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VALIDATION OF THE THEORETICAL MODEL FOR THE STUDY OF DYNAMIC BEHAVIOR ON VERTICAL DIRECTION FOR RAILWAY VEHICLES

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Abstract: This paper focuses on a complex theoretical model regarding the dynamic behavior of the railway vehicles in the vertical plan while travelling on a track with irregularities, subjected to a validation process. The reference ground for this process is constituted by the results derived from experiments made at low velocities. The validation criteria to be implemented aim at the main vehicle resonance frequency and the scale of the vertical acceleration in two reference points of the carbody. The correlations of quality and quantity nature between the rms acceleration spectra coming from the experimental data and numerical simulations will allow the validation of the suggested theoretical model.

Keywords: railway vehicle, dynamic behavior, theoretical model, validation, low velocity

1. INTRODUCTION

The research concerning the dynamic behavior of the railway vehicles involve the use, as an investigation instrument, of the numerical simulation programmes developed on the support of the vehicle theoretical model. Such models, irrespective of their complexity, start from a series of simplifying hypotheses and, hence, the results can deviate to a certain extent from the physical reality [1]. The exploitation of the results is not therefore sufficient and satisfactory, especially for a complex system as the railway vehicle, without them being experimentally validated.

Despite the fact that the numerical simulations of the dynamic behavior in the railway vehicles are widely used, even in the designing stage and in the research activity, the validation methodologies of the theoretical models are rather limited [2, 3]. They firstly aim at the implementation of the numerical simulations in the homologation process of the railway vehicles [4, 5].

While there are no clear specifications related to the validation process for a model at the railway vehicle, as well as a formal definition of this process or a quantity definition of the acceptable errors, the validation process relies on representative comparisons between the results of the experiments made on the vehicle and the ones obtained from the numerical simulations [6]. Their role is to certify the fact that the theoretical model precisely represents the railway vehicle, that the soft in use is appropriate for the application and the specific conditions of evaluating the dynamic behavior have been taken into account [7 - 9].

The validation of a model on which the vehicle dynamic behavior is based can prompt, as case may be, the necessity of completing both the dynamic and static attempts. Likewise, a series of particular trials can be carried out, such as the low velocity ones, and the derived results are useful for validating certain dynamic features of the vehicle's theoretical model [4].

The paper deals with a complex theoretical model of the vehicle-track system, which can be used to evaluate the dynamic behavior of the railway vehicles on the vertical direction, while travelling on a track with longitudinal irregularities in compliance with several criteria – the ride quality, the vibrating comfort and the fatigue stress on the track [10 - 12]. This model is subjected to the

validation process, based on comparisons between the acceleration spectra numerically derived and the ones from the experimental data that had been reached during the track trials at low velocities of a passenger railway vehicle, fitted with Y 32R bogies. The validation criteria aim at the main vehicle resonance frequency and the magnitude of the vertical acceleration in two reference points of the carbody, namely in its center and above the bogie. The correlations between the results from the two methods, theoretical and experimental, will eventually allow the validation of the presented model.

2. THE THEORETICAL MODEL OF THE VEHICLE – TRACK SYSTEM

To gather reliable quantity results for the dynamic behavior at the railway vehicles, mainly for high velocities, requires to adopt certain complex models, which will consider a series of important factors that affect the vibration behavior of the vehicle [13]. On the one hand, the carbody modelling as a flexible body is essential in the numerical simulation of the dynamic behavior at the railway vehicles, mainly for the comfort analysis during rolling [14 - 17]. Another essential factor in the vehicle's vertical dynamics is the system of taking over the longitudinal efforts between the carbody and the bogies, by which the carbody bending vibrations are transmitted to the bogies, thus simultaneously exciting their pitch and bounce vibrations. And these vibrations move forward via the elastic steering in the axles, drawing them into a longitudinal vibration. On the other hand, the axles steering system, basically located in their plan, is at a different height than the bogie center of gravity, which leads to the emergence of a moment that excites the bogie pitch movement, coupled with the axles' rebound. All these effects will visibly influence the dynamic behavior in the vertical plan of the entire vehicle, which justifies the adoption of a complex model, as shown in figure 1 [10 - 12]. It is about a discrete and continuum model of the vehicle-track system, which includes an Euler-Bernoulli beam for the carbody, a system of rigid bodies, namely the axles and the suspended masses of the bogies, plus a system equivalent with the concentrated parameters for the track.

The model parameters are described in Table 1. For the vehicle carbody, the first two natural modes of bending in a vertical plan (symmetrical and antisymmetrical) are being considered, as well as the rigid vibration modes, i.e. bounce z_c and pitch θ_c . The carbody displacement $w(x, t)$ is given by the overlapping of the rigid vibration and bending

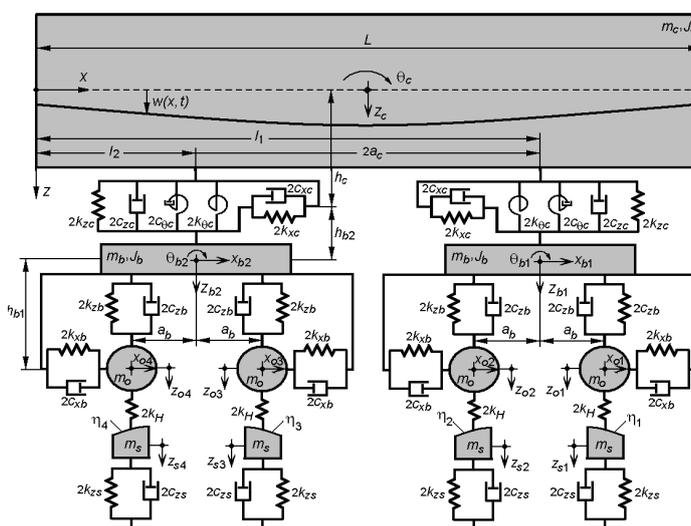


Figure 1. The theoretical model of the vehicle-track system

and bending

$$w(x, t) = z_c(t) + \left(x - \frac{L}{2}\right)\theta_c(t) + \sum_{n=2}^3 X_n(x)T_n(t) \quad (1)$$

where $X_n(x)$, with $n = 2, 3$, are the eigenfunctions of the first two carbody vertical bending modes, and $T_n(t)$ is the time coordinate of the natural bending mode n [11, 12].

The suspended masses of the bogies can perform the following movements: bounce z_{bi} , rebound x_{bi} and pitch θ_{bi} , with $i = 1, 2$. The vehicle's axles have two degrees of freedom, thus being able to complete translation movements on a vertical direction $z_{oj,(j+1)}$ and longitudinal $x_{oj,(j+1)}$, with $j = 2i-1$, and $i = 1, 2$, while each bogie is fitted with the axles j and $j+1$.

The two vehicle suspension stages are modelled via certain Kelvin-Voigt systems. The primary suspension is modelled by two such systems that operate upon the translation on the vertical and

longitudinal directions, while the secondary suspension works through three Kelvin-Voigt systems – two for the translation (vertical and longitudinal) and one for the pitching. The elements of the Kelvin-Voigt system that take over the relative angle travelling between the carbody and bogie will take into account the influence of the secondary suspension. The fact is that the springs of the secondary suspension bring a pitching couple when the layout plans are not parallel [6]. The Kelvin-Voigt system on the longitudinal direction between the carbody and bogie models the transmission system of the longitudinal forces, and the Kelvin-Voigt longitudinal system located in the axles' plan models their elastic steering.

While neglecting the coupling effects between the wheels in the frequency range that is specific to the vehicle vertical vibrations, due to the transmission of the bending waves through the rails, an equivalent model with concentrated parameters will be adopted for the track. To evaluate the wheel-rail contact forces, the magnitude of evaluating the fatigue stress of the track, it is required that the selected model consider the elasticity of the track and of the wheel-rail contract.

Table 1. The parameters of the theoretical model of the vehicle-track system

| | |
|---|------------------------------------|
| Carbody mass | $m_c = 34000 \text{ kg}$ |
| Carbody bending mode | $EI = 3.2 \cdot 10^9 \text{ Nm}^2$ |
| Carbody length | $L = 26.4 \text{ m}$ |
| Carbody wheelbase | $2a_c = 19 \text{ m}$ |
| Carbody pitch inertia moment | $J_c = 7.6 \text{ kgm}^2$ |
| Vertical rigidity of the secondary suspension | $4k_{zc} = 2.4 \text{ MN/m}$ |
| Longitudinal rigidity of the secondary suspension | $2k_{xc} = 4 \text{ MN/m}$ |
| Carbody-bogie pitch angle rigidity | $2k_{\theta c} = 256 \text{ kNm}$ |
| Vertical damping of the secondary suspension | $4c_{zc} = 68.88 \text{ kNs/m}$ |
| Longitudinal damping of the secondary suspension | $2c_{xc} = 50 \text{ kNs/m}$ |
| Carbody-bogie pitch angle damping | $2c_{\theta c} = 2 \text{ kNm}$ |
| Bogie mass | $m_b = 3200 \text{ kg}$ |
| Bogie pitch inertia moment | $J_b = 2048 \text{ kgm}^2$ |
| Bogie wheelbase | $2a_b = 2.56 \text{ m}$ |
| Vertical rigidity of the primary suspension | $4k_{zb} = 4.4 \text{ MN/m}$ |
| Longitudinal rigidity of the primary suspension | $2k_{xb} = 70 \text{ MN/m}$ |
| Vertical damping of the primary suspension | $4c_{zb} = 52.21 \text{ kNs/m}$ |
| Longitudinal damping of the primary suspension | $2c_{xb} = 50 \text{ kNs/m}$ |
| Axle mass | $m_o = 1650 \text{ kg}$ |
| Rail mass (under the axle) | $m_s = 175 \text{ kg}$ |
| Track vertical rigidity | $k_{zs} = 70 \text{ MN/m}$ |
| Track vertical damping | $c_{zs} = 20 \text{ kNs/m}$ |
| Wheel – rail contact rigidity | $k_H = 1500 \text{ MN/m}$ |
| Height of the carbody centre of mass compared to the secondary suspension | $h_c = 1.3 \text{ m}$ |
| Distance between the bogie centre of mass and the primary suspension plan | $h_{b1} = 0.25 \text{ m}$ |
| Distance between the bogie centre of mass and the secondary suspension plan | $h_{b2} = 0.2 \text{ m}$ |
| Load on the wheel | $Q_o = 58.75 \text{ kN}$ |

Thus, against each axle, the track is represented by an oscillating system with one degree of freedom that can move on the vertical direction, where the corresponding displacement is $z_{sj,(j+1)}$, where $j = 2i-1$. The elasticity of the wheel-rail contact is modelled by introducing certain elastic elements with a Hertzian type linear feature, of rigidity k_H .

The track irregularities against the axles are described by the functions $\otimes_{j,(j+1)}$, where $j = 2i-1$, for $i = 1,2$, depending on the distance along the track [11, 12].

The vibrations of the vehicle-track system detailed in a system comprising the movement equations of the carbody, bogie i and of their corresponding axles j , and $j+1$, respectively, where $j = 2i-1$, as well as of the rails':

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \mu I \frac{\partial^5 w(x,t)}{\partial x^4 \partial t} + \rho_c \frac{\partial^2 w(x,t)}{\partial t^2} = \sum_{i=1}^2 F_{zi} \delta(x - l_i) - \sum_{i=1}^2 (M_i - h_c F_{xi}) \frac{d\delta(x - l_i)}{dx} \quad (2)$$

$$m_b \ddot{z}_{bi} = \sum_{j=2i-1}^{2i} F_{z bj} - F_{zi} \quad (3)$$

$$J_b \ddot{\theta}_{bi} = a_b \sum_{j=2i-1}^{2i} (-1)^{j+1} F_{zbj} - h_{b1} \sum_{j=2i-1}^{2i} F_{xbj} - M_i - h_{b2} F_{xi}; \quad (4)$$

$$m_b \ddot{x}_{bi} = \sum_{j=2i-1}^{2i} F_{xbj} - F_{xi}; \quad (5)$$

$$m_o \ddot{z}_{oj,(j+1)} = 2\Delta Q_{j,(j+1)} - F_{zbj,(j+1)}; \quad (6)$$

$$m_o \ddot{x}_{oj,(j+1)} = -F_{xbj,(j+1)}; \quad (7)$$

$$m_s \ddot{z}_{sj,(j+1)} = F_{zsj,(j+1)} - 2\Delta Q_{j,(j+1)}, \quad (8)$$

where $\rho_c = m_c/L$, $\delta(\cdot)$ is Dirac's function, F_{xi} , F_{zi} and M_i – the forces and moments derived from the secondary suspension of the bogie i , $F_{zbj,(j+1)}$ – the forces from the primary suspension, $F_{xbj,(j+1)}$ – the forces related to the axles' elastic steering system, $F_{zsj,(j+1)}$ – the forces from the track and $\Delta Q_{j,(j+1)}$ – the vertical dynamic forces of the wheel-rail contact

$$\Delta Q_{j,(j+1)} = -k_H [z_{oj,(j+1)} - z_{sj,(j+1)} - \eta_{j,(j+1)}]. \quad (9)$$

While applying the modal analysis method, the carbody equation is changed into four movement equations that correspond to the rigid modes of carbody vibration (bounce and pitch) and to the first two natural modes of bending, symmetrical and antisymmetrical.

Consequently, the vibrations of the vehicle-track systems are expressed by a system of 22 coupled equations. An appropriate selection of the coordinates and a processing of the system of equations, this will decompose itself into two independent systems of 10 equations each and two more decoupled movement equations. The two systems describe the symmetrical and antisymmetrical movements of the vehicle-track system [18].

3. EXPERIMENTAL DETERMINATIONS

To validate the theoretical models of the railway vehicles, a series of distinct trials are performed, including the trials during low velocity circulation. The results thus derived can be used to validate certain dynamic characteristics of the vehicle model [4].

This section describes the experimental determinations that help validate the theoretical model of the railway vehicle previously introduced. The test velocities were of 5...7 km/h on a passenger vehicle class I, series 10-90. The experimental determinations aimed to measure the vertical accelerations in the vehicle carbody, and the accelerations spectra based on the registered data are the reference base during the validation process of the vehicle's theoretical model. Similarly, to determine the excitations due to the irregularities in the track it is required to measure the vertical accelerations in the axles' boxes, as they are further used in the numerical simulations of the railway vehicle's dynamic behavior as input values that define the system disturbance.

The system used for the experimental measurements (figure 2) includes the components of the accelerations measuring system, of the system of data acquisition and processing, as well as the GPS receiver meant to monitor and register the velocity.



Figure 2. The system of measuring, acquisition and processing the experimental data.



Figure 3. The mounting accelerometer to measure the acceleration in the carbody.

To measure the accelerations, Brüel & Kjær type 4514 piezoelectric accelerometers have been used. They have been mounted, as case may be, on a metallic support of a specially designed shape for measuring the accelerations on the vertical and horizontal direction in the carbody (figure 3), as well as on the axles' boxes, for measuring the vertical accelerations at their level (figure 4).



Figure 4. Mounting the accelerometers on the axle boxes

The equipment for the acquisition and processing of the experimental data is represented by the assembly comprising the chassis of the acquisition and processing the NI cDAQ-9174 type data where the NI 9234 serial module is being mounted that is meant to take over and integrate the data flow from the accelerometers. The system of measuring the accelerations is connected to a laptop featuring the LabVIEW software.

The NL-602U type GPS receiver is integrated into the measuring system, so as to monitor and register the vehicle velocity. It is connected to the u-Center navigation software, which provides a series of facilities of monitoring, registration and rendering the navigation data for various test scenarios.

Figure 5 shows the graphical interface of the application made in the LabVIEW environment for recording the experimental data. Here, the two diagrams exhibit the vertical accelerations measured on the two axle boxes.

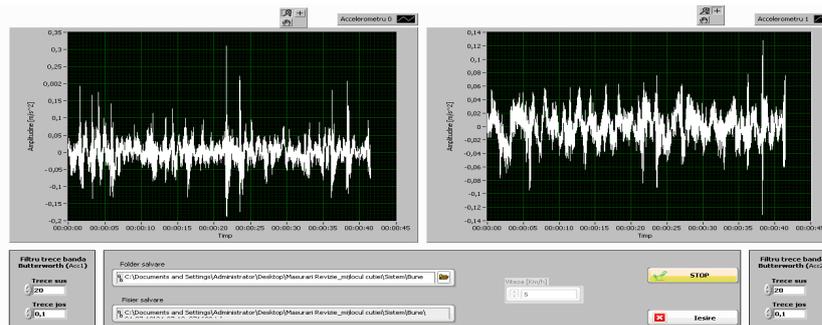


Figure 5. The graphical interface of the application for recording the experimental data

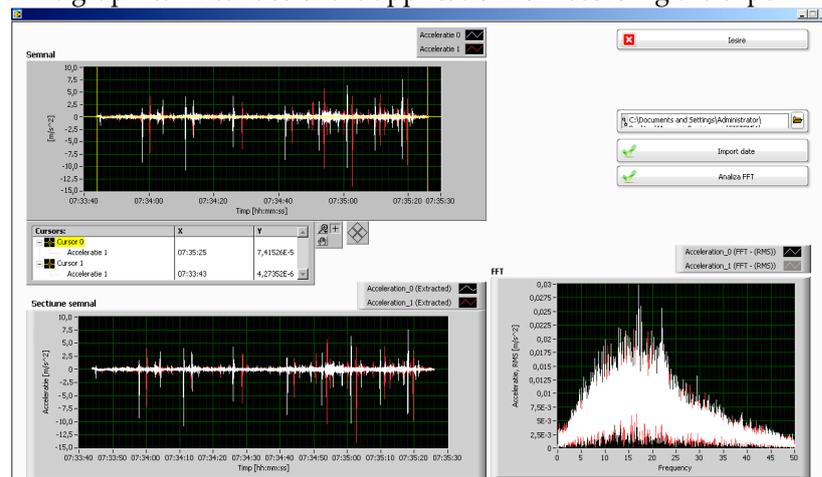


Figure 6. The graphical interface of the application for the analysis of the experimental data

The recorded experimental data are analysed via an application, also made in the LabVIEW environment, whose graphical interface is presented in figure 6. The diagram on the first line contains a 2-minute sequence, extracted from the recordings of the accelerations' amplitudes measured on the two axle boxes. The two recordings can be told apart by the color thus assigned: white – the acceleration measured at the axle box in the traffic direction, on the left side; red – the acceleration measured at the axle box in the traffic direction, on the right side. The sequence

selected from the recording, displayed in the diagram on the second line, is subjected to the fast Fourier transform (FTT), thus obtaining the rms acceleration spectrum that can be visualized in the diagram in the window right corner. In this case, it is about two spectra overlapping in the same diagram, corresponding to the accelerations recorded for the two axle boxes.

4. THE VALIDATION OF THE VEHICLE THEORETICAL MODEL

This section deals with the validation of the theoretical model adopted for the study of the railway vehicle dynamic behavior on the vertical direction during the circulation on an irregular track. The validation process requires the comparison of the results derived from the numerical simulation programs of the vehicle dynamic behavior and those coming from the analysis of the experimental data recorded during travelling at a low velocity.

The validation criteria aim the vehicle resonance frequencies and the magnitude of the vertical acceleration in two reference points of the carbody, namely in its center and above the bogie. To this purpose, the rms acceleration spectra from experiments and theory are compared, noticing that the reference elements in the validation process are represented by the resonance frequency and the accelerations synthetized from the results of the experimental measurements.

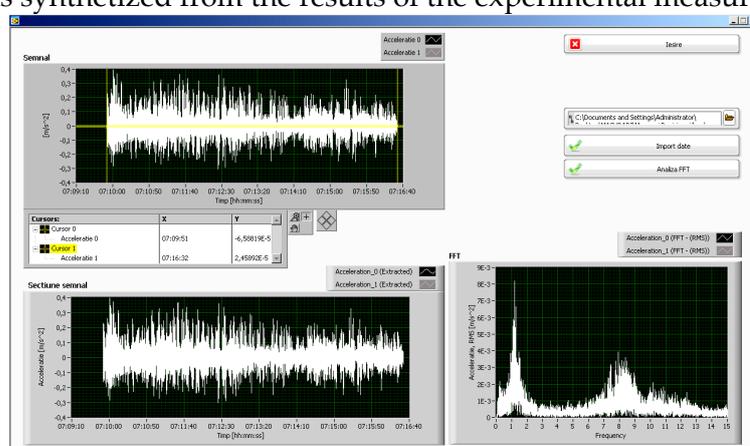


Figure 7. The analysis of the vertical acceleration in the carbody center

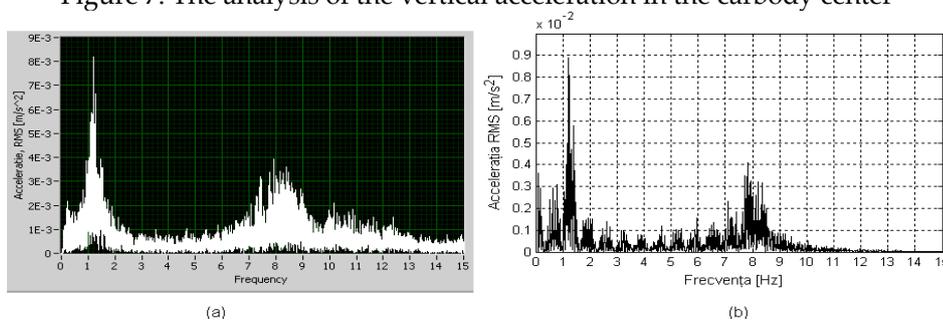


Figure 8. The rms acceleration spectrum in the carbody center: a) experimental; b) theoretical

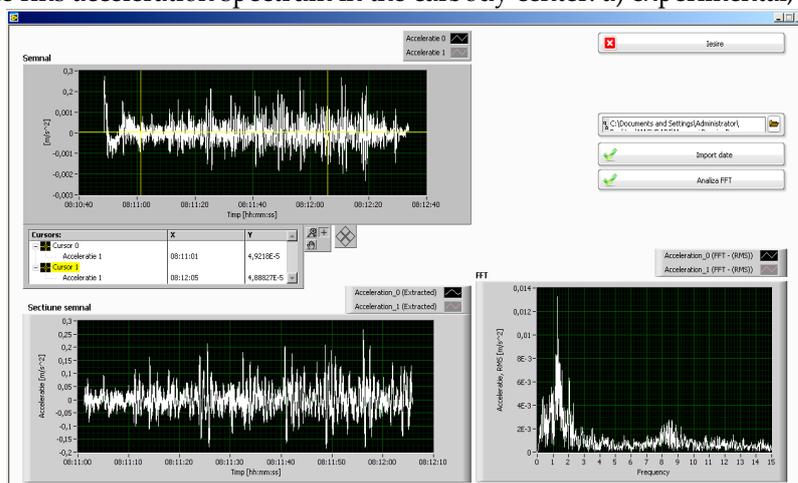


Figure 9. The analysis of the carbody vertical acceleration measured above the bogie

There has been considered a recording of the vertical acceleration measured in the carbody center at velocity of 6 km/h, by applying FFT, and we have the rms acceleration spectrum, as in figure 7. Further on, the figure 8, a, features the spectra of the rms vertical acceleration obtained from the experimental data, with the help of the numerical calculation program (8, b). Upon comparing the two diagrams, the peaks of the dominant resonance frequencies can be noticed on both spectra, namely the low resonance frequency of the bounce movement at 1.17 Hz and the frequency of the carbody symmetrical bending at around 8.2 Hz. Similarly, the resonance frequency of the pitch movement in the bogie at circa 9 Hz is visible.

To apply the validation criterion of the acceleration magnitude, the values corresponding to the peaks of resonance frequencies of the carbody symmetrical bounce and pitch will be compared. It should be noticed that the accelerations above have the same value range on both spectra, also close in values $0,9 \cdot 10^{-2} \text{ m/s}^2$, at the resonance frequency of the low bounce; $0,4 \cdot 10^{-2} \text{ m/s}^2$, at the resonance frequency of the carbody symmetrical bending. Further on, a sequence in a recording of the vertical acceleration is being taken into account, measured above the front bogie in the traffic direction (see figure 9). The diagram featuring the rms acceleration spectrum is extracted and presented in figure 10, a, along with the rms acceleration diagram derived from the vehicle theoretical model (figure 10, b) for the comparison purpose.

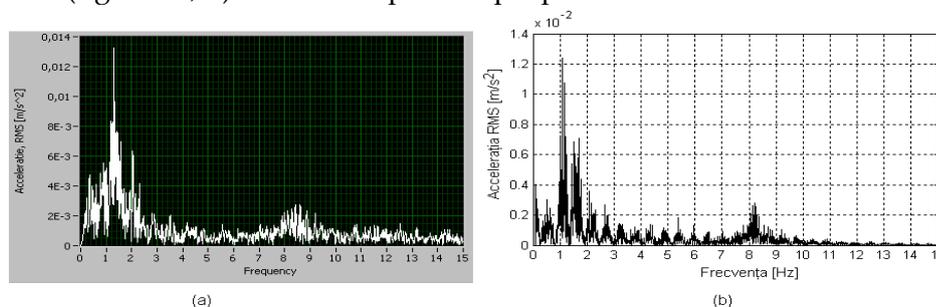


Figure 10. The rms acceleration spectrum above the bogie: a) experimental; b) theoretical

A parallel analysis of the two diagrams will identify the main vehicle resonance frequencies in each of them. On the one hand, it is about the resonance frequencies of the symmetrical movements – the low frequency of the bounce movement at 1.17 Hz and the frequency of the carbody symmetrical bending at 8.2 Hz and, on the other hand, at the resonance frequencies of the antisymmetrical movements – the low frequency of the pitch movement at around 1.8 Hz and the high frequency of the pitch movement at circa 9.3 Hz.

Later on, the second validation criterion of the vehicle theoretical model is being applied, aiming at the magnitude of the vertical accelerations. For this purpose, the values of the accelerations corresponding to the peaks of the dominant frequencies are dealt with, namely the low frequency of the pitch and the symmetrical bending frequency. While examining the two diagrams, it can be noticed that the rms acceleration value is around $1,2 \cdot 10^{-2} \text{ m/s}^2$ for the bounce resonance frequency. Further, the value of the rms acceleration can be read on both diagrams and corresponds to the peak of the low resonance pitch frequency, which is around $0,6 \cdot 10^{-2} \text{ m/s}^2$. Finally, the rms acceleration against the peak of the resonance frequency of the carbody symmetrical bending is identified at circa $0,3 \cdot 10^{-2} \text{ m/s}^2$. The observations that are based on the comparisons between the rms acceleration on the vertical direction from the analysis of the experimental data and the ones from the numerical simulations will allow the validation of the vehicle's theoretical model.

5. CONCLUSIONS

The dynamic behavior of the railway vehicle can be assessed via two different approaches, experimental and theoretical, complementary in their nature. The theoretical studies are developed on an equivalent model of the vehicle, which includes the mechanical model and the numerical model, validated by comparing the results derived from numerical simulations with the ones from experimental studies made on the vehicle under discussion or a similar one.

The paper introduces a complex theoretical model of the vehicle-track system, which looks at a series of factors that influence the dynamic behavior on the vertical direction of the railway vehicles. While travelling on an irregular track, this model can help evaluate the dynamic behavior of the railway vehicles in terms of the ride quality, the vibrating comfort and the fatigue stress on the track.

The theoretical model to be recommended is validated according to the results derived from dynamic trials at train low velocity by measuring the vertical accelerations in the vehicle carbody. The comparison elements that regulate the validation criteria, namely the main vehicle resonance frequencies and the magnitude of the rms vertical acceleration in two reference points of the carbody – in the center of the carbody and above the bogie, are extracted from the acceleration spectra coming from experiments and numerical simulations. The correlations of quality and quantity nature between the results coming from the experimental and theoretical data will allow the validation of the presented theoretical model.

REFERENCES

- [1.] Sebeșan, I., *Dinamica vehiculelor feroviare*, Editura Matrix Rom, București, 2011.
- [2.] Evans, J., Berg, M., Challenges in simulation of rail vehicle dynamics, *Vehicle System Dynamics*, Vol. 47, 2009, pp. 1023–1048.
- [3.] Jönsson, L.O., Nilstam, N., Persson, I., Using simulations for approval of railway vehicles: A comparison between measured and simulated track forces, *Vehicle System Dynamics*, Vol. 46, 2008, pp. 869–881.
- [4.] UIC Leaflet 518, Testing and approval of railway vehicles from the point of view of their dynamic behaviour – Safety – Track fatigue – Running behaviour, 4th edition, Sept. 2009.
- [5.] EN 14363, Railway Applications – Testing for the Acceptance of Running Characteristics of Railway Vehicles – Testing of Running Behaviour and Stationary Tests, CEN, Brussels, 2005.
- [6.] Bruni, S., Vinolas, J., Berg, M., Polach, O., Stichel, S., Modelling of suspension components in a rail vehicle dynamics context, *Vehicle System Dynamics*, Vol. 49, No. 7, 2011, pp. 1021–1072.
- [7.] Diana, G., Cheli, F., Collina, A., Corradi, R., Melzi, S., The development of a numerical model for railway vehicles comfort assessment through comparison with experimental measurements, *Vehicle System Dynamics*, Vol. 38, no. 3, 2002, pp. 165–183.
- [8.] Evans, J., Rogers, P.J., Validation of dynamic simulations of rail vehicles with friction damped Y25 bogies, *Vehicle System Dynamics*, Vol. 28, Supplement, 1998, pp. 219–233.
- [9.] Sayyaadi, H., Shokouhi, N., A new model in rail-vehicles dynamics considering nonlinear suspension components behavior, *International Journal of Mechanical Sciences*, Vol. 51, 2009, pp. 222–232.
- [10.] Dumitriu, M., Modeling of railway vehicles for virtual homologation from dynamic behavior perspective, *Applied Mechanics and Materials* Vol. 371, Volume Innovative Manufacturing Engineering, Trans Tech Publications, Switzerland, 2013, pp 647–651.
- [11.] Dumitriu, M., Evaluation of the comfort index in railway vehicles depending on the vertical suspension features, *Annals of Faculty Engineering Hunedoara – International Journal of Engineering*, Tome XI, Fascicule 4, 2013, pp. 23–32.
- [12.] Dumitriu, M., Modelling of vehicle-track system to study the vertical vibrations, *Annals of the “Constantin Brâncuși” University of Târgu Jiu*, Issue 4, 2012, pp. 53–64.
- [13.] Sebeșan, I., Mazilu, T., *Vibrațiile vehiculelor feroviare*, Editura Matrix Rom, București, 2010.
- [14.] Carlbom, P., Carbody and passengers in rail vehicle dynamics, Doctoral Thesis, Department of Vehicle Engineering, Royal Institute of Technology (KTH), Stockholm, Sweden, 2000.
- [15.] Schandl, G., Lugner, P., Benatzky, C., Kozek, M., Stribersk, M., Comfort enhancement by an active vibration reduction system for a flexible railway car body, *Vehicle System Dynamics* Vol. 45, No. 9, 2007, pp. 835–847.
- [16.] Tomioka, T., Takigami, T., Suzuki, Y., Numerical analysis of three-dimensional flexural vibration of railway vehicle car body, *Vehicle System Dynamics*, Vol. 44, 2006, pp. 272–285.
- [17.] Zhou, J., Goodall, R., Ren, L., Zhang, H., Influences of car body vertical flexibility on ride quality of passenger railway vehicles, *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, Vol. 223, 2009, pp. 461–471.
- [18.] Dumitriu, M., Influence of the vertical suspension on the vibration behavior in the railway vehicles, *Annals of the University of Petroșani, Mechanical Engineering*, vol. 13, 2011, pp. 35–50.