



¹. Ljiljana VELJOVIĆ, ². Dragan MILOSAVLJEVIĆ,
³. Gordana BOGDANOVIĆ, ⁴. Aleksandar RADAKOVIĆ

MODELING AND ANALYSIS FOR THE VIBRATION OF A GYRO-ROTOR

¹⁻³. Faculty of Engineering, University of Kragujevac, Kragujevac, SERBIA
⁴. State University of Novi Pazar, Novi Pazar, SERBIA

Abstract: Dynamic behaviors of a gyro-rotor are analyzed by a theoretical approach. A discrete dynamic model for a gyro-rotor is established by considering the eccentricity, angle inclination and positions of rotating axes. The dynamic model consists of the gyro-rotor which rotates around axes with or without intersection. Using vector method, the nonlinear equations of motion are derived. The dynamic behaviors of the gyro-rotor system are investigated with the time responses computed from the equations of motion. Dynamic behaviors of a gyro-rotor are analyzed by a theoretical approach. A discrete dynamic model for a gyro-rotor is established by considering the eccentricity, angle inclination and positions of rotating axes. The dynamic model consists of the gyro-rotor which rotates around axes with or without intersection. Using vector method, the nonlinear equations of motion are derived. The dynamic behaviors of the gyro-rotor system are investigated with the time responses computed from the equations of motion.

Keywords: Gyro-rotor, Eccentricity, Angle of inclination, rotating axes, computed responses

1. INTRODUCTION

A gyroscope is a body of rotation (for example, a massive disc) which is set spinning at a large angular velocity around its axis of symmetry. First acquaintance with a gyroscope occurs usually in early childhood, when the child watches the unusual behaviour of a widely known toy – a spinning top. A toy gyroscope seems to be able to defy gravity. As long as the top is spinning fast enough, it remains staying steadily (“sleeping”) on the lower sharp end of the axis avoiding tipping over or falling down to the ground and preserving the vertical position of the axis in spite of the high position of its centre of mass – the centre of gravity of a spinning top can be located far above its supporting pivot. If the axis of a spinning gyroscope is inclined to the vertical, the axis generates in space a vertical circular cone, so that the angle between the gyroscope axis and the vertical remains constant during rotation.

The most important practical applications of gyroscopes are met in devices for measuring the orientation or maintaining the stability of airplanes, spacecraft, and submarines – vehicles in general. Various gyroscopes are used as sensors in inertial guidance systems.

The single track gyroscopic vehicle problem is first considered in 1905 by Louis Brennan [1]. Many extensions were later developed, including the work by Shilovskii and several prototypes were built. The differences in the various schemes lie in the number of gyroscopes employed, the direction of the spin axes relative to the rail, and in the method used to produce the acceleration of the spin axle. Other important application of gyroscopic stabilizers include to ships and ocean vehicles and robotics [1].

In recent years, many advanced techniques have been made in modern control theory, and they have been successfully applied to the study of control of a wide variety of engineering systems.

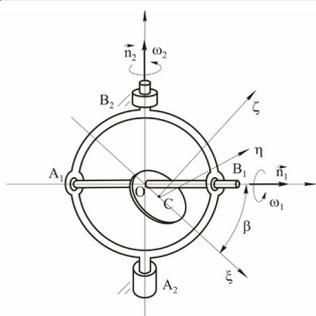


Figure 1. Model of gyro rotor with two component rotation around orthogonal axes without intersections

Then, control of tall buildings and towers, long bridges and other flexible civil engineering structures has generated much interest. Environmental loads on flexible civil structures, such as those stemming from wind and earthquake can cause human discomfort, motion sickness and sometimes endanger structural safety and integrity. Passive and active control schemes are becoming an integral part of the structural systems over the last two decades. Passive devices, such as base isolation system, viscoelastic dampers and tuned mass damper, are widely accepted by the engineering community as a means for mitigating the effect of dynamic loading on structures. However, these passive device methods are unable to adapt to structural changes and to varying usage patterns and loading conditions.

In the last two decades, many other active control devices have been developed and conducted by many researchers for civil engineering applications. One of active control devices used in navigational, aeronautical and space engineering is gyroscope system. The gyroscope system is commonly used for the attitude control of an unstructured object. The gyroscope system also can be used for vibration control of structured objects like buildings, towers and bridges. this system is more compact and has smaller mass than other control devices with the same ability to control. This paper is an attempt to develop some clarity regarding the old problem of precession and especially nutation of gyroscopes

The scientific study of coupled oscillators started with Christian Huygens' observations in the seventeenth century of mutual synchronization of pendulum clocks connected by a beam [1]. More recently, it has been recognized that mutual synchronization of coupled oscillators—the adjustment of rhythms of oscillating systems due to their weak interactions—occurs in many biological systems.

In this paper research results of influences the eccentricity, angle inclination and positions of rotating axes at nonlinear dynamics qualitative properties are presented. Also, the properties of nonlinear dynamics of a gyro-rotor are investigated by using the corresponding equations of phase trajectories of corresponding basic scleronomic nonlinear model to the dynamic model consists of the gyro-rotor which rotates around axes with or without intersection. By using Math-Cad program for drawing families of phase portraits visualization of nonlinear phenomena in dynamic of gyro-rotor model are presented, also on that graphics it is barely noticeable a influence of the eccentricity, angle inclination and positions of rotating axes.

2. THE MODEL OF THE GYRO-ROTOR SYSTEM AND BASIC EQUATIONS

Here is presented eccentric disc (eccentricity is e), with mass m and radius r which is inclined to the axes of its own rotation by the angle β . The angle of own rotation around moveable axis oriented by the unit vector \vec{n}_1 is φ_1 and the angular velocity is ω_1 . The angle of rotation around the shaft support axis oriented by the unit vector \vec{n}_2 is φ_2 and the angular velocity is ω_2 (see Figure 1). When the support shaft is vertical and the rotor shaft is horizontal and when they are without intersection we obtain that angular velocity of rotor is: $\vec{\omega} = \omega_1 \vec{n}_1 + \omega_2 \vec{n}_2 = \dot{\varphi}_1 \vec{n}_1 + \dot{\varphi}_2 \vec{n}_2$

The angles φ_1 and φ_2 are generalized coordinates in case when we investigate system with two degrees of freedom. In this case φ_1 is generalized coordinate. The second angle φ_2 is a rheonomic coordinate which is defined by a time function.

Using the theorem of angular momentum derivative the differential vector equation of the rotation around the rotor shaft in case when the support shaft is vertical and the rotor shaft is horizontal and when they are with or without intersection is obtained [3-5]. Scalar form of equation in a general form is:

$$\ddot{\varphi}_2 + \Omega^2(\lambda - \cos \varphi_2) \sin \varphi_2 + \Omega^2 \psi \cos \varphi_2 = 0 \tag{1}$$

For considering an eccentric disc constants in differential equation (1) are:

$$\Omega^2 = \omega_2^2 \frac{(\varepsilon \sin^2 \beta - 1)}{(\varepsilon \sin^2 \beta + 1)}, \quad \lambda = \frac{g(\varepsilon - 1) \sin \beta}{e \omega_2^2 (\varepsilon \sin^2 \beta - 1)}, \quad \psi = \frac{2me\ell \sin \beta}{J_u - J_v}, \quad \varepsilon = 1 + \left(2\frac{e}{r}\right)^2 \tag{2}$$

The motion of gyro-rotor was presented by means of phase trajectories and that is done for different cases of disk eccentricity and angle of skew. For that reason it is necessary to find first integral of the differential equation (1). After integration of the differential equation (1) the non-linear equation of the phase trajectories of the gyro rotor dynamics with the initial conditions $t_0 = 0, \varphi_1(t_0) = \varphi_{10}, \dot{\varphi}_1(t_0) = \dot{\varphi}_{10}$ is obtained in a form:

$$\dot{\varphi}_1^2 = \dot{\varphi}_{10}^2 + 2\Omega^2 \left[\lambda(\cos \varphi_1 - \cos \varphi_{10}) + \frac{1}{2}(\cos^2 \varphi_{10} - \cos^2 \varphi_1) + \Psi(\sin \varphi_1 - \sin \varphi_{10}) \right] \tag{3}$$

All parameters are in function of eccentricity e and angle β . So, it was very interesting to analyze the influence of these parameters on nonlinear dynamics behavior of system. The solution of this equation is obtained using Math Cad.

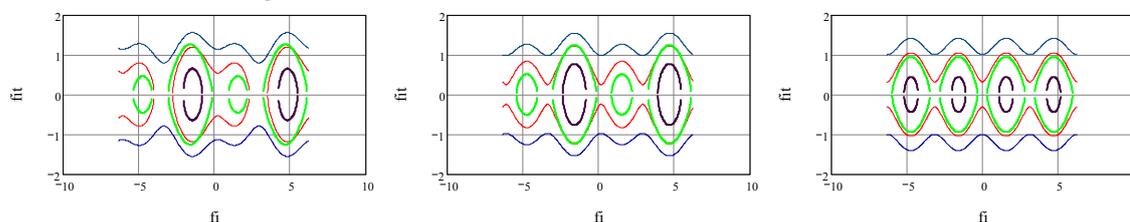


Figure 2. Transformation of a phase trajectory for different cases of rotating and for a corresponding initial condition

In Figure 2a transformation of a phase trajectory in a case when gyro rotor is rotating around axes without intersection with distance between axes is $a = 2mm$ and for different initial conditions. Figure 2b shows transformation of a phase trajectory in a case when gyro rotor is rotating around axes that are in intersection and when gyro rotor is not inclined ($\beta=0$) and in Figure 2c transformation of a phase trajectory in a case when gyro rotor is not eccentric ($e = 0$).

For gyro rotor which rotates around axes that are without intersection and for rotor that is inclined at rotating axis it can be seen that there are stable and not stable dynamical equilibrium positions. Also stable dynamical equilibrium position can be lost with some changes of parameters of the system, such as angle of inclination or eccentricity. That means that dynamic unstable position can be made stable by changing some parameters of the system.

In Figure 3, it can be noticed that the phase trajectories change shapes when system parameters are changing. Also, analysis of rotation of a heavy gyro-rotor show that in graphical presentations of the system kinetic parameters exits a set of the fixed points not depending of change of rigid body eccentricity or angle of inclination or of the orthogonal distance between axes of rigid body coupled rotations. Thus, a closed curve can cross the open i.e. oscillatory motion become non oscillatory. Also, it can be seen trigger of coupled singularities which was the subject of Professor K. Hearth's research [2].

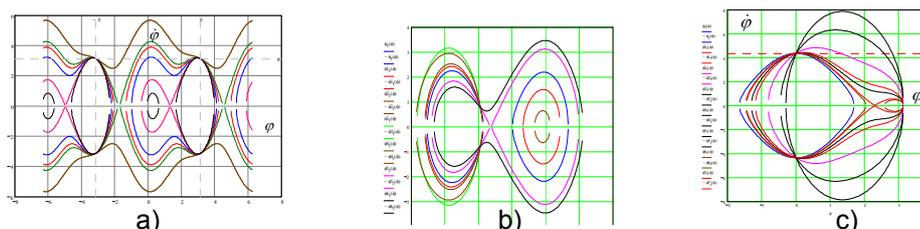


Figure 3. Transformation of a phase trajectory for different values of disk inclination angle β to the axis of self rotation (a and b) and for different values of normal distance between axes and for a corresponding initial condition(c)

Differential equation (1) is obtained by use vector expressions for linear momentum and for angular momentum in a form defined in [2] using mass moment vectors. The body mass inertia moment vectors and vector rotators are very suitable for obtaining linear momentum and angular momentum and their derivatives of the system with coupled numerous rotations around axes without intersections, as well as for vector method analysis of system vector parameter dynamics.

3. CONCLUSION

Applications of the new mass moment vectors for obtaining vector expressions of the kinetic parameters of the nonlinear dynamics of the rigid body coupled rotations around two axes without intersections, show that vector method as well as applications of the mass moment vectors and vector rotators show us a simplest way for analysis characteristic vector structures of coupled rotation kinetic properties. Vector components of complex dynamic structure of rigid body coupled rotations about two axes without intersections is suitable for analysis interactions between some of these vector component parameters and nature of each of these, no visible in the scalar forms.

Using the derived analytical expressions of the gyro-rotor-disk coupled rotations and by standard software tools, the numerous visualizations of phase trajectories are presented. Special attentions are focused to the possibility that by changing system parameters unstable dynamical equilibrium position parameters leads to a stable dynamic equilibrium position.

This method and mass moment vectors can be applied to the rigid body multi-coupled rotations as well as for applications to the coupled multi-body system dynamics with multi-coupled rotations. Vector form of system kinetic parameter expressions based on the mass moment vectors coupled to axis and pole open new simpler possibilities for analysis of the system complex dynamics.

ACKNOWLEDGMENT

Our sincerely thanks to Professor Katica R. (Stevanović) Hedrih, Leader of the Project OI174001 and my supervisor for all her comments, valuable consultations and motivation that she gave me to submit this paper. Parts of this research were supported by the Ministry of Sciences and Environmental Protection of Republic of Serbia through Mathematical Institute SANU Belgrade Grant ON174001 and Faculties of Mechanical Engineering in Niš and Kragujevac

REFERENCES

- [1.] Буляков, Б. В., (1976), Прикладная теория гироскопов, Издательство Московского университета, Russia
- [2.] Hedrih, K.S., (2001), The Vector Method of the Heavy Rotor Kinetic Parameter Analysis and Nonlinear Dynamics, Monograph, University of Niš, pp. 252
- [3.] Hedrih, K.S., Veljović, Lj., (2011), Vector Rotators of a Rigid Body Dynamics with Coupled Rotations around Axes without Intersection, Mathematical Problems in Engineering, Vol 2011, article ID 351269, 26pages, doi:10.1155/2011/351269
- [4.] Hedrih, K.S., Veljović, Lj., (2011) Vector method of kinetic parameter analysis of a rigid body rotation/oscillations around two axes without Intersection, ENOC, 24-29 July 2011, Rome, Italy doi:10.1155/2011/351269
- [5.] Veljović, LJ, (2011) Nelinearne oscilacije giro-rotora, Doktorska disertacija, Mašinski Fakultet, Niš



ANNALS of Faculty Engineering Hunedoara
– International Journal of Engineering



copyright © University Politehnica Timisoara,
Faculty of Engineering Hunedoara,
5, Revolutiei, 331128, Hunedoara, ROMANIA
<http://annals.fih.upt.ro>