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ANALYSIS OF FORCES AND REACTION MOMENTS ACTING ON TRACTOR SEMITRAILER AGREGATE ON SLOPED ROADS

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Abstract: Agricultural semi-tractor aggregate is mainly used to transport agricultural products to processing and storage places. Forces and moments acting on the tractor under full braking on slope depend on tractor mass and slope angle on which it moves. Those are influenced by the rolling track profile, its degree of irregularity, weight of the tractor or tractor-semitrailer, tire pressure and a lot of other factors that less or more influence the unit displacement, its braking stability and efficiency. This paper presents an analysis of the forces and reaction moments which act on the tractor aggregate running on slope and dynamics of tractor-semitrailer aggregate itself.

Keywords: Braking, motion, strength

1. INTRODUCTION

Agricultural semi tractor unit is mainly used to transport agricultural products to processing and storage places. Forces and moments acting on the tractor under full braking on slope depend on tractor mass and slope angle on which it moves. Those are influenced by the rolling track profile, its degree of irregularity, weight of the tractor or tractor-semitrailer, tire pressure and a lot of other factors that less or more influence the unit displacement, its braking stability and efficiency [3, 4, 5, 6, 8 and 10].

2. MATERIAL AND METHOD

In figure 1 it is being shown the tractor with the forces and reaction moments acting on it, operating weight of the tractor, being applied to mass centre G_t determined by the coordinates at b_t and h_t .

This force is decomposed into two components: $G_t \sin \alpha$ - parallel to the ground surface and $G_t \cos \alpha$ - normal to the surface. Traction qualities and aggregate stability in work unit depend on the weight of distribution on the tractor shaft [1], [2].

Soil reaction to rolling track profile on tractor wheels, is determined by the equation of moments of forces system against theoretical point of contact between the wheels and the ground. Those determine the rolling resistance force F_f .

$$F_f = f(Z_1 + Z_2) = X_1 + X_2 \quad (1)$$

where: f - rolling resistance coefficient

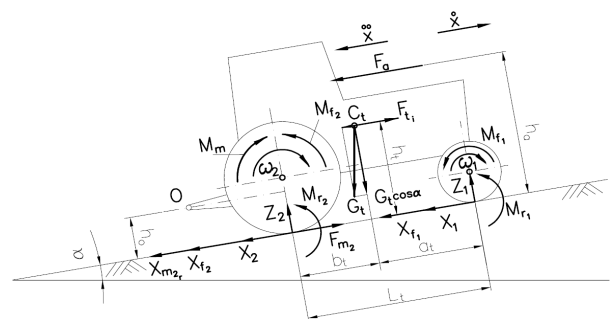


Fig. 1 - The system of forces and moments acting on full braking tractor traveling downhill

For certain greater or less values of tangential reactions on tractor wheels X_1 and X_2 , could result in slip / skid of the wheels according to the coefficient of adhesion (ϕ) [7], [8].

Moment transmitted to the driving wheels, M_m , determines the driving force, F_{m_2} , which is being developed in the interaction with the road surface of driving system:

$$F_{m_2} = \frac{M_m}{r_2} = \frac{M_e i_{tr} \eta_{tr}}{r_2} \quad (2)$$

where: M_e – engine actual moment; i_{tr} – transmission overall ratio; η_{tr} – overall efficiency of transmission; r_2 – dynamic range of the drive wheel; F_{m_2} – tangential traction force.

Moments of rolling resistance (rolling friction moments) M_{r_1} and M_{r_2} , respectively for the front wheels and the rear wheels have the next relations:

$$M_{r_1} = s_1 Z_1 \quad \text{and} \quad M_{r_2} = s_2 Z_2 \quad (3)$$

where: s_1, s_2 - Are the coefficients of rolling friction of the front and rear wheels.

The global resistance force acting on the tractor, implemented by the resistance force of that which acts to the pressure center is expressed by the relation:

$$F_a = \pm K A_F \dot{X}_2 \quad [\text{N}] \quad (4)$$

where: K - coefficient that takes into account the aerodynamic shape of the tractor and air density ($K = 0,5 \div 0,8$); A_F – the front surface of the tractor; \dot{X}_2 - the tractor speed relative to the direction and wind speed.

In general, air resistance can be ignored when wind speed is below the limit of 5.5 m / s (about 20 km / h), the wind having a low value [1], [2], [11].

The force because of inertia resistance F_i , represents the resultant of the following forces of inertia:

□ inertia force of the tractor mass:

$$F_i = \frac{G}{g} \cdot \frac{d\dot{x}}{dt} \quad (5)$$

□ inertia force of bodies in rotation with non-connected in terms kinematics to transmission (e.g. driven wheels):

$$M_{i_1} = 2J_1 \frac{d\omega_1}{dt}; \quad F_{i_1} = \frac{2J_1}{r_1} \cdot \frac{d\omega_1}{dt} \quad (6)$$

where: M_{i_1} - inertia couple for starting driven wheels; J_1 - inertia moment of the driven wheel relative to the axis of rotation; ω_1 - the angular speed of the driven wheels; r_1 - dynamic range of the driven wheels; F_{i_1} - inertia force of transmission parts and other parts that are kinematic connected to it (wheel and other parts interposed between the crankshaft of the engine and the drive wheels).

$$M_{i_2} = 2J_m \frac{d\omega_2}{dt} \Rightarrow F_{i_2} = \frac{2J_m}{r_2} \cdot \frac{d\omega_2}{dt} \quad (7)$$

where: M_{i_2} - inertia couple for starting driven wheels; J_m - inertia moment reduced to the driven wheels; ω_2 - wheel angular speed; r_2 - wheel dynamic range; F_{i_2} - wheel inertia force.

$$M_{i_v} = J_M \frac{d\omega}{dt} \Rightarrow F_{i_v} = \frac{J_M i_{tr} \eta_{tr}}{r_2} \cdot \frac{d\omega}{dt} \quad (8)$$

where: M_{i_v} - inertia couple for starting flywheel and other engine parts in rotation and translation; J_M - inertia moment of the moving masses reduced to engine crankshaft; ω - crankshaft angular speed; i_{tr} - the transmission ratio; η - overall efficiency of the transmission; F_{i_v} - the developed inertia force by M_{i_v} .

$$M_{i_x} = J_x \frac{d\omega_x}{dt} \Rightarrow F_{i_x} = \frac{J_x i_{tr} \eta_{tr}}{r_2} \cdot \frac{d\omega_x}{dt} \quad (9)$$

where: M_{i_x} - inertia couple for starting the transmission of a particular part, placed between the engine and wheels; J_x - part inertia moment; ω_x - part angular speed; F_{i_x} - inertia force developed by couple M_{i_x} .

Resultant expression of inertia force is:

$$F_i^R = F_i + F_{i_1} + F_{i_2} + F_{i_v} + \sum F_{i_x} \quad (10)$$

For the power-train engine-transmission-wheel drive can write a series of equality:

$$\frac{d\omega}{dt} = \frac{d(\omega_m \cdot i_{tr})}{dt} = \frac{i_{tr} \cdot d\omega_m}{dt} = \frac{i_{tr}}{r_2} \cdot \frac{dx}{dt} \quad (11)$$

and, by analogy:

$$\frac{d\omega}{dt} = \frac{i_x}{r_2} \cdot \frac{dx}{dt} \quad (12)$$

Making the appropriate substitutions in relation (10) and taking into account relations (11) and (12), [1], [2], [11] follows:

$$F_i^R = m + \frac{dx}{dt} \left\{ 1 + \frac{1}{m} \left[\frac{1}{r_2^2} (J_M i_{tr}^2 \eta_{tr} + \sum J_x i_x \eta_x + 2J_m) + \frac{2J_1}{r_1^2} \right] \right\}$$

Because it is being considered acceleration-deceleration conditions, the parentheses define the concept of coefficient of the masses in rotation:

$$\delta_{rot} = 1 + \frac{1}{m} \left[\frac{1}{r_2^2} (J_M i_{tr}^2 \eta_{tr} + \sum J_x i_x \eta_x + 2J_m) + \frac{2J_1}{r_1^2} \right] \quad (13)$$

Therefore, the resultant forces of inertia are expressed:

$$F_i^R = \pm \left(m \frac{dx}{dt} \delta_{rot} \right) \quad (14)$$

- Under full braking conditions the tractor must have set the following forces and moments [9].
- The braking moment applied to the front wheels M_{f_1} which determines the braking total force X_{f_1} :

$$X_{f_1} = \frac{M_{f_1}}{r_1} \quad (15)$$

- The braking moment applied to the rear wheels M_{f_2} which determines the braking total force X_{f_2} :

$$X_{f_2} = \frac{M_{f_2}}{r_2} \quad (16)$$

- Reduced resistance force from the engine to the rear wheel X_{m_b} can be produced only in the event that no clutch brake appears.

The forces and moments acting on the integral braking tractor and which is reduced at the point O of coupling trailer is summarized in Figure 2.

The forces and moments at the point O are reduced to a torso which is denoted in formula by $\tau_{t_o}(F_i, M_i)$, which belongs to the tractor having reducing elements formed from the resultant \vec{R}_i and the resulting moment \vec{M}_{t_o} in which the forces are contained in the plane XOY, and the moments are perpendicular to the respective plane [1], [2] as follows:

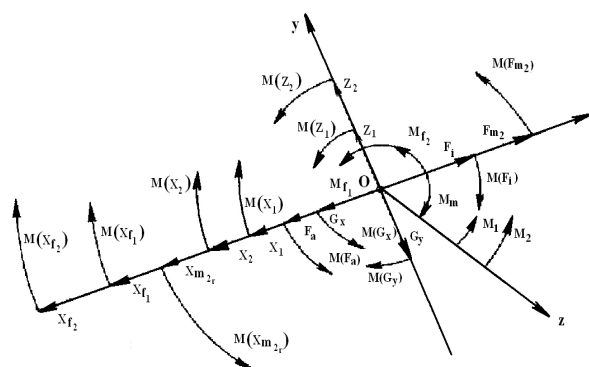


Fig. 2 - The system of forces and moments acting on the tractor with full braking on slope and reduced to point O of coupling with semi trailer [11]

$$R_{ix} = \pm F_i + F_{m_2} - G_x - F_a - X_1 - X_2 - X_{m_{2r}} - X_{f_1} - X_{f_2} \quad (17)$$

$$R = Z_1 + Z_2 - G_y \quad (18)$$

$$\vec{R}_i = R_{ix} \vec{i} + R_{iy} \vec{j} \quad (19)$$

$$M_{iz} = M_m \pm M(F_i) + M(G_y) + M(X_1) + M(X_2) + M(X_{f_1}) + M(X_{f_2}) - M_{f_2} - M(F_{m_2}) - M_{f_1} - M(Z_1) - M(Z_2) - M(G_x) - M(F_a) - M(X_{m_{2r}}) \quad (20)$$

So the torsion low point of forces and moments O , is:

$$\tau_{io}(F_i, M_i) = \left\{ \begin{array}{l} \vec{R}_i = R_{ix} \vec{i} + R_{iy} \vec{j} \\ \vec{M}_{io} = M_{iz} \vec{k} \end{array} \right\} \quad (21)$$

From the torsion element analysis it is deduced that the resultant \vec{R}_i and the resultant moment \vec{M}_{io} are orthogonal because the system of forces is acting in the plane XOY .

Taking into account the road conditions and other random factors, [1], [2], [11], it can lead to a torso with non-orthogonal elements, meaning a certain random torsion with elements:

$$\tau_{io}(F_i, M_i) = \frac{\vec{R}_i}{\vec{M}_{io}} = \frac{R_{ix} \cdot \vec{i} + R_{iy} \cdot \vec{j} + R_{iz} \cdot \vec{k}}{M_{ix} \cdot \vec{i} + M_{iy} \cdot \vec{j} + M_{iz} \cdot \vec{k}} \quad (22)$$

3. RESULTS AND DISCUSSIONS

Forces and moments acting on the semi-trailer in slope driving conditions

In order to determine the resultants \vec{R}_r and \vec{M}_{r_0} which define the torso of reduction system of forces and moments acting on the semi-trailer in relation to the point O the coupling to the tractor, are determined the forces and moments acting on it (fig. 3), where the forces and moments are shown in the figure are:

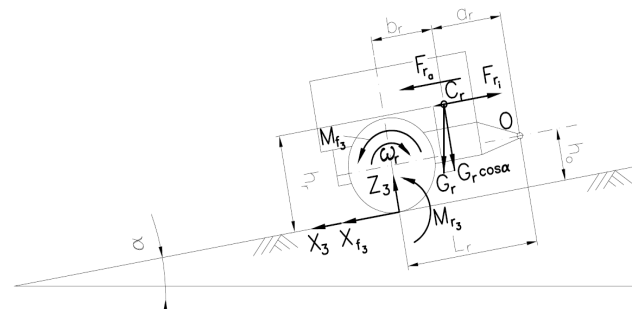


Fig. 3 - The system of forces and moments acting on the trailer

- G_r - operating weight of the semitrailer applied to the center of mass defined by coordinates a_r , b_r and h_r . This force is decomposed into two separate components: $G_r \sin \alpha$ - parallel to the rolling surface, and $G_r \cos \alpha$ - normal to the surface of the rolling paths.
- the rolling resistance moment (rolling frictional moment), M_{r_3} , for the driving system of the trailer, having the next expression:

$$M_{r_3} = s_3 \cdot Z_3 \quad (23)$$

where: Z_3 – reaction of rolling path on the running system of the trailer; s_3 – friction coefficient when rolling.

- The overall force of resistance, F_{r_a} , which acts on the trailer by air resistance;
- The inertia force, F_{r_i} , which includes the inertia force resulting from the translational movement and the inertia force of mass of trailer in rotation movement. These forces are:

- inertia force of semi-trailer mass:

$$F_{r_i} = \frac{G_r}{g} \ddot{x} \quad (24)$$

- the inertia force of the moving rotation parts:

$$M_{r_{3i}} = 2 \frac{J_r \cdot d\omega_3}{dt} \Rightarrow F_{r_{3i}} = \frac{2J_r \cdot d\omega_3}{r_3 dt} \quad (25)$$

- total inertia force of the semi-trailer:

$$F_r^i = \frac{G_r}{g} \ddot{x} + \frac{2J_r \cdot d\omega_3}{r_3 dt} = \frac{G_r}{g} \ddot{x} + \frac{J_r \cdot \ddot{x}}{r_3^2} = \frac{G_r}{g} \ddot{x} \left(1 + \frac{J_r}{r_3^2} \right) = \frac{G_r}{g} \ddot{x} \delta_s = m_r \ddot{x} \delta_s \quad (26)$$

Under the braking of the semitrailer -tractor unit the following forces and moments are highlighted [1], [2], [11]:

- the braking moment applied to the semi-tractor wheels, M_{f_3} , which determines the total braking force, X_{f_3} ;
- tangential reaction on the wheel of trailer X_3 ;

$$X_3 = f \cdot Z_3 \quad (27)$$

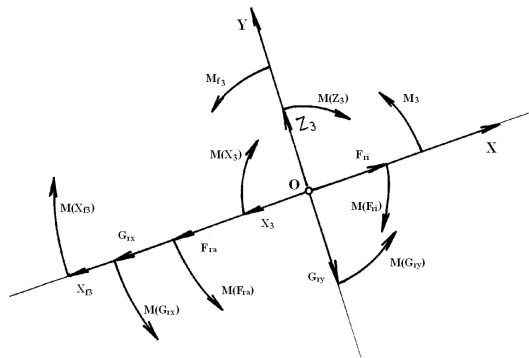


Fig. 4 - The forces and moments acting on the semi-trailer traveling on slope is reduced in the low point O that connects to the tractor [11]

The system of forces and moments that act on the semi-trailer reduced in point O of tractor coupling is summarized in the figure 4.

The system of forces and moments at the point O is reduced to a torso which is denoted by $\tau_{r_o}(F_{ri}, M_{ri})$ which belongs to the semi-trailer, having elements formed from the resultant reduction, \vec{R}_r , and the resultant moment, \vec{M}_{r_o} , in which the forces are contained in the plane XOY, and the moments are perpendicular to the respective plan, thereby:

$$R_{rx} = \pm F_{ri} - G_{rx} - F_{ra} - X_3 - X_{f_3} \quad (28)$$

$$R_{ry} = Z_3 - G_{ry} \quad (29)$$

$$\vec{R}_r = R_{rx} \vec{i} + R_{ry} \vec{j} \quad (30)$$

$$M_{rz} = \pm M(F_{ri}) + M(Z_3) + M(X_3) + M(X_{f_3}) - M_3 - M(G_{rx}) - M_{f_3} - M(F_{ra}) - M(G_{ry}) \quad (31)$$

The torso of forces and moments reduced in O, is:

$$\tau_{r_o}(F_i, M_i) = \begin{cases} \vec{R}_r = R_{rx} \vec{i} + R_{ry} \vec{j} \\ \vec{M}_{r_o} = M_{rz} \vec{k} \end{cases} \quad (32)$$

From the torso elemental analysis, τ_{r_o} , it results that the resultant, \vec{R}_r , and the resultant moment, \vec{M}_{r_o} , are orthogonal because the system of forces is considered to act in the plane XOY.

In reality, semi-trailer-tractor unit is moving on bumpy roads and under the action of random factors, so in these conditions, the system of forces and moments define a random torso for certain elements.

$$\mathcal{C} \tau_{r_o}(F_i, M_i) = \begin{Bmatrix} \vec{R}_r \\ \vec{M}_{r_o} \end{Bmatrix} \quad (33)$$

The resultant, \vec{R}_r , determines the coupling reactions of the semi-trailer and tractor (studied in section), and the resultant moment, \vec{M}_{r_o} , determines the degree of semitrailer mobility in relation with the tractor $\omega_x, \omega_y, \omega_z$.

Dynamics of tractor semi - trailer aggregate

The forces and moments referring to each component of the unit consisting of the tractor and semi-trailer are being shown schematically in Figure 5. The action of semi-trailer on tractor through device / connection coupler is being represented by the resultant, \vec{R}_r , of all forces acting on trailer.

If the system of forces is contained in the plane XOY then the resultant \vec{R}_r is being decomposed in two parts:

$$\vec{R}_r = \vec{F}_x + \vec{F}_y \quad (34)$$

where: F_x – represents the traction force (F_t); F_y – is the pressing force on the tractor, which represents the empty part out of weight part G_r of semi-trailer on hitch .

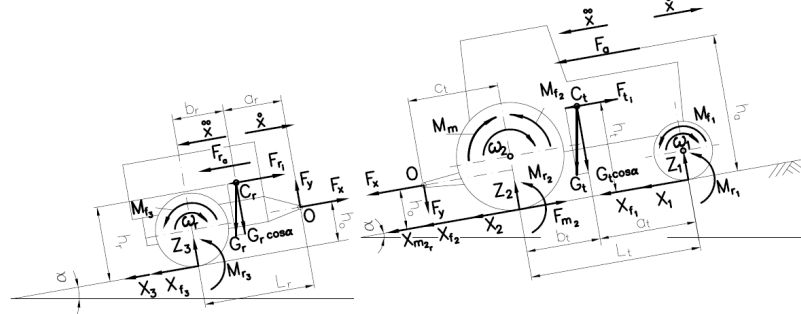


Fig. 5 – The forces and moments acting on the tractor semi-trailer unit

If the system is reduced to a resultant force oriented in space, then the resultant, \vec{R}_r , is being decomposed in three components:

$$\vec{R}_r = \vec{F}_x + \vec{F}_y + \vec{F}_z \quad (35)$$

where: F_z – represents the component which is perpendicular to the plane XOY and is determined by lateral forces that occur in the horizontal plane (e.g. when aggregate moves on level curve and when traveling on bumpy roads).

The component F_x , after the direction of movement of the unit has the expression:

$$F_x = \pm F_{ri} + X_3 + G_{rx} + F_{ra} + X_{f3} \quad (36)$$

where: X_3 – the rolling resistance force; G_{rx} - weight composition of the trailer depending on the direction parallel to the road surface; F_{ra} - air force resistance; X_{f3} – brake force resistance; F_{ri} – inertia force.

If is considered the constant travel speed in non braking displacement and the influence of air resistance is negligible [1], [2], [11], then the component, F_x , will have the next expression:

$$F_x = X_3 + G_{rx} = f \cdot Z_3 + G_r \sin \alpha \quad (37)$$

Which is a traction force, in which Z_3 moments are determined by the equation with relation to the coupling point O.

In the specified conditions (constant speed, air resistance force is negligible) it results:

$$Z_3 = \frac{G_r(a_r \cos \alpha + k_r \sin \alpha)}{a_r + b_r + fh_o} \quad (38)$$

For the components from the relationship (37), significance of forces is:

$$X_3 = f \cdot Z_3 \quad (39)$$

$$G_{rx} = G_r \sin \alpha \quad (40)$$

$$F_{ra} = \pm KA_f \cdot X^2 \quad (41)$$

$$X_{f3} = \frac{M_{r3}}{r_3} \quad (42)$$

$$F_{ri} = m_r \frac{d\dot{x}}{dt} \left[1 + \frac{1}{m_r} \left(\frac{2J_3}{r_3^2} \right) \right] = m_r \frac{d\dot{x}}{dt} \delta_s \quad (43)$$

The component F_x which represents the required traction force for towing a trailer is calculated with:

$$F_x(F_t) = f \cdot Z_3 + G_r \sin \alpha \pm KA_f X^2 + \frac{M_{f3}}{r_3} \pm \frac{G_r}{g} \cdot \frac{d\dot{x}}{dt} \delta_s \quad (44)$$

Under complex conditions of unit movement, component Z_3 has the next expression:

$$Z_3 = \frac{G_r \cos \alpha \cdot a_r + G_r \sin \alpha \cdot h_r \pm F_{ri} \cdot h_r + F_{ra} \cdot h_r - \frac{M_{r3}}{r_3} h_o}{a_r + b_r + fh_o} \quad (45)$$

If it customizes the unit displacement conditions, then the relation (36) reduces to (37). Substituting expression of Z_3 component in (44) it gives traction force for hauling a semitrailer.

The F_y component, in the perpendicular direction to the travel direction of the unit is expressed as:

$$F_y = G_r \cos \alpha - Z_3 \quad (46)$$

or taking into consideration the relationship (4.45), F_y becomes:

$$F_y = \frac{G_r \cos \alpha (a_r + b_r + fh_o) - G_r \cos \alpha \cdot a_r + G_r \sin \alpha \cdot h_r \pm F_{ri} \cdot h_r + F_{ra} \cdot h_r + \frac{M_{r3}}{r_3} h_o}{a_r + b_r + fh_o} \quad (47)$$

Under conditions of constant speed without neglecting the braking force of air resistance, the force expression F_y is:

$$F_y = \frac{G_r \cos \alpha (a_r + b_r + fh_o) - G_r \cos \alpha \cdot a_r + G_r \sin \alpha \cdot h_r}{a_r + b_r + fh_o} \quad (48)$$

The normal reactions, Z_1 , Z_2 of soil on the tractor wheels are obtained from the equations of moments, in relation to the theoretical contact (A, B):

$$Z_1 = \frac{M_m + G_t \cos \alpha \cdot h_t \pm F_a \cdot h_t - F_{ii} \cdot h_t - M_{r2} - F_y \cdot c_t - M_{f1} - G_t \sin \alpha \cdot h_t - M_{f2} - F_x \cdot h_o}{L_t} \quad (49)$$

$$Z_2 = \frac{F_x \cdot h_o + F_y (c_t + L_t) + M_{f2} - M_m - G_t \cos \alpha \cdot h_t \pm F_{ii} \cdot h_t \pm F_a \cdot h_t + M_{r1} + M_{f1} + G_t \sin \alpha \cdot h_t}{L_t} \quad (50)$$

The determination of these reactions implies discussions on the dynamics of the tractor-semitrailer unit ($Z_1 > 0$), regarding tractor manoeuvring and on increasing the wheels adhesion force which influences traction force of the tractor.

CONCLUSIONS

Under integral braking on the tractor act the following forces and moments:

- The braking moment applied to the front wheels M_{f1} ;
- The braking moment applied to the rear wheels M_{f2} ;
- The resistance force from the engine reduced to the rear wheel (only occurs when braking without declutching).

The resultant \vec{R}_t and the resultant moment \vec{M}_0^t to the system of forces and moments acting on the tractor with full braking on downhill travel reduced to the low point O for trailer couplings are orthogonal because the system of forces is considered to act in the plane XOY .

When the semi-trailer-tractor unit brakes the following forces and moments are acting on it:

- The braking moment applied to the semi-tractor wheels, M_{f3} ;
- The tangential reaction on the trailer wheels X_3 ;

In this case, the resultant, \vec{R}_r , and the resultant moment, \vec{M}_{r0} at the system of forces and moments acting on a trailer moving on slope, reduced in O point for coupling to the tractor are orthogonal because the acting system of forces in the plane XOY , are considered.

The resultant, \vec{R}_r , of all forces that are acting on the semi trailer represents the action of semi-trailer on tractor action through device / connection coupler.

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