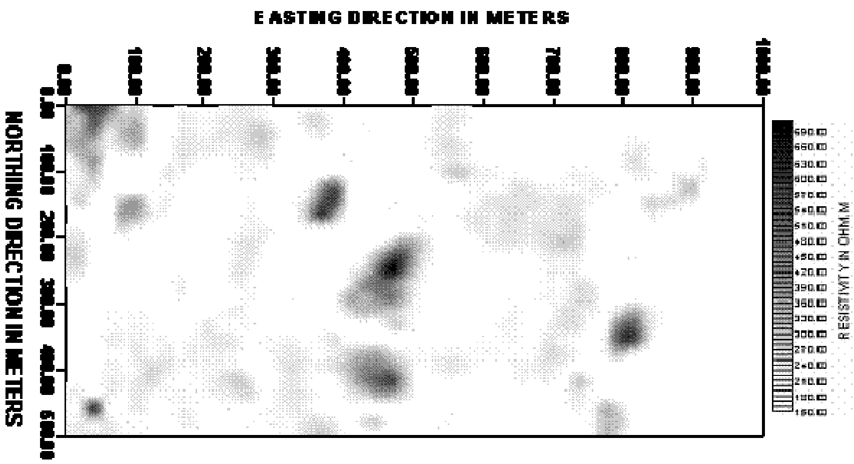

Figure 9: The corresponding output digital hyperbolic tangent functions filter of the resistivity data

Then we deferred all the results to build a regular map which represent in fact digital hyperbolic tangent functions filter map of the phosphate deposit "disturbances" (Figure 10). The advantage of using this filter is the ability to preserve higher moments in the resistivity data and thus reduce smoothing on peak heights. It is a powerful tool particulary suitable in denoising and smoothing resistivity data. Moreover this property is crucial for performing the reconstruction of the filtered geophysical signal corresponding to resitivity data anomaly map of the Moroccan phosphate deposit "disturbances".

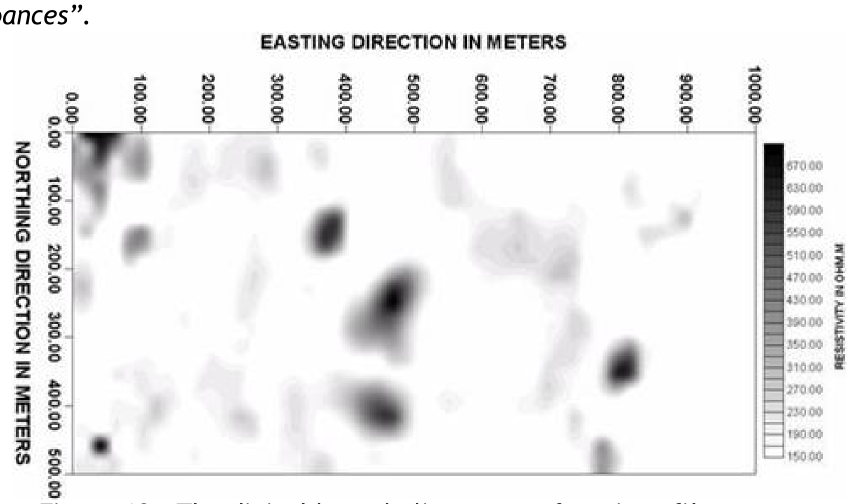

Figure 10: The digital hyperbolic tangent functions filter output of the phosphate deposit "disturbances" map given in Figure 6.

CONCLUSIONS

Figure 10 represents an indicator of the "smoothed" level of variation of the contrast of density between the disturbances and the normal phosphate-bearing rock. The digital hyperbolic tangent functions filter output map corresponding to surface modeling of resistivity anomalies were obtained by AutoSignal routine. The use of digital hyperbolic tangent functions filtering method represents an effective filtering way which makes it possible to attenuate considerably the noise represented by the minor dispersed and random "disturbances". Comparatively to classical approaches used in filtering and denoising geophysical data maps, the advantage of the digital hyperbolic tangent functions filtering method is doesn't introduce significant distorsion to the shape of the original resistivity signal. The proposed optimal smoothing method tends to give a real estimation of the surface of the phosphate deposit "disturbances" zones with a significant suppression of the noise. We have presented an automatic method for smoothing. A particular feature of digital hyperbolic tangent functions filtering method is its ability to adjust to abrupt discontinuities on the data. The results show a significant suppression of the noise and a very good recovery of the resistivity anomalies signal. This technique of smoothing resistivity data map was found to be consistently useful and the corresponding map may be used as auxiliary tools for decision making under field conditions.

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