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1. Juraj KAPUSTA, 2. Juraj CAMBER, 3. Gabriel HULKÓ

APPLICATION OF DISTRIBUTED PARAMETER SYSTEM CONTROL FOR STEEL BILLET INDUCTION HEATING

1-3. INSTITUTE OF AUTOMATION, MEASUREMENT AND APPLIED INFORMATICS, FACULTY OF MECHANICAL ENGINEERING, SLOVAK UNIVERSITY OF TECHNOLOGY, NÁM. SLOBODY 17, 812 31 BRATISLAVA, SLOVAKIA

ABSTRACT: Nowadays, the achievement of proper steel billets temperature profile is not the only design priority of induction heaters for hot forming applications. Due to its high operating costs, its design is constantly improving in terms of electrical and thermal efficiency. Therefore the more efficient multi-coil design starts to be more used in industrial practice. Numerical model of mentioned heater based on partial differential equations were solved by finite element method in virtual software environment. Primary goal of computer modeling was to investigate the thermal dynamics of four-module heater working in steady-state operation regime. Obtained data were applied to design an advanced control circuit based on distributed parameter systems theory. This may open up the opportunity to make further progress in induction heaters design.
KEYWORDS: induction heaters design, numerical model, finite element method, computer modeling

INTRODUCTION - MODELING OF CONTINUOUS INDUCTION HEATING

Induction heating of steel is contactless heating method that combines electrical, magnetic and thermal phenomena. It works on the principle of energy distribution from inductor coil to the heated billet carried by generated electrical and magnetic fields. Eddy currents induced in heated billet flow through its body and generate heat due to electrical resistivity of material. Described phenomenon is called the Joule effect. The mathematical description for continuous induction heating of steel billets can be described by two coupled partial differential equations (PDE) formulations – the electromagnetic part by a time-harmonic reduced PDE of the magnetic vector potential and the thermal part by PDE for temperature fields [6,8]. Electromagnetic field can be calculated by magnetic vector potential A as

$$\sigma\left(\frac{\partial \mathbf{A}}{\partial t}-(\mathbf{v}\times\nabla\times\mathbf{A})\right)+\nabla\times\left(\mu_0^{-1}\mu_r^{-1}\nabla\times\mathbf{A}\right)=\mathbf{J}_{\text{source}}, \tag{1}$$

*where **A** represents magnetic vector potential, **B** $=\nabla\times$ **A** is magnetic flux density [T], ω is angular frequency, ε_0 is permittivity of vacuum, ε_r relative permittivity, **v** [$m\cdot s^{-1}$] velocity and $\mathbf{J}_{source}$ represents the drive current density in the coil. The ability of material to conduct current is specified by electrical conductivity $\sigma = 1/\rho$ [$S\cdot m^{-1}$], where ρ represents the electrical resistivity ρ [Ω·m].*

Electrical conductivity is temperature dependent material property. Relative magnetic permeability μ_r is non-dimensional parameter which indicates the ability of metal to conduct the magnetic flux better than a vacuum (μ_0 represents the permeability of vacuum). This material property is both magnetic field intensity and temperature dependable (Figure 1b) and it has a great effect on all basic induction heating phenomena. The temperature at which billet body becomes non-magnetic is called the Curie temperature T_c (μ_r = 1). The drop of relative permeability μ_r for forging steel is set to T_c =775 [°C]. The phenomenon of non-uniform induced current distribution within the billet cross-section is called the „skin effect". The maximum value of current density is located on the surface of the heated body and the current density decreases from the body surface toward its center. Current penetration depth is frequency dependent, the lower current frequency creates greater penetration depth and vice versa [4,6].

In induction heating, all three modes of heat transfer (conduction, convection and radiation) are present [6,8]. Therefore the temperature distribution profile of heated billet is given by equation

$$\rho(T)C_P(T)\left(\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T\right)-\nabla\cdot\left(\lambda(T)\nabla T\right)=Q_j, \tag{2}$$

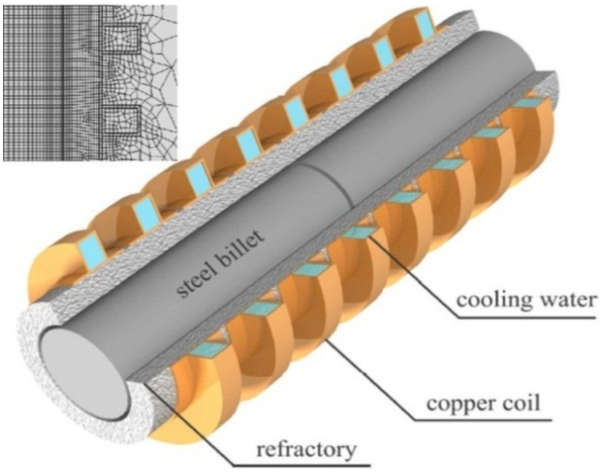
where temperature dependent variables of steel billet are: density ρ [$kg\cdot m^{-3}$], thermal conductivity λ [$W\cdot m^{-1}\cdot K^{-1}$] and specific heat capacity C_P [$J\cdot kg^{-1}\cdot K^{-1}$] of forging steel. Q_j represents the Joule losses in billet generated by eddy currents flow [12]. Joule losses can be rewritten as

$$Q_j = \|\mathbf{J}_{in}\|^2 / \sigma(T), \tag{3}$$

TABLE 1. TEMPERATURE-DEPENDENT MATERIAL PROPERTIES

Temp (K)	σ (S/m).106	λ (W/m.K)	CP (J/kg.K)	ρ (kg/m3)	h (W/m2.K)
273	1.61	45	475	7850	10
373	1.42	44	489	7826	15
473	1.3	42	495	7796	20
673	1.25	37	518	7731	25
873	1.05	33	580	7660	40
1073	0.8	30	585	7585	65
1173	0.7	29	600	7534	80
1273	0.6	29	620	7483	95
1473	0.59	29	650	7380	130

Both convection and radiation are responsible for heat loss from billet surface to the surrounding area. Therefore boundary condition of inward heat flux $Q_{loss}=h(T_{ext} - T)$ has been applied. Temperature dependent combined heat transfer coefficient h carries the influences of radiation and convection to the billet temperature in one complex variable. Text is external temperature near the billet surface and T is billet surface temperature.



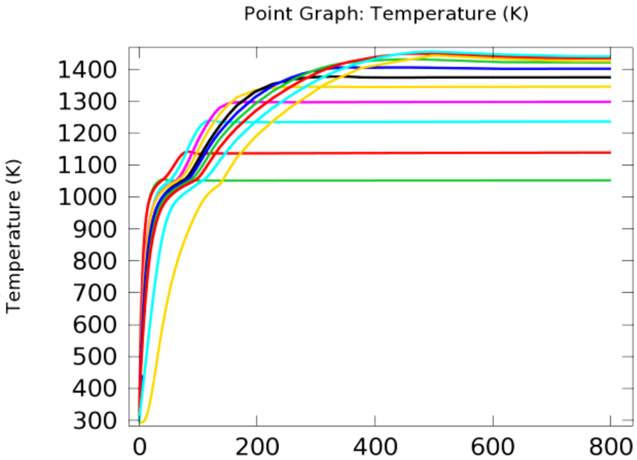


Figure 1. a) Detail of model part with custom mesh, b) Solved steady-state measured by ten virtual probes

The nonlinear, temperature-dependent material parameters of forging steel (see Table. 1) have been used in calculation, Rudnev (2010), Behúlová et. al. (2006). The drop of relative permeability μ_r was set to value of T_c =1020K. Depending on heated material, the heating process of φ80mm steel billets for forging takes several minutes to achieve an optimal steady state. Production rate cycle was set to one properly heated billet per 25 seconds with final temperature T_{out}≈1450K. The four coil currents I_1 - I_4 [kA] was applied with real engineering applications in mind. The billets were continuously moving through the inductor coil with velocity of 10mm/s. Three-dimensional model of inductor coil, kindly provided by Roboterm company, was transformed into 2D axial symmetric space dimension. Four coil modules in serial connection has been used. It created inductor with total of 55 turns and length of 3m. The coil interior was coated by refractory. Custom combination of distributed, mapped and free quadratic mesh was applied to achieve good accuracy-to-computing time ratio. Very dense mesh was created on the billet surface and between coils and billet. The goal was to achieve a precise calculation of electro-magnetic phenomena and „skin effect". Figure 1a shows the small part of model and used mesh style.

In real production demands noted above, the steady-state temperature profile has been solved by COMSOL Multiphysics. Obtained steady-state measured by ten virtual probes shows Figure 1b.

DYNAMICS INSPECTION OF INDUCTION HEATER

Each coil $\{GU_i\}_i$ in continuous modular induction heater (see Figure 2) is powered by its own current $\{UA_i(t)\}_i$ generated by actuating members $\{SA_i\}_i$, therefore it is a typical lumped-input and distributed-parameter-output system (LDS). It causes the change of billets thermal profile $\mathbf{Y}(\mathbf{x},t)$ along the inductor length. It should be noted that current is not actuating variable in real-life control of induction heating coil power.

In practice PLC is operating with coil voltage, which "carries" the current into the coils. The change of coil voltage gives adequate change of coil current as well. With model simplification in mind the coil currents represents the actuating variables of DPS control circuit.

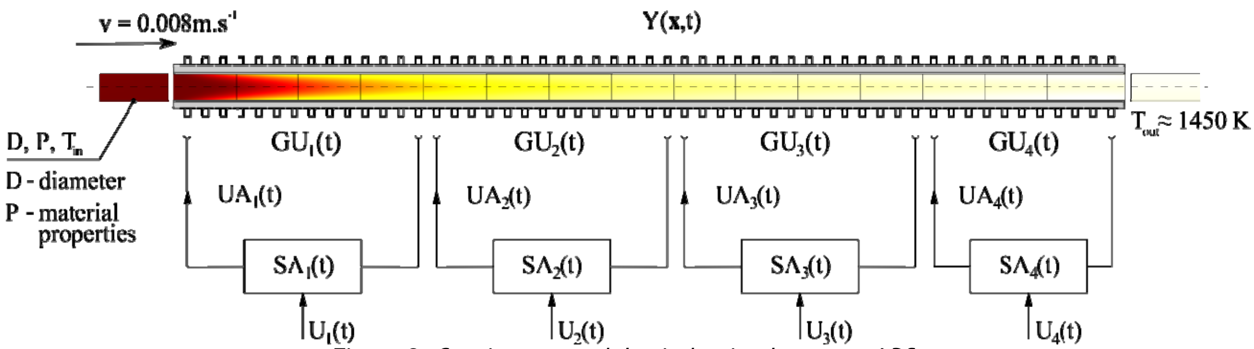


Figure 2. Continuous modular induction heater as LDS.

In this case of numerical simulation of induction heating process, there is necessary to introduce a discrete HLDS model (the same LDS system with input block of zero-hold units $\{H_i\}_i$). It is distributed on the interval of inductor length [0,L] with output quantity $Y(\mathbf{x},k)$. The model is discretized in time relation and continuous in space relation on the same interval. In this case the distributed quantities are used in its discrete form as finite sequences of quantities. The discretization in space domain is usually considered by the computational nodes of the numerical model of the controlled DPS over the complex-shape definition domain.

In the linearized region of solved steady-state operation setpoint, the 10% step to each lumped-input variable (coil currents) has been applied. The distributed step responses $\{\mathbf{H}_i(\mathbf{x},k)\}_{i=1\ldots4}$, had been calculated. The distributed-parameter impulse characteristics $\{\mathbf{G}_i(\mathbf{x},k)\}_{i=1\ldots4}$ were obtained by subtracting of shifted distributed-parameter step characteristics. It has led us to input-to-output relation

$$Y(x,k)=\sum_{i=1}^{4}Y_i(x,k)=\sum_{i=1}^{4}\mathcal{G}_i(x,k)\oplus U_i(k), \qquad (4)$$

where $\{Y_i(x,k)\}_i$ represents four components of HLDS distributed output. When the vector of lumped variables $\{U_i(k)\}_{i=1\ldots4}$, operates as the input of HLDS block, the overall output variable $Y(x,k)$ is obtained. Now let us analyse the distributed transient characteristics for each point $\{x_i\}_{i=1\ldots4}$, which carries the highest amplitude peaks (see Figure 4a). The reduced steady-state distributed parameter transient responses were introduced as

$$\left\{\mathcal{HR}_i(\mathbf{x},\infty)=\mathcal{H}_i(\mathbf{x},\infty)/\mathcal{H}_i(x_i,\infty)\right\}_{i=1\ldots4} \qquad (5)$$

for $\{\mathbf{H}_i(x_i,\infty)\}_{i=1\ldots4}\neq 0$. In the next step of analysis, the discrete transfer functions $\{S_i(x_i,z)\}_{i=1\ldots4}$ were assigned to partial distributed parameter transient responses with maximal amplitudes at points $\{x_i\}_{i=1\ldots4}$ on the inductor length interval [0,L], in their proximity the measurements are performed.

Two crucial components of steady-state HLDS dynamics were obtained:

- *time components of dynamics $\{S_i(x_i,z)\}_{i=1\ldots4}$*
- *space components of dynamics $\mathbf{HR}_i(\mathbf{x},\infty)\}_{i=1\ldots4}$*

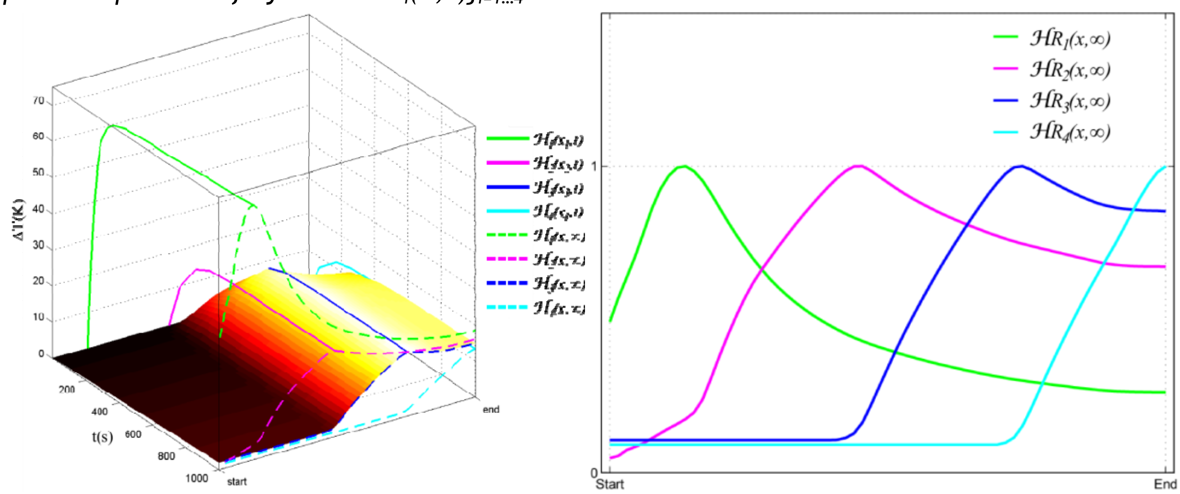


Figure 4. a) Time and spatial distributed transient characteristics to +10% step of each coil current b) Four reduced steady - state distributed parameter transient responses.

The relation between lumped input variables $\{U_i(z)\}_{i=1,4}$ and partial distributed output variables for $\{x_i\}_{i=1,4}$ takes form as follows

$$\left\{Y_i(x_i,z)=S_i(x_i,z)U_i(z)\right\}_{i=1,4}. \qquad (6)$$

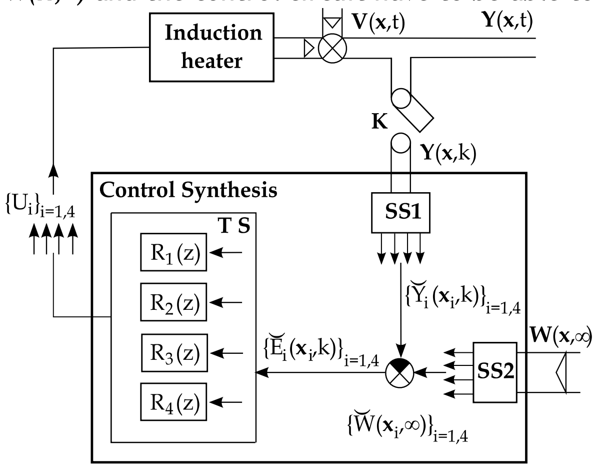
In respect to used variables, the overall distributed steady-state output variable is given by equation

$$Y(x,\infty) = \sum_{i=1}^{4} Y_i(x_i,\infty)\mathcal{H}\mathbf{R}_i(x,\infty) \tag{7}$$

CONTROL OF INDUCTION HEATER AS DISTRIBUTED PARAMETER SYSTEM

According to Figure 2, the Distributed parameter system (DPS) control circuit has been designed with DPS Blockset components in MATLAB & Simulink interface.

In order to control of induction heater in a linearized region of the set-point, the simulation of distributed parameter feedback control is set up (Figure 5.). The objective of control is to hold the proper billet's temperature profile required by technology. It is given by initial reference variable $W(\mathbf{x},\infty)$ and the control circuit have to be able to ensure its optimal approximation.



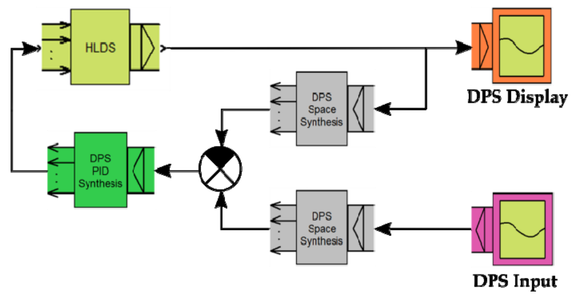


Figure 5. Control circuit, a) schematically, b) circuit developed in MATLAB using DPS Blockset.

The SS1 block is solving

$$\min_{Y_i} \left\| Y(x,k) - \sum_{i=1}^{4} Y_i(x_i,k)\mathcal{H}\mathbf{R}_i(x,\infty) \right\|_2 . \tag{8}$$

and in steady-state (assuming $V(\mathbf{x},t) = 0$) it holds

$$\breve{Y}(x,\infty) = \sum_{i=1}^{4} \breve{Y}_i(x_i,\infty)\mathcal{H}\mathbf{R}_i(x,\infty). \tag{9}$$

The SS2 block is calculating

$$\min_{Y_i} \left\| W(x,\infty) - \sum_{i=1}^{4} W_i(x_i,\infty)\mathcal{H}\mathbf{R}_i(x,\infty) \right\|_2 . \tag{10}$$

The SS2 block calculation output is the best approximation of reference variable

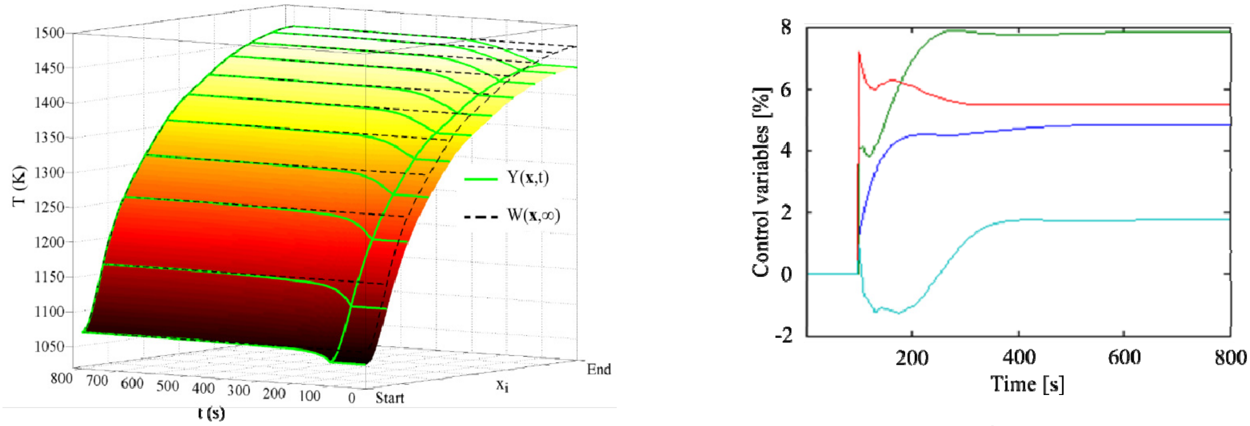
$$\breve{W}(x,\infty) = \sum_{i=1}^{4} \breve{W}_i(x_i,\infty)\mathcal{H}\mathbf{R}_i(x,\infty). \tag{11}$$

The mentioned approximation problems are being solved in a strictly normed function space of distributed parameter quantities with the base $\mathbf{HR}_i(\mathbf{x},\infty)\}_{i=1...4}$. In terms of equation (7), where relation between $\{U_i(k)\}_{i=1...4}$ and $\{Y_i(x_i,k)\}_{i=1...4}$ has been demonstrated using transfer functions $\{S_i(x_i,z)\}_{i=1...4}$, there is four set of variables $\breve{Y}_i(x_i,k)$, $\breve{W}_i(x_i,\infty)$, $\breve{E}(x_i,k)$ and $U_i(k)$ belong to each feedback control circuit. The control synthesis of induction heater model is composed of four identical feedback control circuits.

Generally, there can be used the majority of controlling algorithms (PID, model predictive control, state-space etc.) in the regulator section $R_i(z)$, depending on process control requirements. The conventional PI algorithm was considered as optimal for our model control purposes. Parameters of PI regulators were tuned by method of placing the poles. The goal of control synthesis was to reach the input-output balance. In steady-state it can be described by relations (9) and (11) using the control error approximation

$$\left\{ \breve{E}_i(x_i,\infty) = \breve{W}_i(x_i,\infty) - \breve{Y}_i(x_i,\infty) = 0 \right\}_{i=1...4} . \tag{12}$$

Let us consider a control problem of increasing the steady-state temperature profile of the billet by 30K in full coil length in the time of t=100s, Figure 6a. There is a step change of reference variable performed by actuating variables of four PI regulators in DPS PID Synthesis block. Actuating variables generated by regulators are changing the coil currents, Figure 6b. From the initial steady-state temperature profile, the new one is reached in approximately 400[s]. It corresponds to the considered billet transfer over the entire length of inductor coil.



a) b)

Figure 6. a) Time-spatial step response given by step change at t=100[s], b) Actuating variables (as percentage of coil currents).

CONCLUSIONS - SUMMARY

The achieved results represent the possibilities of numerical PDE solution describing the process of induction heating for control design purposes. The control objective, discussed in this article, is to keep the heated steel billet's temperature field within allowable limits around the set-point. Non uniformity of heated steel properties may cause the temperature variation. Thus induction heating process is subject to uncertainties in physical and chemical properties of the billets material and fluctuations of the electric power delivered to the inductor coils. In the next step, there is ambition to design proposed control circuit that keeps the process running in an emergency setting in case of one inductor module failure. The remaining healthy modules will hold the desired temperature profile within some tolerable limits, depending of course on the machine construction limitations. Inductor heater with this ability placed in fully automatic, high productivity workplace, offers the advantage of continuous operation in case of one inductor module malfunction.

In the case of this paper the numerical solution was done using the virtual software environment COMSOL Multiphysics. It is used for inspection of dynamic characteristics of the induction heating process and building the model in form of lumped-input and distributed-parameter-output system. Interconnecting this model with MATLAB & Simulink via DPS Blockset (Figure 5b) opens new possibilities for simulation studies of steel billet induction heating in the field of control design and process optimization as well. In the industry practice, the number of measuring pyrometers is limited to the reasonably applicable minimum due to high costs Obtained results (Figure 6) indicate the possibility of replacement of several PLCs and pyrometers with one industrial computer equipped by proper software. This opens up new possibilities for induction heating control for the engineering practice.

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