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VISCOELASTIC EFFECTS ON UNSTEADY MHD FREE CONVECTION AND MASS TRANSFER FOR VISCOELASTIC FLUID FLOW PAST A HOT VERTICAL POROUS PLATE WITH HEAT GENERATION/ ABSORPTION THROUGH POROUS MEDIUM

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Abstract: The effects of viscoelasticity on unsteady MHD free convection and mass transfer flow of a viscoelastic, incompressible, electrically conducting fluid past an infinite hot vertical porous plate embedded in porous medium is analyzed. Heat generation/absorption and viscous dissipation effects are included. The temperature of the plate is assumed to be span wise cosinusoidally fluctuating with time. The governing, coupled, non-linear partial differential equations are solved by perturbation technique. The graphical results for transient velocity and transient temperature profiles and tabulated results for skin friction coefficient and Nusselt number are presented and discussed for various viscoelastic parameters with the combination of the other flow parameters.
Keywords: viscoelastic, chemical reaction, heat generation/absorption, unsteady flow

1. INTRODUCTION

The study of coupled heat and mass transfer under the influence of a magnetic field and chemical reaction arise in many transport processes both naturally and in many branches of science and engineering applications. They play an important role in many industries viz. in the chemical industry, power and cooling industry for drying, chemical vapour deposition on surfaces, cooling of nuclear reactors and magnetohydrodynamic (MHD) power generators. In many industrial process involving flow and mass transfer, the diffusing species can be generated or absorbed due to some kind of chemical reaction with the ambient fluid which can greatly affect the flow and hence the properties and quality of the final product. The order of the chemical reaction depends on several factors. One of the simplest chemical reactions is the first-order chemical reaction in which the rate of reaction is directly proportional to the species concentration. Soundalgekar et al. (1979) studied the problem of free convection effects on Stokes problem for a vertical plate with transverse applied magnetic field. Das et al (1994) studied the effect of homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Muthucumaraswamy (2002) investigated the effect of chemical reaction on moving isothermal vertical infinitely long surface with suction. Hady et al. (2006) studied the MHD free convection flow along a vertical wavy surface with heat generation or absorption effect. EL-Kabeir and Abdou (2007) studied chemical reaction effects on MHD flow past a vertical isothermal cone surface in micropolar fluids with heat generation/absorption. The motion and mass transfer with chemically reactive species for two classes of viscoelastic fluid over a porous stretching sheet has been studied by Cortell (2007). Prakash et al. (2008) studied MHD free convection of a chemically reacting micro-polar fluid with mass transfer in the presence of a uniformly applied transverse magnetic field and variable suction. Prasad et al. (2008) studied the radiation and mass transfer effects on an unsteady two dimensional laminar mixed convective boundary layer flow of a viscous fluid along a semi-infinite vertical permeable moving plat. Das et al. (2009) studied the Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Singh et al (2010) investigated the effects of chemical reaction and heat generation/absorption on unsteady MHD free convection heat and mass transfer flow of an electrically conducting, viscous, incompressible fluid past an infinite hot vertical porous plate through porous medium when the plate temperature is span wise cosinusoidally fluctuating with time.

In this study, an attempt has been made to extend the problem studied by Singh and Kumar (2010) to the case of viscoelastic fluid characterised by second-order fluid. The effects of viscoelastic parameter with the combinations of the other flow parameters have been studied thoroughly and presented graphically.

2. MATHEMATICAL FORMULATION

Consider an unsteady free convective viscoelastic fluid flow past an infinite, hot porous plate lying vertically on x* - z* plane. The x* - axis is taken in the direction of the buoyancy force and y* - axis is taken normal to the plane of the plate. A uniform magnetic field is applied in the direction perpendicular to the plane of the plate. The transverse magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible. It is assumed that there is no applied voltage which implies the absence of an electric field. Also, it is assumed that there exists a first-order chemical reaction between the fluid and species concentration. Let (u*,v*,w*) be the components of velocity in the (x*,y*, z*) directions respectively. Since the plate is considered infinite in x* - direction, hence all physical variables will be independent of x*. Further, since the plate is subjected to a constant suction velocity, i.e. v* = - V, w* is independent of z* and so we can assume w* = 0 throughout.

The temperature of the plate is considered to vary span wise cosinusoidally fluctuating with time and assumed to be of the form

$$T_w^*(z^*,t^*) = T_0^* + \varepsilon\left(T_0^* - T_\infty^*\right)\cos\left(\frac{\pi z^*}{l} - \omega^* t^*\right) \tag{1}$$

where T_0^*, T_∞^* and T_w^* are the mean, ambient temperature and wall temperature of the plate respectively, w* is frequency, t* is time, l is the wave length and ε is a small parameter, $\varepsilon \ll 1$.

The constitutive equation for the incompressible second-order fluid is

$$S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \tag{2}$$

where S is the stress tensor, p is the hydrostatic pressure, A_n, n=1,2 are the kinematic Rivlin-Ericksen tensors, μ_1, μ_2 and μ_3 are the material co-efficients describing the viscosity, visco-elasticity and cross-viscosity respectively, where μ_1 and μ_3 are positive and μ_2 is negative (Coleman and Markovitz (1964)). The equation (2) was derived by Coleman and Noll (1960) from that of the simple fluids by assuming that the stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

Applying the Boussinesq and boundary layer approximation, the flow is governed by the following equations:

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -V, \quad V > 0, \tag{3}$$

$$\rho\left(\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*}\right) = \mu_1\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) + \mu_2\left(\frac{\partial^3 u^*}{\partial y^{*2}\partial t^*} + v^*\frac{\partial^3 u^*}{\partial y^{*3}} + \frac{\partial^3 u^*}{\partial z^{*2}\partial t^*} + v^*\frac{\partial^3 u^*}{\partial y^*\partial z^{*2}}\right)$$

$$+\rho g\beta\left(T^* - T_\infty^*\right) + \rho g\beta_c\left(C^* - C_\infty^*\right) - \sigma B_0^2 u^* - \frac{\mu_1 u^*}{K^*} \tag{4}$$

$$\rho C_p\left(\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = k\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) + \mu_1\left\{\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial u^*}{\partial z^*}\right)^2\right\}$$

$$+\mu_2\left(v^*\frac{\partial^2u^*}{\partial y^{*2}}\frac{\partial u^*}{\partial y^*}+v^*\frac{\partial u^*}{\partial z^*}\frac{\partial^2u^*}{\partial y^*\partial z^*}+\frac{\partial^2u^*}{\partial y^*\partial t^*}\frac{\partial u^*}{\partial y^*}+\frac{\partial u^*}{\partial z^*}\frac{\partial^2u^*}{\partial z^*\partial t^*}\right)+Q^*\left(T^*-T^*_\infty\right) \tag{5}$$

$$\frac{\partial C^*}{\partial t^*}+v^*\frac{\partial C^*}{\partial y^*}=D\left(\frac{\partial^2C^*}{\partial y^{*2}}+\frac{\partial^2C^*}{\partial z^{*2}}\right)-k_1\left(C^*-C^*_\infty\right) \tag{6}$$

where g, T*, C*, B_0, D, σ, k, K*, k_1, C_p, ρ, β, β_c and Q* are acceleration due to gravity, fluid temperature, species concentration, magnetic permeability, chemical molecular diffusivity, electrical conductivity, thermal conductivity, permeability of the porous medium, chemical reaction parameter, specific heat at constant pressure, density, coefficient of volume expansion for heat transfer, volumetric coefficient of expansion with species concentration, heat generation/absorption coefficient, respectively.

The boundary conditions of the problem are

$$u^*=0,\ T^*=T_0^*+\varepsilon\left(T_0^*-T_\infty^*\right)\cos\left(\frac{\pi z^*}{l}-\omega^*t^*\right),\ C^*=C_0^*\quad \text{at } y=0$$

$$u^*=0,\ T^*=T^*_\infty,\ C^*=C^*_\infty\quad \text{as } y\to\infty \tag{7}$$

where C_0^* and C_∞^* the species concentration at the wall and at infinity, respectively.

Introduce the following non-dimensional parameters:

$$y=\frac{y^*}{l},\ z=\frac{z^*}{l},\ u=\frac{u^*}{V},\ \theta=\frac{T^*-T^*_\infty}{T^*_0-T^*_\infty},\ \phi=\frac{C^*-C^*_\infty}{C^*_0-C^*_\infty},\ t=\omega^*t^*,\ \Pr=\frac{\mu_1C_p}{k},$$
$$Ec=\frac{V^2}{C_p\left(T_0^*-T_\infty^*\right)},\ \mathrm{Re}=\frac{Vl}{\upsilon_1},\ Gr=\frac{\upsilon_1 g\beta\left(T_0^*-T_\infty^*\right)}{V^3},\ Gm=\frac{\upsilon_1 g\beta_c\left(C_0^*-C_\infty^*\right)}{V^3},$$
$$\omega=\frac{\omega^*l^2}{\upsilon_1},\ Sc=\frac{\upsilon_1}{D},\ M^2=\frac{\sigma B_0^2l^2}{\mu},\ \gamma=\frac{l^2k_1}{\upsilon_1},\ Q=\frac{Q^*}{\upsilon_1\rho C_p},\ k=\frac{k^*}{l^2} \tag{8}$$

In view of the above non-dimensional parameters, the equations (4) – (6) can be expressed in non-dimensional form as

$$\omega\frac{\partial u}{\partial t}-\mathrm{Re}\frac{\partial u}{\partial y}=\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}+\alpha\left[\omega\frac{\partial^3u}{\partial y^2\partial t}+\omega\frac{\partial^3u}{\partial z^2\partial t}-\mathrm{Re}\left(\frac{\partial^3u}{\partial y^3}+\frac{\partial^3u}{\partial y\partial z^2}\right)\right]+\mathrm{Re}^2\left(Gr\theta+Gm\phi\right)-\left(M^2+\frac{1}{k}\right)u \tag{9}$$

$$\omega\frac{\partial\theta}{\partial t}-\mathrm{Re}\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)+Ec\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right]+\alpha\left[\omega Ec\left(\frac{\partial^2u}{\partial y\partial t}\frac{\partial u}{\partial y}+\frac{\partial^2u}{\partial z\partial t}\frac{\partial u}{\partial z}\right)-\left(\frac{\partial^2u}{\partial y^2}\frac{\partial u}{\partial y}+\frac{\partial^2u}{\partial y\partial z}\frac{\partial u}{\partial z}\right)\right]+Q\theta \tag{10}$$

$$\omega\frac{\partial\phi}{\partial t}-\mathrm{Re}\frac{\partial\phi}{\partial y}=\frac{1}{Sc}\left(\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)-\gamma\phi \tag{11}$$

where $\alpha=\dfrac{\mu_2}{\rho l^2}$.

The boundary conditions (7) become

$$u=0,\ \theta=1+\varepsilon\cos(\pi z-t),\ \varphi=1\ \text{at } y=0$$
$$u=0,\ \theta=0,\ \varphi=0\quad \text{as } y\to\infty \tag{12}$$

3. SOLUTION OF THE PROBLEM:

Assume

$$f(y,z,t)=f_0(y)+\varepsilon f_1(y)e^{i(\pi z-t)} \tag{13}$$

where f stands for u, θ and φ Then substituting equation (13) into equations (9) to (11) and equating the like powers of ε the following equations are obtained:

The zeroth- order equations are:

$$\alpha\,\mathrm{Re}\,u_0'''-u_0''-\mathrm{Re}\,u_0'+\left(M^2+\frac{1}{k}\right)u_0=\mathrm{Re}^2\left(Gr\,\theta_0+Gm\,\phi_0\right) \tag{14}$$

$$\theta_0''+\mathrm{Re}\,\Pr\,\theta_0'+Q\Pr\,\theta_0+Ec\Pr\left(u_0'\right)^2=\alpha\Pr\,\omega Ec\,u_0''u_0' \tag{15}$$

$$\varphi_0''+\mathrm{Re}\,Sc\,\varphi_0'-\gamma Sc\,\varphi_0=0 \tag{16}$$

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The corresponding boundary conditions are

$$u_0 = 0, \theta_0 = 1, \varphi_0 = 1 \quad \text{at} \ y = 0$$
$$u_0 = 0, \theta_0 = 0, \varphi_0 = 0 \ \text{as} \ y \to \infty \tag{17}$$

The first-order equations are:

$$\alpha \operatorname{Re} u_1''' - (1 - i\alpha\omega)u_1'' - \operatorname{Re}(1+\alpha\pi^2)u_1' + \left(\pi^2 + M^2 + \frac{1}{k} - i\omega - i\omega\pi^2\right)u_1 = \operatorname{Re}^2\left(\operatorname{Gr}\theta_1 + \operatorname{Gm}\varphi_1\right) \tag{18}$$

$$\theta_1'' + \operatorname{Re}\Pr\theta_1' - \left(\pi^2 - Q\Pr - i\omega\Pr\right)\theta_1 = -2Ecu_0'u_1' + \alpha\,[i\omega Ecu_0'u_1' + \omega Ec\,(u_0''u_1' + u_1''u_0')] \tag{19}$$

$$\varphi_1'' + \operatorname{Re}\operatorname{Sc}\varphi_1' - (\pi^2 + \gamma\operatorname{Sc} - i\omega\operatorname{Sc})\varphi_1 = 0 \tag{20}$$

where prime denotes differentiation with respect to y.
The corresponding boundary conditions are

$$u_1 = 0,\ \theta_1 = 1,\ \varphi_1 = 0 \quad \text{at} \ y = 0,$$
$$u_1 = 0,\ \theta_1 = 0,\ \varphi_1 = 0 \quad \text{as} \ y \to \infty. \tag{21}$$

The equations (16) and (20) are ordinary second order differential equations and solved under the boundary conditions given in (17) and (21), respectively. Hence the expressions of $\varphi_0(y)$ and $\varphi_1(y)$ are given by

$$\varphi_0(y) = e^{-m_1 y} \tag{22}$$
$$\varphi_1(y) = 0 \tag{23}$$

Since equations (14), (15), (18), and (19) are coupled differential equations, approximate solution is obtained for small values of Ec as the Eckert number is small for incompressible fluid flows. Hence assuming

$$F_0 = F_{00} + EcF_{01} + o(Ec^2); \qquad F_1 = F_{10} + EcF_{11} + o(Ec^2) \tag{24}$$

where F stands for u and θ. Using (24) in (14), (15), (18) and (19), and equating like powers of Ec the following equations are obtained:

- The zeroth-order equations are:

$$\alpha \operatorname{Re} u_{00}''' - u_{00}'' - \operatorname{Re} u_{00}' + \left(M^2 + \frac{1}{k}\right)u_{00} = \operatorname{Re}^2\left(Gr\theta_{00} + Gme^{-m_1 y}\right) \tag{25}$$

$$\alpha \operatorname{Re} u_{10}''' - (1 - i\alpha\omega)u_{10}'' - \operatorname{Re}(1+\alpha\pi^2)u_{10}' + (\pi^2 + M^2 + \frac{1}{k} - i\omega - i\alpha\omega\pi^2)u_{10} = \operatorname{Re}^2 Gr\theta_{10} \tag{26}$$

$$\theta_{00}'' + \operatorname{Re}\Pr\theta_{00}' + Q\Pr\theta_{00} = 0 \tag{27}$$

$$\theta_{10}'' + \operatorname{Re}\Pr\theta_{10}' - (\pi^2 - Q\Pr - i\omega\Pr)\theta_{10} = 0 \tag{28}$$

- The corresponding boundary conditions are

$$u_{00} = 0,\ \ \theta_{00} = 1,\ \ u_{10} = 0,\ \theta_{10} = 0 \ \text{at} \ y = 0,$$
$$u_{00} = 0,\ \ \theta_{00} = 0,\ \ u_{10} = 0,\ \theta_{10} = 0 \ \text{as} \ y \to \infty. \tag{29}$$

- The first-order equations are:

$$\alpha \operatorname{Re} u_{01}''' - u_{01}'' - \operatorname{Re} u_{01}' + \left(M^2 + \frac{1}{k}\right)u_{01} = \operatorname{Re}^2 Gr\theta_{01} \tag{30}$$

$$\alpha \operatorname{Re} u_{11}''' - (1 - i\alpha\omega)u_{11}'' - \operatorname{Re}(1+\alpha\pi^2)u_{11}' + \left(\pi^2 + M^2 + \frac{1}{k} - i\omega - i\alpha\omega\pi^2\right)u_{11} = \operatorname{Re}^2 \operatorname{Gr}\theta_{11} \tag{31}$$

$$\theta_{01}'' + \operatorname{Re}\Pr\theta_{01}' + Q\Pr\theta_{01} = -\Pr u_{00}'^2 + \alpha\Pr\omega u_{00}''u_{00}' \tag{32}$$

$$\theta_{11}'' + \operatorname{Re}\Pr\theta_{11}' - (\pi^2 - Q\Pr - i\omega\Pr)\theta_{11} = -2u_{00}'u_{10}' + [i\omega u_{00}'u_{10}' + \omega u_{00}''u_{10}' + u_{10}''u_{00}'] \tag{33}$$

The corresponding boundary conditions are

$$u_{01} = 0,\ \ \theta_{01} = 0,\ \ u_{11} = 0,\ \theta_{11} = 0 \ \text{at} \ y = 0,$$
$$u_{01} = 0,\ \ \theta_{01} = 0,\ \ u_{11} = 0,\ \theta_{11} = 0 \ \text{as} \ y \to \infty \tag{34}$$

Solving (27) and (28) under boundary conditions (29), we get

$$\theta_{00} = e^{-m_2 y} \tag{35}$$

$$\theta_{10} = e^{-m_3 y} \tag{36}$$

The equations (25), (26), (30), (31), (32) and (33) are still coupled and non-linear. The approximate solution is obtained by perturbation technique for small values of α as for small shear rate $|\alpha| < 1$. Hence assuming

$$u_{00} = u_{000} + \alpha u_{001} \;,\; u_{01} = u_{010} + \alpha u_{011} \;,\; u_{10} = u_{100} + \alpha u_{101} \;,\; u_{11} = u_{110} + \alpha u_{111} \;, \tag{37}$$
$$\theta_{01} = \theta_{010} + \alpha\theta_{011} \;,\; \theta_{11} = \theta_{110} + \alpha\theta_{111} \;,$$

The zeroth-order equations are:

$$u''_{000} + \mathrm{Re}\, u'_{000} - \left(M^2 + \frac{1}{k}\right)u_{000} = -\mathrm{Re}^2\left(Gr\theta_{00} + Gme^{-m_1 y}\right) \tag{38}$$

$$u''_{010} + \mathrm{Re}\, u'_{010} - \left(M^2 + \frac{1}{k}\right)u_{010} = -\mathrm{Re}^2\, Gr\theta_{010} \tag{39}$$

$$u''_{100} + \mathrm{Re}\, u'_{100} - \left(\pi^2 + M^2 + \frac{1}{k} - i\omega\right)u_{100} = -\mathrm{Re}^2\, Gr\theta_{10} \tag{40}$$

$$u''_{110} + \mathrm{Re}\, u'_{110} - \left(\pi^2 + M^2 + \frac{1}{k} - i\omega\right)u_{110} = -\mathrm{Re}^2\, Gr\theta_{110} \tag{41}$$

$$\theta''_{010} + \mathrm{Re}\,\mathrm{Pr}\,\theta'_{010} + Q\,\mathrm{Pr}\,\theta_{010} = -\mathrm{Pr}\, u'^2_{000} \tag{42}$$

$$\theta''_{110} + \mathrm{Re}\,\mathrm{Pr}\,\theta'_{110} + (\pi^2 - Q\,\mathrm{Pr} - i\omega\,\mathrm{Pr})\theta_{110} = -2u'_{000}u'_{100} \tag{43}$$

The corresponding boundary conditions are

$$u_{000} = 0,\; u_{010} = 0,\; u_{100} = 0,\; u_{110} = 0,\; \theta_{010} = 0,\; \theta_{110} = 0 \quad \text{at } y = 0,$$
$$u_{000} = 0,\; u_{010} = 0,\; u_{100} = 0,\; u_{110} = 0,\; \theta_{010} = 0,\; \theta_{110} = 0 \text{ as } y \to \infty. \tag{44}$$

The first-order equations are:

$$u''_{001} + \mathrm{Re}\, u'_{001} - \left(M + \frac{1}{k^2}\right)u_{001} = \mathrm{Re}\, u'''_{000} \tag{45}$$

$$u''_{011} + \mathrm{Re}\, u'_{011} - \left(M + \frac{1}{k^2}\right)u_{011} = \mathrm{Re}\, u'''_{010} - \mathrm{Re}^2\, Gr\theta_{011} \tag{46}$$

$$u''_{101} + \mathrm{Re}\, u'_{101} - \left(\pi^2 + M + \frac{1}{k^2} - i\omega\right)u_{101} = i\omega u''_{100} - i\omega\pi^2 u_{100} + \mathrm{Re}\, u'''_{100} - \mathrm{Re}\,\pi^2 u'_{100} \tag{47}$$

$$u''_{111} + \mathrm{Re}\, u'_{111} - \left(\pi^2 + M + \frac{1}{k^2} - i\omega\right)u_{111} = -\mathrm{Re}^2\, Gr\theta_{111} + i\omega u''_{110} - i\omega\pi^2 u_{110} + \mathrm{Re}\, u'''_{110} - \mathrm{Re}\,\pi^2 u'_{110} \tag{48}$$

$$\theta''_{011} + \mathrm{Re}\,\mathrm{Pr}\,\theta'_{011} + Q\,\mathrm{Pr}\,\theta_{011} = -2u'_{000}u'_{001} + \mathrm{Pr}\,\omega u''_{000}u'_{000} \tag{49}$$

$$\theta''_{111} + \mathrm{Re}\,\mathrm{Pr}\,\theta'_{111} - \left(\pi^2 - Q\,\mathrm{Pr} - i\omega\right)\theta_{111} = i\omega u'_{000}u'_{100} + \omega u''_{000} + u''_{100}u'_{000} - 2u'_{000}u'_{101} - 2u'_{001}u'_{100} \tag{50}$$

The corresponding boundary conditions are

$$u_{001} = 0,\; u_{011} = 0,\; u_{101} = 0,\; u_{111} = 0,\; \theta_{011} = 0,\; \theta_{111} = 0 \quad \text{at } y = 0,$$
$$u_{001} = 0,\; u_{011} = 0,\; u_{101} = 0,\; u_{111} = 0,\; \theta_{011} = 0,\; \theta_{111} = 0 \text{ as } y \to \infty. \tag{51}$$

The solutions of the equations (38)-(43), (45)-(50) are obtained under the boundary conditions (44) and (51) respectively, but not presented here due to brevity.

From the equation (13), the expressions for velocity and temperature profiles are given by

$$f(y,z,t) = f_0(y) + \varepsilon\left[f_r \cos(\pi z - t) - f_i \sin(\pi z - t)\right] \tag{52}$$

where $f_1 = f_r + if_i$

Hence the expressions for the transient velocity and transient temperature profiles from (52) for z = 0 and $t = \frac{\pi}{2}$ can be written as

$$f\left(y, 0, \frac{\pi}{2}\right) = f_0(y) + \varepsilon f_1 \tag{53}$$

4. RESULTS AND DISCUSSION

The purpose of this study is to bring out the effects of the viscoelastic parameter α on the governing flow. The effects of viscoelastic parameter on the transient velocity, transient temperature, co-efficient of skin friction and Nusselt number are computed for different values of

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the parameters involved in the problem. The corresponding results for Newtonian fluid can be deduced from the above results by setting α=0, and it is worth mentioning here that these results coincide with that of Singh and Kumar (2010). The real parts of the results are considered throughout for numerical validation.

Figures 1-5 and 6-10 depicts the variation of transient velocity and transient temperature to observe the viscoelastic effects for $\mathrm{Re} = 2$, $\mathrm{Pr} = 5$, $\mathrm{Sc} = 0.6$, $Q = 0.5$, $\mathrm{Ec} = 0.01$, $\omega = 5$, $\gamma = 0.2$, $\varepsilon = 0.2$, $t = \frac{\pi}{2}$, $z = 0$ with various sets of values of Grashof number for mass transfer (Gm), Grashof number for heat transfer (Gr), permeability parameter (k), and magnetic field parameter (M). It is evident from the figures 1-5 that the transient velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach a zero value.

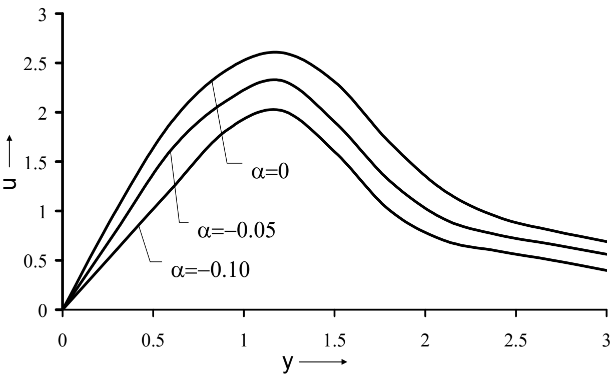

Figure 1: Variation of transient velocity against y for Gm = 5, Gr = 5, k = 0.5, M = 1

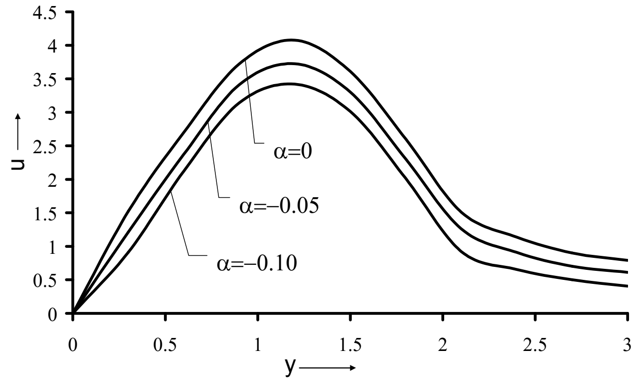

Figure 2: Variation of transient velocity against y for Gm = 10, Gr = 5, k = 0.5, M = 1

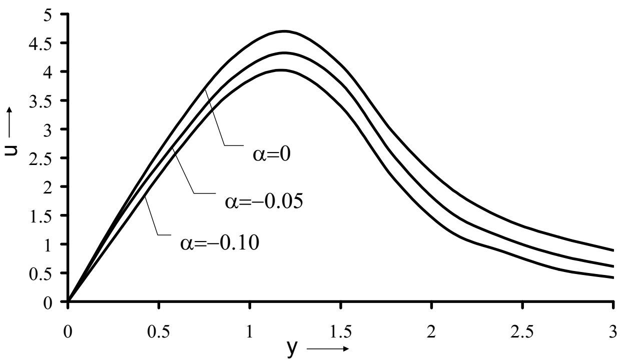

Figure 3: Variation of transient velocity against y for Gm = 5, Gr = 105, k = 0.5, M = 1

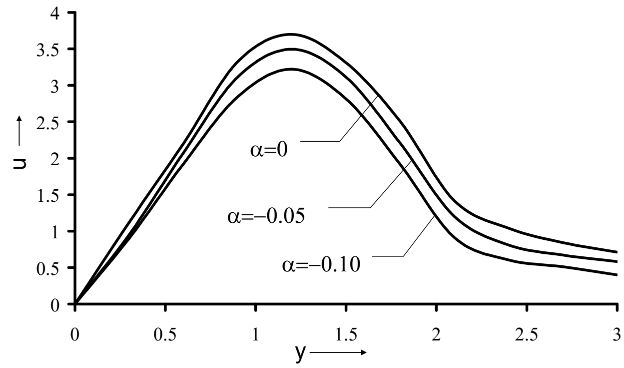

Figure 4: Variation of transient velocity against y for Gm = 5, Gr = 5, k = 1, M = 1

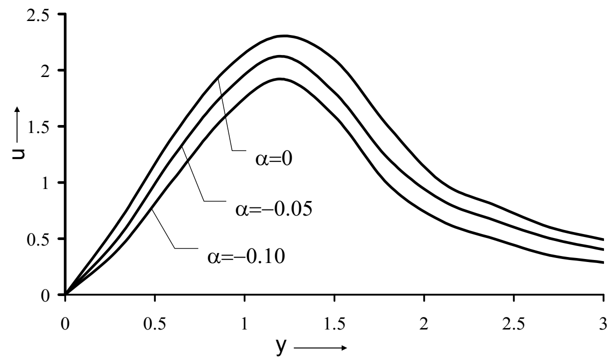

Figure 5: Variation of transient velocity against y for Gm = 5, Gr = 5, k = 0.5, M = 2

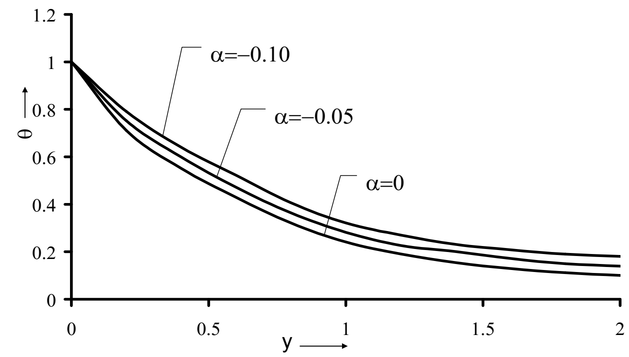

Figure 6: Variation of transient temperature against y for Gm = 5, Gr = 5, k = 0.5, M = 1

From the figures 1-5, it is observed that the transient velocity decrease with increasing values of the visco-elastic parameter $|\alpha|$, $(0, -0.05, -0.1)$ in comparison with Newtonian fluid. But the transient temperature increase (Figures 6-10) with increasing values of the visco-elastic parameter $|\alpha|$, $(0, -0.05, -0.1)$ in comparison with Newtonian fluid.

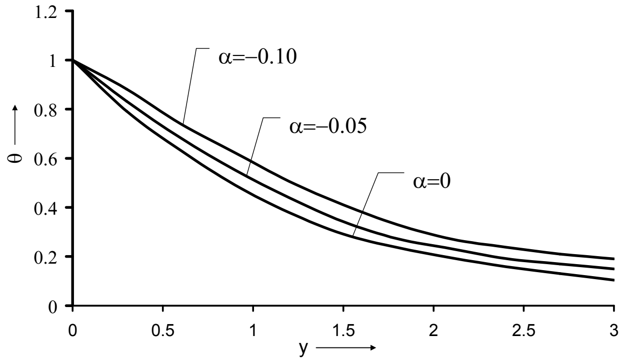

Figure 7: Variation of transient temperature against y for Gm = 10, Gr = 5, k = 0.5, M = 1

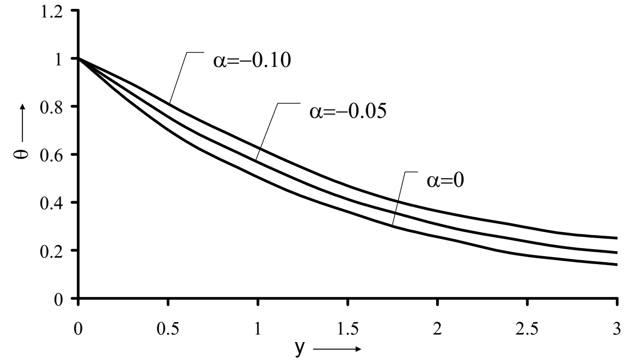

Figure 8: Variation of transient temperature against y for Gm = 5, Gr = 10, k = 0.5, M = 1

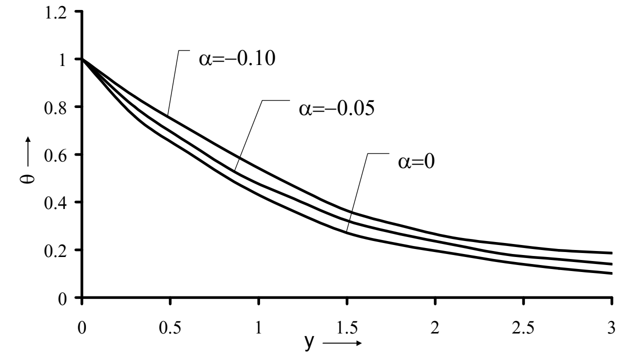

Figure 9: Variation of transient temperature against y for Gm = 5, Gr = 5, k = 1, M = 1

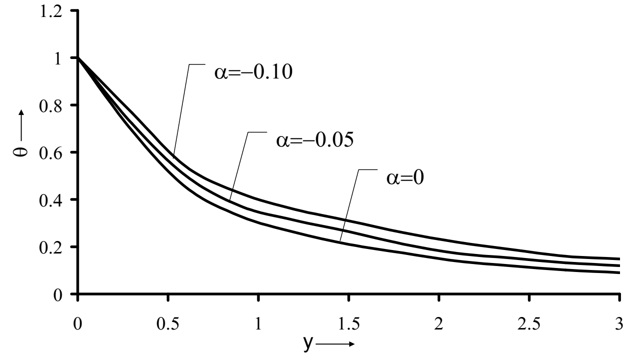

Figure10: Variation of transient temperature against y for Gm = 5, Gr = 5, k = 0.5, M = 2

It is noticed that transient velocity and transient temperature increase with the increase of Grashof number for mass transfer (Figures 1 and 2; Figures 6 and 7), Grashof number for heat transfer (Figures 1 and 3; Figures 6 and 8) and permeability parameter (Figures 1 and 4; Figures 6 and 9) for both Newtonian and non-Newtonian cases. But the figures 1, 5 and 6, 10 show the transient fluid velocity and transient temperature decreases with the increase of magnetic field for both Newtonian and non-Newtonian fluid cases.

The expression for the skin friction coefficient C_{fx} at the plate in the x* - direction is given by

$$C_{fx} = \left[\frac{\partial u}{\partial y} + \alpha \left(\omega \frac{\partial^2 u}{\partial y \partial t} - \mathrm{Re} \frac{\partial^2 u}{\partial y^2} \right) \right]_{y=0}$$

The rate of heat transfer coefficient in terms of the Nusselt number Nu at the plate is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y} \right)_{y=0}$$

The effects of mass Grashof number, heat Grashof number, porous parameter and magnetic field parameter on the skin friction co-efficient and Nusselt number are shown in Table 1. It is observed that the values of skin friction co-efficient and Nusselt number increases as the values of viscoelastic parameter $|\alpha|$, $(\alpha = 0, -0.05, -0.1)$ increase in comparison with Newtonian fluid. It is also observed that the values of skin friction coefficient increases whereas the values of Nusselt number decreases for increasing values of Gm (cases I, II, III and

Table 1: Values of skin friction coefficient at the plate with $\mathrm{Re} = 2$, $\mathrm{Pr} = 5$, $\mathrm{Sc} = 0.6$, $\mathrm{Q} = 0.5$, $\mathrm{Ec} = 0.01$, $\omega = 5$, $\gamma = 0.2$, $\varepsilon = 0.2$, $t = \frac{\pi}{2}$, $z = 0$.

Cases	*Gm*	*Gr*	*k*	*M*	*α*	C_{fx}	*Nu*
I	5	5	0.5	1	0	19.7204	0.6945
II	5	5	0.5	1	– 0.05	20.9205	0.7146
III	5	5	0.5	1	– 0.10	21.7840	0.7231
IV	10	5	0.5	1	0	29.0120	0.3678
V	10	5	0.5	1	– 0.05	31.1124	0.3864
VI	10	5	0.5	1	– 0.10	33.2318	0.3975
VII	5	10	0.5	1	0	31.8204	0.2894
VIII	5	10	0.5	1	– 0.05	33.1224	0.3145
IX	5	10	0.5	1	– 0.10	35.2206	0.3542
X	5	5	1	1	0	22.6228	0.5782
XI	5	5	1	1	– 0.05	24.1228	0.5945
XII	5	5	1	1	– 0.10	26.0214	0.6122
XIII	5	5	0.5	2	0	15.8801	0.8764
XIV	5	5	0.5	2	– 0.05	17.7204	0.8962
XV	5	5	0.5	2	– 010	19.0212	0.9122

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IV, V, VI), Gr (cases I, II, III and VII, VIII, IX) and k (cases I, II, III and X, XI, XII). But the values of skin friction coefficient decreases and Nusselt number increases with the increase of M (cases I, II, III and XIII, XIV, XV) for both Newtonian and non-Newtonian cases.

5. CONCLUSION

The above study brings out the following results of physical interest:

(i) The velocity decrease with the increasing values of the viscoelastic parameter.

(ii) The temperature increase with the increasing values of the viscoelastic parameter.

(iii) The velocity, temperature, skin friction coefficient increase and Nusselt number decrease with the increase of Grashof number for mass transfer, Grashof number for heat transfer and permeability parameter for both Newtonian and non-Newtonian fluid. However, magnetic fields have opposite effect on velocity, temperature, skin friction and Nusselt number for both Newtonian and non-Newtonian fluid.

(iv) Skin friction coefficient and Nusselt number increases with the increasing values of the viscoelastic parameter.

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