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THE EFFECT OF FLUID INERTIA ON THE FILM PRESSURE BETWEEN TWO AXIALLY OSCILLATING PARALLEL CIRCULAR PLATES WITH A VISCO-ELASTIC FLUID AS LUBRICANT

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Abstract: The present theoretical study investigates the effects of elasticity and fluid inertia on the squeeze-film between two parallel circular plates of which the top plate is axially oscillating. The originality of the present study is to use non-Newtonian visco-elastic lubricant (Walters B' model) which is a realistic model representing many liquids such as synovial fluid in human joints and industrial fluids. Flow phenomena are characterized by non-dimensional parameters such as Reynolds number (R_e) and elastic parameter (R_c). It is remarked that the lubricant with low elasticity is favourable in enhancing the load bearing capacity in case of axially oscillating parallel plate type bearing irrespective of fluid inertia.
Keywords: Circular plates, visco-elastic fluid, fluid inertia, pressure

1. INTRODUCTION

In the field of hydrodynamic slider bearings and hydrostatic bearings, in which certain periodic vibrations of a supported load occur, the inertia of the fluid can influence the load capacity. Fuller [1] gave a solution for the pressure in the fluid film between oscillating parallel circular plates without considering the inertia terms. Archibald [2] investigated the behaviour of the squeeze film between various geometrical configurations of flat surfaces.

The effects of fluid inertia between axially oscillating parallel circular plates were studied both experimentally and theoretically by Kuhn and Yates [3]. Prakash and Vij [4] have studied the load carrying capacity and time-height relations for the squeeze film between porous annular plates. Murti [5] discussed the behaviour of squeeze film trapped between curved circular plates describing the film thickness by an expression of exponential form. Gupta and Vora [6] modified and developed the approach of Murti [5] to study the squeeze film behaviour between curved annular plates. Patel and Deheri [7] investigated the problem of Gupta and Vora [6] by considering the lower plate along a surface generated by a hyperbolic form. Patel and Deheri [8] analyzed the behaviour of squeeze film between curved annular plates considering curvature of secant form to represent the film thickness.

It is a well known fact that the bearing surfaces after having some run-in and wear develop roughness. Bujurke and Kudenatti [9] have studied the effects of surface roughness on the squeeze film performance between two rectangular plates with an electrically conducting fluid in the presence of a transverse magnetic field. Recently, Patel et al. [10] analized the magnetic fluid based squeeze film between rotating porous rough circular plates.

Magnetohydrohynamic (MHD) flow of a fluid in squeeze film lubrication prevents the unexpected variation of lubricant viscosity with temperature under severe operating conditions. Hamza [11] has shown the effects of MHD on a fluid film squeezed between two rotating surfaces. Naduvinamani et al. [12] have undertaken a detailed study of magnetic effects in rectangular plates and reported that bearing characteristics such as pressure distribution, load capacity and squeezing time seem to increase for increasing the Hartman number. Bujurke and Naduvinamani [13] have studied the effect of roughness on squeeze film characteristics between two rectangular plates of which the upper plate has roughness structure and the lower plate has a porous material. This study shows that the effect of a roughness is to increase the load capacity of the bearing compared to smooth case.

Kudenatti et al. [14] have presented numerical solution of the MHD Reynolds equation for squeeze film lubrication between porous and rough rectangular plates. They have applied finite difference based multigrid method for the solution of modified Reynolds equation to investigate the combined effects of surface roughness, magnetic field, couple-stress fluid and permeability. Naduvinamani and Marali [15] have studied dynamic Reynolds equation for micropolar fluids and analyzed the case of plane inclined slider bearings with squeezing effect. Siddiqui et al. [16] have studied non-Newtonian fluids, namely a Sisco fluid and an Oldroyd 6-constant fluid on a vertically moving belt. However, most of the above referred studies are related to Newtonian lubricants. But in the present study we have considered the non-Newtonian visco-elastic fluid as lubricant.

The aim of the present study is to generalize the work of Dash and Kamila [17] with visco-elastic fluid Walters B' model [18] as lubricant. Since to the best of the authers knowledge no work has been reported so far on the specific model considered here used as lubricant and the method of calculating pressure vis-a vis predicting load bearing capacity of the lubricant. Evidences of liquid elasticity in an 1.5 percent starch solution can be observed by Hess's experiment [19] and by the recoil of air bubbles in a mixture of polymethyl methacrylate and cyclohexanone (made by dissolving 3 gms Perspex in 100 ml of solvent) contained in a bottle which has been suddenly turned and then brought to rest. When visco-elastic liquid is in flow, a certain amount of energy is stored up in the material as strain energy in addition to viscous dissipation. In an inelastic viscous liquid we are concerned only with the rate of strain, but in elastic liquids we cannot neglect the strain, however small it may be, as it is responsible for the recovery to the original state and for the reverse flow that follows the removal of stress. Only in visco-elastic liquid there is a degree of recovery from the strain when the stress is removed where as in other liquids the whole strain remains. The novelty of the present study is to investigate the effect of elasticity on the bearing characteristics i.e. load bearing capacity, amount of pressure built up inside the bearing set up.

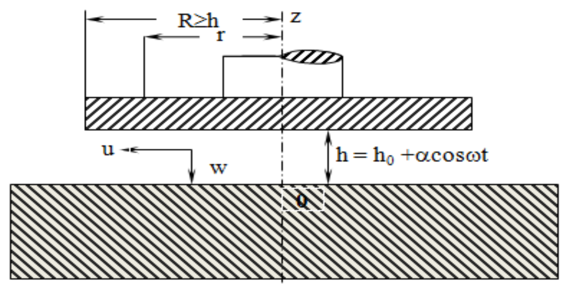

Figure 1. The geometry of bearing

2. ANALYSIS OF THE PROBLEM

We consider a visco-elastic incompressible fluid between two parallel circular plates initially separated by a small distance h_0. There is a sinusoidal axial vibration of amplitude α of the top plate. Cylindrical polar co-ordinates are used to describe the flow phenomena. The velocity components in the radial r and in the axial z directions are assumed to be u and w respectively. The geometry of the problem is shown in Fig.1 and the velocity field can be taken as

$$u = u(r,z,t),\; w = w(z,t) \tag{1}$$

The surviving stress components of Walters B' model [18] in cylindrical polar co-ordinates are

$$p^{rr} = 2\eta_0 \frac{\partial u}{\partial r} - 2\kappa_0 \left[u\frac{\partial^2 u}{\partial r^2} + w\frac{\partial^2 u}{\partial r \partial z} - 2\left\{ \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial z}\right)^2 \right\} + \frac{\partial^2 u}{\partial t \partial r} \right] \tag{2}$$

$$p^{\theta\theta} = 2\eta_0 \frac{u}{r} - 2\kappa_0 \left[\frac{u}{r}\frac{\partial u}{\partial r} + \frac{w}{r}\frac{\partial u}{\partial z} - 3\frac{u^2}{r^2} + \frac{1}{r}\frac{\partial u}{\partial t} \right] \tag{3}$$

$$p^{zz} = 2\eta_0 \frac{\partial w}{\partial z} - 2\kappa_0 \left[w\frac{\partial^2 w}{\partial z^2} - 2\left(\frac{\partial w}{\partial z}\right)^2 + \frac{\partial^2 w}{\partial z \partial t} \right] \tag{4}$$

$$p^{zr} = \eta_0 \frac{\partial u}{\partial z} - \kappa_0 \left[u\frac{\partial^2 u}{\partial r \partial z} + w\frac{\partial^2 u}{\partial z^2} - 2\frac{\partial u}{\partial z}\frac{\partial w}{\partial z} + \frac{u}{r}\frac{\partial u}{\partial z} + 2\frac{\partial^2 u}{\partial z \partial t} \right] \tag{5}$$

The equations of momentum and continuity take the forms

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \frac{\partial p^{rr}}{\partial r} + \frac{\partial p^{rz}}{\partial z} + \frac{p^{rr} - p^{\theta\theta}}{r} \tag{6}$$

$$\rho\left(\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \frac{\partial p^{zz}}{\partial z} + \frac{\partial p^{rz}}{\partial r} + \frac{p^{rz}}{r} \tag{7}$$

$$\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r} = 0 \tag{8}$$

The momentum and continuity equations, with relatively thin film and neglecting the variation of axial velocity in the radial direction and the pressure gradient along z-axis can be written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\eta_0}{\rho}\frac{\partial^2 u}{\partial z^2} - \frac{\kappa_0}{\rho}\left[w\frac{\partial^3 u}{\partial z^3} + 2\frac{\partial^3 u}{\partial z^2 \partial t} + \frac{4u}{r}\frac{\partial^2 u}{\partial z^2} \right] \tag{9}$$

$$\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z} = \frac{\eta_0}{\rho}\frac{\partial^2 w}{\partial z^2} - \frac{\kappa_0}{\rho}\left[w\frac{\partial^3 w}{\partial z^3} - 3\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z^2} \right] \tag{10}$$

$$u = -\frac{r}{2}\frac{\partial w}{\partial z} \tag{11}$$

where η_0 and κ_0 are the coefficients of viscosity and elasticity of the fluid respectively.

3. SOLUTION OF THE EQUATIONS

An iteration technique has been used to solve (9). Following Kahlert [20], equation (9) can be written as

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta_0}\frac{\partial p}{\partial r} + \frac{\rho}{\eta_0} G(r,z,t) \tag{12}$$

where

$$G(r,z,t) = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} + \frac{\kappa_0}{\rho}\left[w\frac{\partial^3 u}{\partial z^3} + 2\frac{\partial^3 u}{\partial z^2 \partial t} + \frac{4u}{r}\frac{\partial^2 u}{\partial z^2} \right] \tag{13}$$

The boundary conditions are:

$$\left.\begin{aligned} u = 0,\ w = 0 \ \text{ at } z = 0 \\ u = 0,\ w = v \text{ at } z = h \end{aligned}\right\} \tag{14}$$

where v is the velocity of the top plate.

The first iterate solution u_1 is obtained by putting $G(r,z,t) = 0$ in (12) and using the boundary conditions (14), we get

$$u_1 = \frac{3vr}{h^3}\left(z^2 - hz\right) \tag{15}$$

$$w_1 = -\frac{v}{h^3}\left(2z^3 - 3hz^2\right) \tag{16}$$

With these values of the velocities, equation (13) becomes

$$G(r,z,t) = 3r\frac{\partial v}{\partial t}\left[\frac{z^2}{h^3} - \frac{z}{h^2} + \frac{4\kappa_0}{\rho h^3} \right] + 3v^2 r\left[\frac{2z^3}{h^5} - \frac{z^4}{h^6} + \frac{6\kappa_0}{\rho}\left(\frac{4z^2}{h^6} - \frac{4z}{h^5} \right) \right] \tag{17}$$

Substituting equation (17) into equation (12) and using (14), the second iterate solution u_2 of the radial velocity is obtained as

$$u_2 = \frac{1}{2\eta_0}\frac{\partial p}{\partial r}\left(z^2 - hz\right) + \frac{3r\rho}{\eta_0}\frac{\partial v}{\partial t}\left[\frac{z^4}{12h^3} - \frac{z^3}{6h^2} + \frac{z}{12} + \frac{2\kappa_0}{\rho}\left(\frac{z^2}{h^3} - \frac{z}{h^2}\right)\right]$$
$$+ \frac{3v^2 r\rho}{\eta_0}\left[\frac{z^5}{10h^5} - \frac{z^6}{30h^6} - \frac{z}{15h} + \frac{6\kappa_0}{\rho}\left(\frac{z^4}{3h^6} - \frac{2}{3}\frac{z^3}{h^5} + \frac{z}{3h^3}\right)\right] \quad (18)$$

If v_0 is the maximum velocity of the top plate then from equations (11) and (18), we get

$$\frac{\partial w_2}{\partial z} = -\frac{1}{\eta_0 r}\frac{\partial p}{\partial r}\left(z^2 - hz\right) - \frac{6v_0^2\rho}{\eta_0}\left[\frac{z^5}{10h^5} - \frac{z^6}{30h^6} - \frac{z}{15h} + \frac{6\kappa_0}{\rho}\left(\frac{z^4}{3h^6} - \frac{2}{3}\frac{z^3}{h^5} + \frac{z}{3h^3}\right)\right] \quad (19)$$

Integration of equation (19) with boundary conditions $w = 0$ at z = 0 and $w = -v_0$ at $z = h = h_0$ gives an expression for the radial pressure gradient at any r at the time of maximum velocity of the top plate.

$$\frac{\partial p}{\partial r} = -\frac{6\eta_0 r v_0}{h_0^3} - \frac{36\eta_0 r v_0^2}{h_0^3}\left(\frac{\rho}{\eta_0}\right)\left[\frac{3h_0}{140} - \frac{2}{5}\frac{\kappa_0}{\rho h_0}\right] \quad (20)$$

Integrating (20) by using boundary condition $p = 0$ at $r = R$ where R is the radius of the upper plate, the pressure at the maximum velocity of the top plate is

$$p^1 = \frac{3\eta_0 v_0}{h_0^3}\left(R^2 - r^2\right)\left[1 + \frac{9}{70}h_0 v_0\left(\frac{\rho}{\eta_0}\right) - \frac{12}{5}\left(\frac{\kappa_0 v_0}{\eta_0 h_0}\right)\right]$$

The non-dimensional form of pressure is

$$p = 3R_e\left(1 - \frac{r^2}{R^2}\right)\left[1 + R_e\left(\frac{9}{70} - \frac{12}{5}R_c\right)\right]$$

where $R_e = \frac{\rho h_0 v_0}{\eta_0}$, Reynolds number, $R_c = \frac{\kappa_0}{\rho h_0^2}$, Elastic parameter and $p = \frac{\rho h_0^4 p^1}{\eta_0^2 R^2}$, non-dimensional pressure.

4. RESULTS AND DISCUSSION

The following discussion reveals the effect of Reynolds number and elastic parameter on pressure variations at the central point. Figure 2 shows the pressure variation at the central point r = 0 of bottom plate for different values of Reynolds number(R_e) and Elastic parameter(R_c).

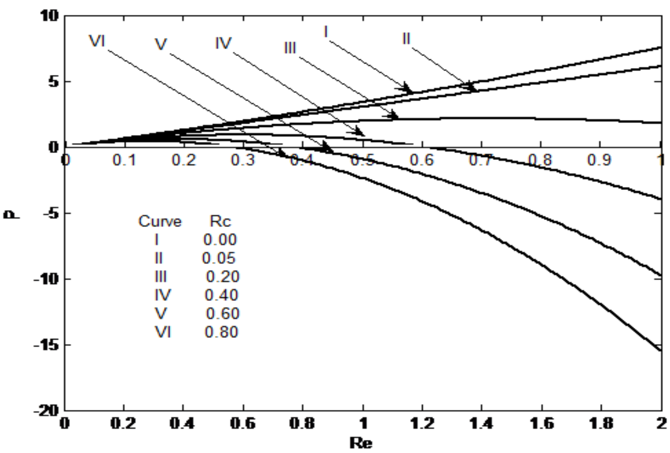

Figure 2. Pressure variation at the central point (r=0) of bottom plate for different values of R_e and R_c

From figure 2 it is observed that pressure decreases with an increase in the value of elastic parameter (R_c). Further, it is interesting to note that pressure remains positive and increases as the Reynolds number increases in case of viscous fluid $(R_c = 0.0)$ as well as visco-elastic fluid $(R_c = 0.05)$. One striking feature of the pressure is marked for higher value of elastic parameter i.e $R_e \geq 0.4$. The elasticity property of the fluid is characterized by the storage of strain energy when the fluid is subject to stress. On the removal of the stress, the stored strain energy is released, that contribute to memory effect. The greater elasticity property contribute to build up the negative pressure for low/higher value of Reynolds number i.e irrespective of dominance of inertia effect on the flow i.e

$R_e > 1$ or $R_e < 1$. Thus, it is concluded that the choice of lubricant should be such that the fluid must posses low elastic property, so that building up of negative pressure may be avoided to enhance the load bearing capacity of the bearing set-up. Further, on careful observation it is revealed that an increase in negative pressure is accelerated with the increase of elastic property of the fluid.

5. CONCLUSIONS

Since higher pressure generation vis-à-vis load bearing capacity is desirable within the bearing set - up, it is inferred that lubricant with high elasticity is counterproductive in case of axially oscillating parallel plate type bearing . This should be a vital point for selection of the lubricant.

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