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[1.] S. MAKHIJA, [2.] A.K. AGGARWAL

EFFECT OF SUSPENDED PARTICLES, ROTATION AND MAGNETIC FIELD ON THERMOSOLUTAL CONVECTION IN RIVLIN-ERICKSEN FLUID WITH VARYING GRAVITY FIELD IN POROUS MEDIUM

[1-2.] Department of Mathematics, Jaypee Institute of Information Technology, Noida (UP), INDIA

Abstract: The thermosolutal convection in Rivlin-Ericksen elastic-viscous fluid in porous medium is considered to include the effect of suspended particles in the presence of uniform magnetic field, uniform rotation and variable gravity field. It is found that for stationary convection, the stable solute gradient has stabilizing effect on the system. Rotation has stabilizing effect as gravity increases upward and destabilizing effect as gravity decreases upward whereas suspended particles have destabilizing effect as gravity increases upward and stabilizing effect as gravity decreases upward. The medium permeability has stabilizing/destabilizing effect depending on the rotation parameter. The magnetic field has a stabilizing effect in the absence of rotation whereas in the presence of rotation it has a stabilizing/destabilizing effect under certain conditions. The principle of exchange of stabilities is satisfied in the absence of magnetic field, rotation and stable solute gradient. The presence of magnetic field, rotation and stable solute gradient introduces oscillatory modes into the system.
Keywords: Thermosolutal convection; suspended particles; magnetic field; rotation; porous medium

1. INTRODUCTION

A detailed account of the theoretical and experimental study of thermal instability (Bénard convection) in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [3]. The use of Boussinesq approximation has been made throughout, which states that the density may be treated as a constant in all the terms in equations of motion except the external force term. Chandra [2] observed a contradiction between the theory and his experiment for the onset of convection in fluids heated from below. Scanlon and Segel [9] studied the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. Sharma [10] has studied the thermal instability of a layer of viscoelastic (Oldroydian) fluid acted on by a uniform rotation and found that rotation has destabilizing as well as stabilizing effects under certain conditions in contrast to that of a Maxwell fluid where it has a destabilizing effect.

The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [17]. In such situation, buoyancy forces can arise not only from density differences due to variations in temperature, but also from those due to variations in solute concentration. The conditions under which convective motions are important in geophysical situations are usually far removed from the consideration of single component fluid and therefore it is desirable to consider a fluid acted on by a solute gradient. Thermosolutal convection problems arise in oceanography, limnology and engineering. Examples of particular interest are provided by ponds built to trap solar heat (Taber and Matz, [16]) and some Antarctic lakes (Shirtcliffe, [14]). The physics is quite similar in the stellar case, in that helium acts like salt in raising the density and in diffusing more slowly than heat.

There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. Two such classes of fluids are Rivlin-Ericksen and Walter's (model B') fluids. Rivlin and Ericksen [8] have proposed a theoretical model for such one class of elastico-viscous fluids. Sharma and Aggarwal [11] have studied the effect of compressibility and suspended particles on thermal convection in a Walters' (model B') elastico-viscous fluid in hydromagnetics.

The medium has been considered to be non-porous in all above studies. In recent years, the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. The stability of the flow through a porous medium taking into account the Darcy resistance was considered by Lapwood [4] and Wooding [18]. When the fluid permeates a porous medium, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous and viscoelastic terms in the equation of motion are replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\mathbf{q}\right]$, where μ and μ' are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid, k_1 is the medium permeability and $\boldsymbol{q}$ is the Darcian (filter) velocity of the fluid. A comprehensive review of the literature concerning thermal convection in a fluid-saturated porous medium may be found in the book by Nield and Bejan [5]. Sharma et al.[12] have studied thermosolutal convection in Rivlin-Ericksen fluid in porous medium in hydromagnetics and Sunil et al. [15] considered thermal convection in porous medium permeated with suspended particles. Effect of rotation on thermosolutal convection in a Rivlin-Ericksen fluid permeated with suspended particles in porous medium is investigated by Aggarwal [1]. Thermal stability of a fluid layer under variable gravitational field heated from below or above is investigated analytically by Pradhan and Samal [6]. Although the gravity field of the earth is varying with height from its surface, we usually neglect this variation for laboratory purposes and treat the field as a constant. However, this may not be the case for large-scale flows in the ocean, the atmosphere or the mantle. It can become imperative to consider gravity as a quantity varying with distance from the centre. Recently, Rana and Kumar [7] have studied the stability of Rivlin-Ericksen elastico-viscous rotating fluid permeating with suspended particles under variable gravity field in porous medium.

In the present paper, we have considered the effect of suspended particles, rotation and magnetic field on thermosolutal convection in Rivlin-Ericksen elastico-viscous fluid in porous medium. Here, we have extended the results reported by Sharma and Rana [13] to include the effect of magnetic field for Rivlin-Ericksen fluid.

2. MATHEMATICAL FORMULATION

Consider an infinite horizontal fluid particle layer of Rivlin-Ericksen elastico-viscous fluid of thickness d bounded by the planes $z=0$ and $z=d$ in porous medium, is acted upon by a uniform rotation $\boldsymbol{\Omega}(0,0,\Omega)$, a uniform magnetic field $\boldsymbol{H}(0,0,H)$ and variable gravity $\boldsymbol{g}(0,0,-g)$, where $g=\lambda g_0, g_0(>0)$ is the value of g at $z=0$ and λ can be positive or negative as gravity increases or decreases upwards from its value g_0 (see figure 1).This layer is heated from below and subjected to stable

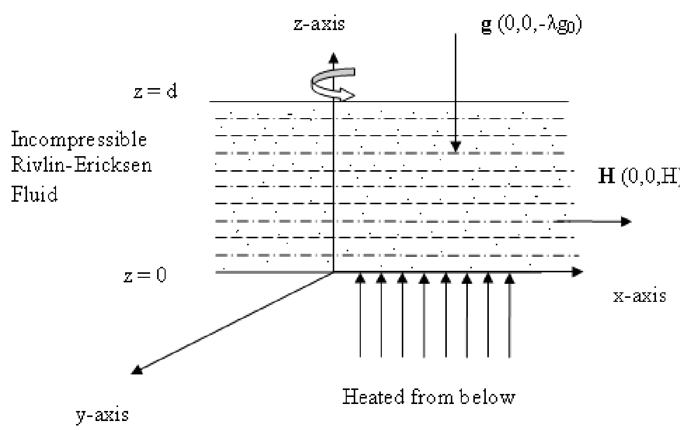

Figure 1: Geometrical Configuration

solute gradient such that a uniform temperature gradients $\beta = \left| dT/dz \right|$ and solute concentration gradients $\beta' = \left| dC/dz \right|$ are maintained across the layer.

Then the equations of motion and continuity governing the flow are

$$\frac{1}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q}\cdot\nabla)\boldsymbol{q}\right] = -\frac{1}{\rho_0}\nabla p + \boldsymbol{g}\left(1+\frac{\delta\rho}{\rho_0}\right) + \frac{KN}{\varepsilon\rho_0}(\boldsymbol{q}_{\mathbf{d}} - \boldsymbol{q}) - \frac{1}{k_1}\left(\upsilon + \upsilon'\frac{\partial}{\partial t}\right)\boldsymbol{q} \tag{1}$$

$$+\frac{2}{\varepsilon}(\boldsymbol{q}\times\boldsymbol{\Omega}) + \frac{\mu_e}{4\pi\rho_0}[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}],$$

$$\nabla\cdot\boldsymbol{q} = 0 \tag{2}$$

where ρ, p and $\boldsymbol{q}$ denote respectively, the density, the pressure and the filter velocity of the pure fluid; $\boldsymbol{q}_d$ and $N(\bar{x},t)$ denote the velocity and number density of the suspended particles, respectively. $K = 6\pi\rho\upsilon\eta$, where η is the particle radius, is the Stokes' drag coefficient, $\bar{x} = (x,y,z)$. Let ε, k_1, υ, υ' stand for medium porosity, medium permeability, kinematic viscosity of fluid and kinematic viscoelasticity of fluid, respectively.

In writing equation (1) we have assumed uniform size of fluid particles, spherical shape and small relative velocities between the fluid and particles. Then the net effect of the suspended particles on the fluid through porous medium is equivalent to an extra body force term per unit volume $\frac{KN}{\varepsilon}(\boldsymbol{q}_d - \boldsymbol{q})$. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equation of motion of the particles. The distances between particles are assumed to be so large compared with their diameter that interparticle reactions need not be accounted for. The effects of pressure, gravity and Darcian force on the suspended particles (assumed large distances apart) are negligibly small and therefore ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are

$$mN\left[\frac{\partial \boldsymbol{q}_d}{\partial t} + \frac{1}{\varepsilon}(\boldsymbol{q}_d\cdot\nabla)\boldsymbol{q}_d\right] = KN(\boldsymbol{q} - \boldsymbol{q}_d) \tag{3}$$

$$\varepsilon\frac{\partial N}{\partial t} + \nabla.(N\boldsymbol{q}_d) = 0 \tag{4}$$

Let c, c_{pt}, T, C, q' and q'' denote respectively the specific heat of fluid at constant pressure, specific heat of fluid particles, temperature, solute concentration, effective thermal conductivity of the pure fluid and an analogous effective solute conductivity. If we assume that the particles and the fluid are in thermal and solute equilibrium, then the equation of heat and solute conduction gives

$$[\rho c\varepsilon + \rho_s c_s(1-\varepsilon)]\frac{\partial T}{\partial t} + \rho c(\boldsymbol{q}\cdot\nabla)T + mNc_{pt}\left(\varepsilon\frac{\partial}{\partial t} + \boldsymbol{q}_d\cdot\nabla\right)T = \mathrm{q}'\nabla^2 T \tag{5}$$

$$[\rho c\varepsilon + \rho_s c_s(1-\varepsilon)]\frac{\partial T}{\partial t} + \rho c(\boldsymbol{q}\cdot\nabla)T + mNc_{pt}\left(\varepsilon\frac{\partial}{\partial t} + \boldsymbol{q}_d\cdot\nabla\right)C = \mathrm{q}''\nabla^2 C \tag{6}$$

where ρ_s, c_s are the density and the specific heat of the solid material respectively, and ε is medium porosity. The equation of state for the fluid is given by

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(C - C_0)], \tag{7}$$

where ρ_0, T_0 and C_0 are the density, temperature and solute concentration of the fluid at bottom surface $z = 0$, α is the coefficient of thermal expansion and α' is the analogous coefficient of solvent expansion.

3. THE PERTURBATION EQUATIONS

The initial state of the system, denoted by subscript 0, is taken to be a quiescent layer (no setting) with a uniform particle distribution N_0 i.e. $\boldsymbol{q} = (0,0,0), \boldsymbol{q}_d = (0,0,0)$ and $N = N_0$ is a constant. The

character of the equilibrium of this initial static state can be determined, as usual, by supposing that the system is slightly disturbed and then by following its further evolution.
The steady state solution to the governing equations is

$$\boldsymbol{q} = (0,0,0), \boldsymbol{q}_d = (0,0,0), T = T_0 - \beta z, C = C_0 - \beta' z, \rho = \rho_0 (1 + \alpha\beta z - \alpha'\beta' z)$$

Let $\delta\rho, \delta p, \theta, \boldsymbol{q}(u,v,w), \boldsymbol{q}_d(l,r,s), \gamma$ and N denote respectively the small perturbations in density ρ, pressure p, temperature T, fluid velocity $\boldsymbol{q}(0,0,0)$ fluid particle velocity $\boldsymbol{q}_d(0,0,0)$, solute concentration C and number density N_0. Then the linearized perturbation equations of motion, continuity, heat conduction and solute conduction for Rivlin-Ericksen elastic-viscous fluid-particle layer are

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{q}}{\partial t} = -\frac{1}{\rho_o}\nabla\delta p - g_0(\alpha\theta - \alpha'\gamma)\lambda + \frac{KN}{\varepsilon\rho_o}(\boldsymbol{q}_d - \boldsymbol{q}) - \frac{1}{K_1\rho_0}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\boldsymbol{q} + \frac{2}{\varepsilon\rho_o}(\boldsymbol{q}\times\boldsymbol{\Omega}) + \frac{\mu_e}{4\pi\rho_o}(\nabla\times\boldsymbol{h})\times\boldsymbol{H} \tag{8}$$

$$\nabla\cdot\boldsymbol{q} = 0 \tag{9}$$

$$\varepsilon\frac{\partial N}{\partial t} + N_0(\nabla\cdot\boldsymbol{q}_d) = 0 \tag{10}$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)\boldsymbol{q}_d = \boldsymbol{q} \tag{11}$$

$$(E + h\varepsilon)\frac{\partial\theta}{\partial t} = \beta(w + hs) + \kappa\nabla^2\theta \tag{12}$$

$$(E' + h\varepsilon)\frac{\partial\gamma}{\partial t} = \beta'(w + hs) + \kappa'\nabla^2\gamma \tag{13}$$

where, $h = \frac{fc_{pt}}{c}, f = \frac{mN_0}{\rho_0}, \kappa = \frac{q'}{\rho_0 c}, \kappa' = \frac{q''}{\rho_0 c}, \nu = \frac{\mu}{\rho_0}$, and $\nu' = \frac{\mu'}{\rho_0}$

In writing equation (8), use has been made of the Boussinesq equation of state

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma).$$

4. EXACT SOLUTION AND DISPERSION RELATION

Analyzing the perturbations into normal modes by seeking solutions in the form

$$[w,\theta,\gamma,h_z,\xi,\zeta] = [W(z),\Theta(z),\Gamma(z),K(z),Z(z),X(z)].\exp(ik_x x + ik_y y + nt) \tag{14}$$

where k_x, k_y are the wave numbers along x and y directions respectively, and $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number of the disturbance and n is the growth rate which is , in general , a complex constant and $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ stand for the z-components of vorticity and current density, respectively. Eliminating the physical quantities using the non-dimensional parameters $a = kd$, $\sigma = \frac{nd^2}{\upsilon}$, $p_1 = \frac{\upsilon}{\kappa}$, $p_1' = \frac{\upsilon}{\kappa'}$, $\tau_1 = \frac{\tau\upsilon}{d^2}$, $M = \frac{mN_0}{\rho_0}$, $p_2 = \frac{\upsilon}{\eta}$, $p_l = \frac{k_1}{d^2}$, $F = \frac{\upsilon'}{d^2}$, $E_1 = E + h\varepsilon$, $E_2 = E' + h\varepsilon, H = 1 + h, D^* = dD$ and dropping (*) for convenience, the linearized dimensionless perturbation equations are

$$\left[\frac{\sigma}{\varepsilon}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1+F\sigma}{P_l}\right](D^2 - a^2)W = -\frac{ga^2d^2}{\upsilon}(\alpha\Theta - \alpha'\Gamma) - \frac{2\Omega d^3}{\upsilon\varepsilon}DZ + \frac{\mu_e Hd}{4\pi\rho_0\upsilon}(D^2 - a^2)DK \tag{15}$$

$$\left[\frac{\sigma}{\varepsilon}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1+F\sigma}{P_l}\right]Z = \left(\frac{2\Omega d}{\varepsilon\upsilon}\right)DW + \frac{\mu_e Hd}{4\pi\rho_0\upsilon}DX \tag{16}$$

$$\left(D^2-a^2-\sigma E_1 p_1\right)\Theta=-\left(\frac{\beta d^2}{\kappa}\right)\left(\frac{H+\tau_1\sigma}{1+\tau_1\sigma}\right)W \tag{17}$$

$$\left(D^2-a^2-\sigma E_2 p_1'\right)\Gamma=-\left(\frac{\beta' d^2}{\kappa'}\right)\left(\frac{H+\tau_1\sigma}{1+\tau_1\sigma}\right)W \tag{18}$$

$$\left(D^2-a^2-p_2\sigma\right)K=-\frac{Hd}{\eta}DW \tag{19}$$

$$\left(D^2-a^2-p_2\sigma\right)X=-\frac{Hd}{\eta}DZ \tag{20}$$

Eliminating Z, K, Θ, Γ and X between the equations (15) – (20), we obtain

$$\begin{aligned}&\left\{\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_l}\right]\left(D^2-a^2-p_2\sigma\right)+QD^2\right\}.\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_l}\right]\left(D^2-a^2-\sigma E_1p_1\right)\\&.\left(D^2-a^2-\sigma E_2p_1'\right)\left(D^2-a^2-p_2\sigma\right)\left(D^2-a^2\right)W\\&=\left(\frac{H+\tau_1\sigma}{1+\tau_1\sigma}\right)a^2\left(D^2-a^2-p_2\sigma\right).\left\{\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_l}\right]\left(D^2-a^2-p_2\sigma\right)+QD^2\right\}\\&\cdot\left\{R\lambda\left(D^2-a^2-\sigma E_2p_1'\right)-S\lambda\left(D^2-a^2-\sigma E_1p_1\right)\right\}\\&-\frac{T_A}{\varepsilon^2}\left(D^2-a^2-\sigma E_1p_1\right)\left(D^2-a^2-\sigma E_2p_1'\right).\left(D^2-a^2-p_2\sigma\right)^2D^2W\\&+Q\left\{\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_l}\right]\left(D^2-a^2-p_2\sigma\right)+QD^2\right\}\cdot\left(D^2-a^2-\sigma E_1p_1\right)\left(D^2-a^2-\sigma E_2p_1'\right)\left(D^2-a^2\right)D^2W\end{aligned} \tag{21}$$

where, $R=\dfrac{g\alpha\beta d^4}{\nu\kappa}$ is the thermal Rayleigh number, $S=\dfrac{g\alpha'\beta' d^4}{\nu\kappa'}$ is the analogous solute Rayleigh number, $T_A=\left(\dfrac{2\Omega d^2}{\nu}\right)^2$ is the Taylor number and $Q=\dfrac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ is the Chandrasekhar number.

Consider the case where both the boundaries are free from stresses and perfect conductors of heat and solute, the medium adjoining the fluid is perfectly conducting and temperatures at the boundaries are kept fixed. The boundary conditions appropriate for the problem are (Chandrasekhar [3])

$$W=D^2W=0,\ \Theta=\Gamma=0,\ DK=0 \text{ when } z=0 \text{ and } 1 \tag{22}$$

Proper solution of W characterizing the lowest mode is

$$W=W_0\sin\pi z \text{ where } W_0 \text{ is a constant.} \tag{23}$$

Substituting the proper solution (23) in equation (21), we obtain the dispersion relation

$$\begin{aligned}R_1x\lambda=&\left(\frac{1+x+iE_1p_1\sigma_1}{1+x+iE_1p_1'\sigma_1}\right)S_1x\lambda+\left[\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+i\tau_1\sigma_1\pi^2}\right)+\left(\frac{1+iF\sigma_1\pi^2}{P}\right)\right].\left[\frac{1+i\tau_1\sigma_1\pi^2}{H+i\tau_1\sigma_1\pi^2}\right].\\&\cdot\left(1+x+iE_1p_1\sigma_1\right)\left(1+x\right)+\frac{T_{A_1}}{\varepsilon^2}\frac{\left(1+i\tau_1\sigma_1\pi^2\right)}{\left(H+i\tau_1\sigma_1\pi^2\right)}.\frac{(1+x+i\sigma_1p_2)(1+x+iE_1p_1\sigma_1)}{\left\{\left[i\frac{\sigma_1}{\varepsilon}\left(1+\frac{M}{1+i\tau_1\sigma_1\pi^2}\right)+\frac{1+iF\sigma_1\pi^2}{P}\right](1+x+i\sigma_1p_2)+Q_1\right\}}\\&+Q_1\frac{\left(1+i\tau_1\sigma_1\pi^2\right)}{\left(H+i\tau_1\sigma_1\pi^2\right)}(1+x)\frac{\left(1+x+E_1p_1\sigma_1\right)}{\left(1+x+i\sigma_1p_2\right)}\end{aligned} \tag{24}$$

where $R_1=\dfrac{R}{\pi^4},\ S_1=\dfrac{S}{\pi^4},\ Q_1=\dfrac{Q}{\pi^2},\ T_{A_1}=\dfrac{T}{\pi^4},\ x=\dfrac{a^2}{\pi^2},\ i\sigma_1=\dfrac{\sigma}{\pi^2}$ and $P=\pi^2P_l$.

Equation (24) is the required dispersion relation including the effects of magnetic field, rotation, medium permeability, kinematic viscosity, stable solute gradient and variable gravity field on the thermosolutal instability of Rivlin-Ericksen fluid in porous medium. The dispersion relation vanishes to the one derived by Sharma and Rana [13] if the magnetic field parameter is vanishing.

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5. THE STATIONARY CONVECTION

For stationary convection, the marginal state will be characterized by $\sigma = 0$. Thus equation (24) reduces to

$$R_1 = S_1 + \frac{(1+x)}{xH\lambda}\left(\frac{1+x}{P} + Q_1\right) + \frac{T_{A_1}}{x\lambda H\varepsilon^2}\frac{(1+x)^2}{\left(\frac{1+x}{P} + Q_1\right)} \tag{25}$$

The above relation expresses the modified Rayleigh number R_1 as a function of the modified solute gradient parameter S_1, modified magnetic field parameter Q_1, suspended particles parameter H, modified rotation parameters T_{A_1}, medium permeability parameter P and dimensionless wave number x. The parameter F accounting for the kinematic viscoelasticity vanishes for the stationary convection and the Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid. To study the effect of stable solute gradient, magnetic field, suspended particles, rotation and permeability, we examine the nature of $\frac{dR_1}{dS_1}, \frac{dR_1}{dQ_1}, \frac{dR_1}{dH}, \frac{dR_1}{dT_{A_1}}$ and $\frac{dR_1}{dP}$ analytically.

Equation (25) yields

$$\frac{dR_1}{dS_1} = 1 \tag{26}$$

which is positive implying thereby that the effect of stable solute gradient is to stabilize the system. This is in agreement with the result of figure 2 where R_1 is plotted against S_1 for $H = 60$, $P = 0.001$, $T_{A_1} = 20$, $\varepsilon = 0.15$, $Q_1 = 200$, $\lambda = 2$, $x = 2,4,6,8,10$ and figure 3 where R_1 is plotted against x for $S_1 = 20, 40, 60, 80$. This stabilizing effect of stable solute gradient is in good agreement with the earlier works of Sharma and Rana [13].

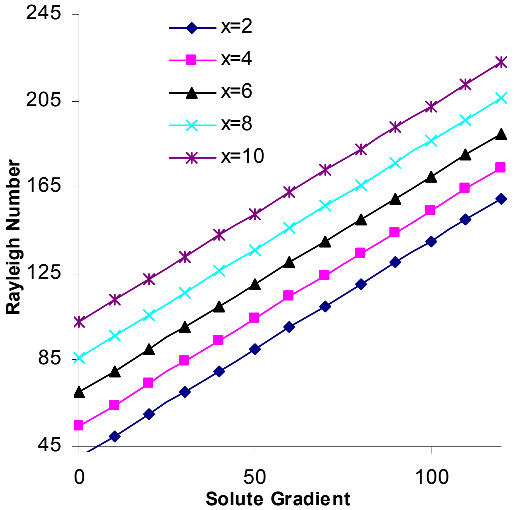

Figure 2: Variation of R_1 with S_1

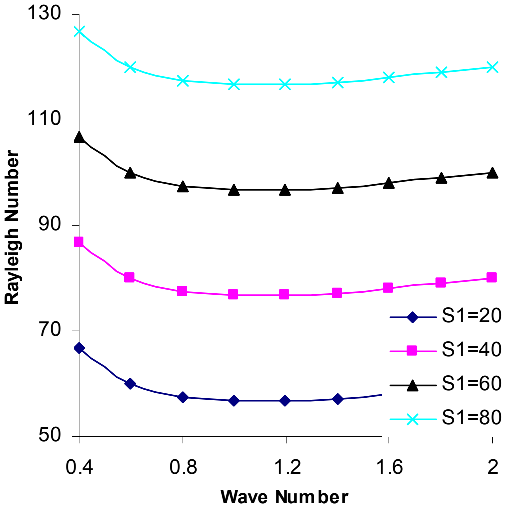

Figure 3: Variation of R_1 with x

From equation (25), we get

$$\frac{dR_1}{dT_{A_1}} = \frac{1}{\varepsilon^2 Hx\lambda}\frac{(1+x)^2}{\left(\frac{1+x}{P} + Q_1\right)} \tag{27}$$

which shows that rotation has a stabilizing effect on the system in porous medium when gravity increases upwards from its value g_0 (i.e. $\lambda > 0$) and destabilizes the system when gravity decreases upwards. Also figure 4 and 5 confirms the above result numerically for fixed $H = 10$, $P = 0.2$, $S_1 = 40$, $\varepsilon = 0.15$, $Q_1 = 200$, $\lambda = 2$ and various values of x and T_{A_1} respectively. This stabilizing effect of rotation is in good agreement with earlier works of Sharma and Rana [13] and Rana and Kumar [7].

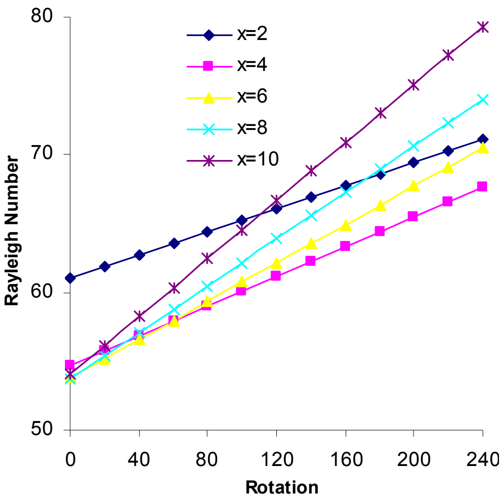

Figure 4: Variation of R_1 with T_{A_1}

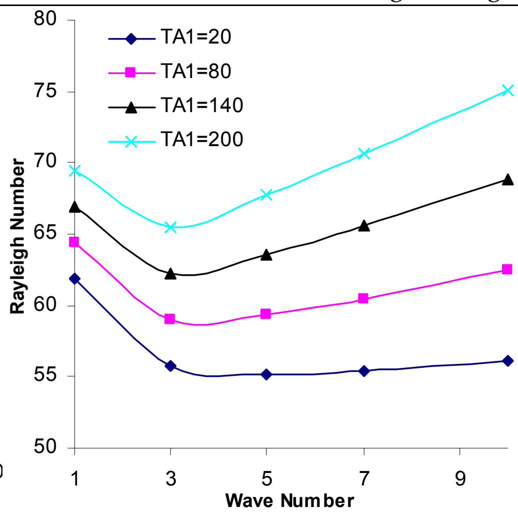

Figure 5: Variation of R_1 with x

Also from equation (25), we get

$$\frac{dR_1}{dH} = -\frac{(1+x)}{x\lambda H^2}\left[\frac{(1+x)}{P} + \frac{T_{A_1}}{\varepsilon^2}\frac{(1+x)}{\left(\frac{1+x}{P}+Q_1\right)} + Q_1\right] \quad (28)$$

which is negative, implying thereby that the effect of suspended particles is to destabilize the system when gravity increases upwards (i.e. $\lambda > 0$) and stabilizes the system when gravity decreases upwards. Also in figures 6 and 7, R_1 decreases with the increase in H which confirms the above result numerically. This result is in agreement with the result of Sharma and Aggarwal [11] in which effect of compressibility and suspended particles is investigated on thermal convection in a Walters' B' elastico-viscous fluid in hydromagnetics.

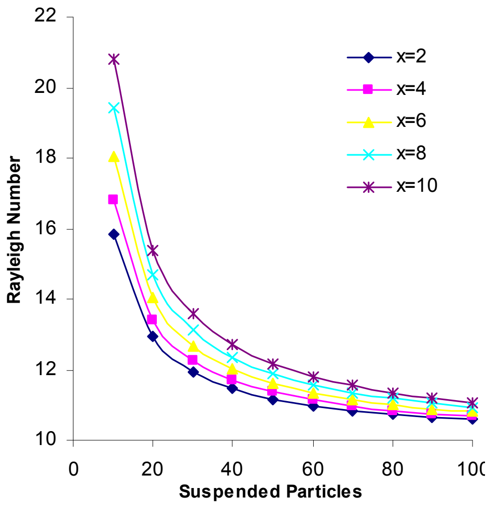

Figure 6: Variation of R_1 with H

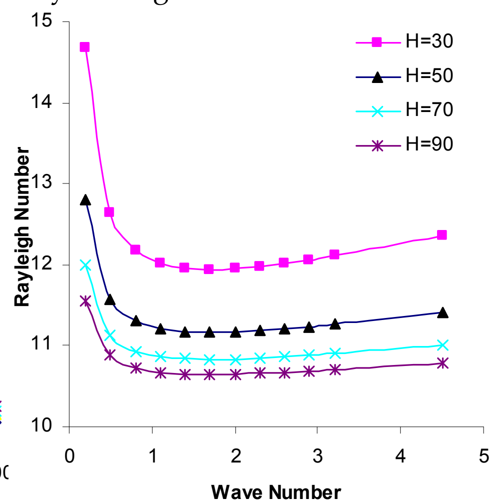

Figure 7: Variation of R_1 with x

It is evident from equation (25) that

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{x\lambda H}\left[\frac{1}{P^2} - \frac{T_{A_1}}{\varepsilon^2}\frac{(1+x)}{(1+x+PQ_1)^2}\right] \quad (29)$$

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{Hx\lambda}\left[1 - \frac{T_{A_1}}{\varepsilon^2}\cdot\frac{(1+x)P^2}{(1+x+PQ_1)^2}\right] \quad (30)$$

In the absence of rotation ($T_{A_1} = 0$), $\frac{dR_1}{dP}$ is always negative and $\frac{dR_1}{dQ_1}$ is always positive, which means that medium permeability has a destabilizing effect, whereas magnetic field has a stabilizing effect on the system when gravity increases upwards from its value g_0. For a rotating system, when gravity increases upwards, the medium permeability has a destabilizing (stabilizing) effect and magnetic field has a stabilizing (destabilizing) effect if

$$T_{A_1} < (\text{or} >) \frac{(1 + x + PQ_1)^2}{P^2(1+x)} \varepsilon^2 \tag{31}$$

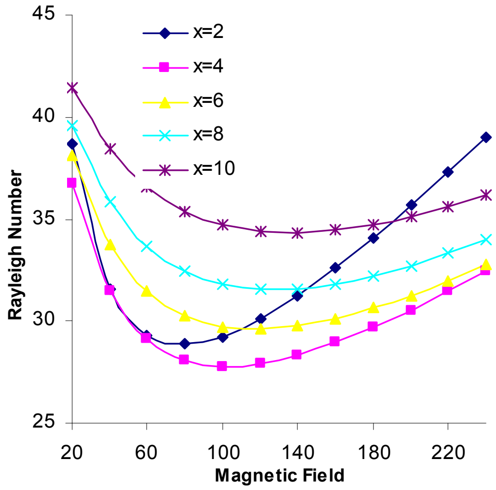

Figure 8: Variation of R_1 with Q_1

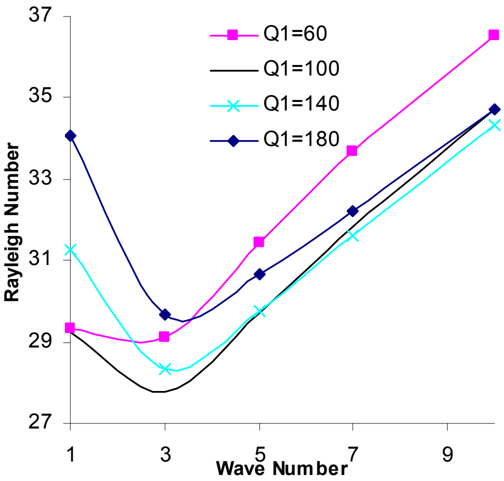

Figure 9: Variation of R_1 with x

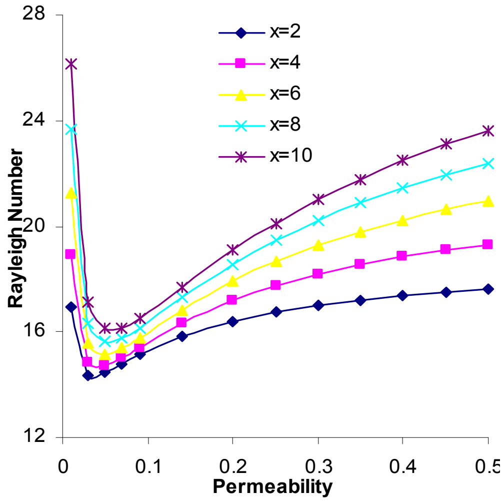

Figure 10: Variation of R_1 with P

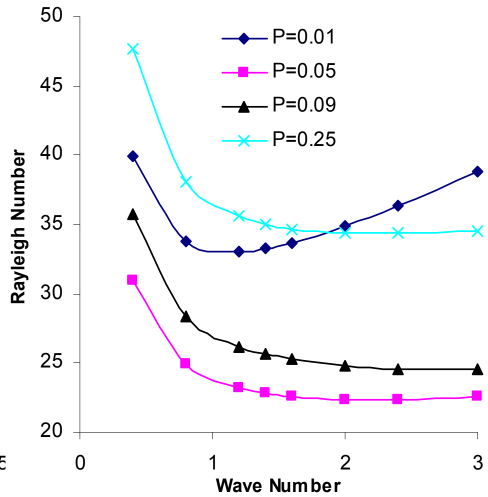

Figure 11: Variation of R_1 with x

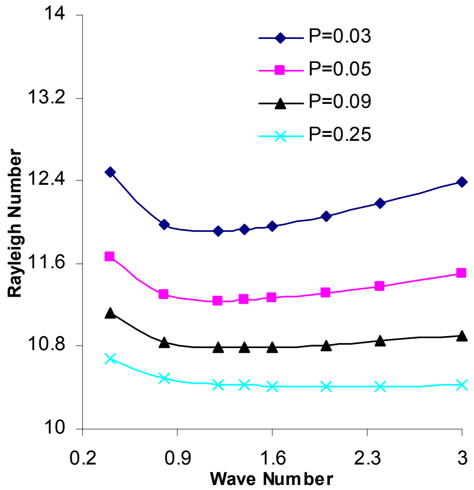

Figure 12: Variation of R_1 with x for $T_{A_1} = 0$

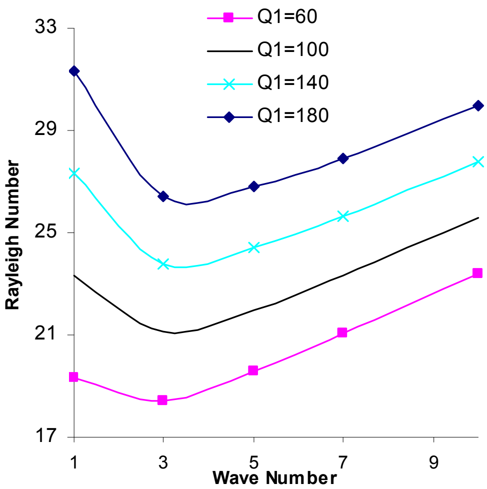

Figure 13: Variation of R_1 with x for $T_{A_1} = 0$

In figure 8, R_1 is plotted against Q_1 for $H = 10$, $P = 0.13$, $S_1 = 10$, $\varepsilon = 0.15$, $\lambda = 2$ and $x = 2, 4, 6, 8, 10$. Figure 9 represents the graph of Rayleigh number R_1 versus x for $Q_1 = 60, 100, 140, 180$. Here, we find that magnetic field has stabilizing as well as destabilizing effect under condition (31). In figure 10, R_1 is potted against P for $T_{A_1}=100, H=40,$ $Q_1=30, S_1=10, \varepsilon=0.15, \lambda=2$ and $x=2,4,6,8,10$ whereas figure 11 represents R_1 versus x for $P=0.01,\ 0.05,\ 0.09,\ 0.25$. These figures show that the permeability has destabilizing (stabilizing) effect in the presence of rotation under condition (31).

In figure 12, R_1 is plotted against wave number x for $S_1=10, T_{A_1}=0$, $H=10, \varepsilon=0.15, Q_1=30,$ $\lambda=2$ and different values of permeability parameter $P=0.03,\ 0.05,\ 0.09,\ 0.25$ and figure 13 shows the graph of R_1 versus x for $S_1=10, H=100, T_{A_1}=0, \varepsilon=0.15, \lambda=2$ and $Q_1=60,100,140,180$. Figure 12 shows stabilizing effect of magnetic field whereas figure 13 shows the destabilizing effect of medium permeability in the absence of rotation .

6. STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Multiplying equation (15) by W^*, the complex conjugate of W, and integrating over the range of z

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_e}\right]\int_0^1 W^*(D^2-a^2)W dz+\frac{\lambda g_0 a^2 d^2}{\nu}\int_0^1 W^*(\alpha\Theta-\alpha'\Gamma)dz$$
$$+\frac{2\Omega d^3}{\varepsilon\nu}\int_0^1 W^* D z dz-\frac{\mu_e H d}{4\pi\rho_0\nu}\int_0^1 W^*(D^2-a^2)DK dz=0 \quad (32)$$

Integrating equation (32) and using boundary conditions (22) - (23) together with equations (16) - (20), we obtain

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1+F\sigma}{P_e}\right]I_1-\frac{\lambda g_0 a^2}{\nu}\left(\frac{1+\tau_1\sigma^*}{H+\tau_1\sigma^*}\right)\left[\frac{\alpha\kappa}{\beta}(I_2+\sigma^* E_1 p_1 I_3)-\frac{\alpha'\kappa'}{\beta'}(I_4+\sigma^* E_2 p_1' I_5)\right]$$
$$+d^2\left[\frac{\sigma^*}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma^*}\right)+\frac{1+F\sigma^*}{P_e}\right]I_6-\frac{\mu_e\eta d^2}{4\pi\rho_0\nu}[I_7+p_2\sigma I_8]+\frac{\mu_e\eta}{4\pi\rho_0\nu}[I_9+p_2\sigma^* I_{10}]=0 \quad (33)$$

where, $I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz$, $I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz$, $I_3=\int_0^1|\Theta|^2 dz$, $I_4=\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz$, $I_5=\int_0^1|\Gamma|^2 dz$,

$I_6=\int_0^1|Z|^2 dz$, $I_7=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz$, $I_8=\int_0^1|X|^2 dz$, $I_9=\int_0^1\left(|D^2K|^2+2a^2|DK|^2+a^4|K|^2\right)dz$, $I_{10}=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz.$

and σ^* is the complex conjugate of σ. The integrals I_1-I_{10} are all positive definite. Putting $\sigma=i\sigma_i\left(\sigma^*=-i\sigma_i\right)$ in equation (33) and equating imaginary parts, we obtain

$$\sigma_i\left\{\begin{array}{c}\left[\frac{1}{\varepsilon}\left(1+\frac{M}{1+\sigma_i^2\tau_i^2}\right)+\frac{F}{P_l}\right]I_1+\frac{\lambda g_0 a^2\tau_1}{\nu}\cdot\left(\frac{H-1}{H^2+\sigma_i^2\tau_i^2}\right)\left[\frac{\alpha\kappa}{\beta}I_2-\frac{\alpha'\kappa'}{\beta'}I_4\right]\\ -\frac{\lambda g_0 a^2}{\nu}\left(\frac{H+\sigma_i^2\tau_i^2}{H^2+\sigma_i^2\tau_i^2}\right)\cdot\left\{-\frac{\alpha\kappa}{\beta}E_1 p_1 I_3+\frac{\alpha'\kappa'}{\beta'}E_2 p_1' I_5\right\}\\ -d^2\left[\frac{1}{\varepsilon}\left(1+\frac{M}{1+\sigma_i^2\tau_i^2}\right)+\frac{F}{P_l}\right]I_6-\frac{\mu_e\eta d^2}{4\pi\rho_0\nu}p_2 I_8-\frac{\mu_e\eta}{4\pi\rho_0\nu}p_2 I_{10}\end{array}\right\}=0 \quad (34)$$

In the absence of rotation, magnetic field and solute gradient, equation (34) reduces to

$$\sigma_i\left\{\begin{array}{c}\left(\frac{1}{\varepsilon}\left(1+\frac{M}{1+\sigma_i^2\tau_i^2}\right)+\frac{F}{P_l}\right)I_1+\frac{\lambda g_0 a^2\tau_1}{\nu}\cdot\left(\frac{H-1}{H^2+\sigma_i^2\tau_i^2}\right)\frac{\alpha\kappa}{\beta}I_2\\ +\frac{\lambda g_0 a^2}{\nu}\frac{\alpha\kappa}{\beta}E_1 p_1\left(\frac{H+\sigma_i^2\tau_i^2}{H^2+\sigma_i^2\tau_i^2}\right)I_3\end{array}\right\}=0 \quad (35)$$

Equation (35) implies that σ_i is zero (as the terms in the bracket are positive definite) in the absence of rotation, magnetic field and stable solute gradient, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied. It is evident from equation (34) that the presence of magnetic field, rotation and stable solute gradient introduces oscillatory modes (as σ_i may not be zero).

7. CONCLUSIONS

In this section, we have studied the effect of magnetic field, rotation, variable gravity field and suspended particles on thermosolutal convection in Rivlin-Ericksen elastico-viscous fluid saturating a porous medium. The principal conclusions from the analysis are as follows:

- ✓ For stationary convection, the suspended particles have destabilizing effect for $\lambda > 0$ and stabilizing effect for $\lambda < 0$.
- ✓ When gravity increases upwards (i.e. $\lambda > 0$), the medium permeability has a destabilizing effect in the absence of rotation whereas in the presence of rotation it has a destabilizing effect when $T_{A_1} < \frac{(1+x+PQ_1)^2}{P^2(1+x)}\varepsilon^2$ and has stabilizing effect when $T_{A_1} > \frac{(1+x+PQ_1)^2}{P^2(1+x)}\varepsilon^2$.
- ✓ The magnetic field has a stabilizing effect in the absence of rotation when gravity increases upwards whereas in the presence of rotation it has a stabilizing effect when $T_{A_1} < \frac{(1+x+PQ_1)^2}{P^2(1+x)}\varepsilon^2$ and has destabilizing effect when $T_{A_1} > \frac{(1+x+PQ_1)^2}{P^2(1+x)}\varepsilon^2$.
- ✓ For stationary convection, the stable solute gradient is found to have stabilizing effect whereas effect of rotation is stabilizing for $\lambda > 0$ and destabilizing for $\lambda < 0$.
- ✓ The principle of exchange of stabilities is satisfied in the absence of magnetic field, rotation and stable solute gradient. The presence of magnetic field, rotation and stable solute gradient introduces oscillatory modes (as σ_i may not be zero).

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