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KINETICAL INVESTIGATION OF MULTI DEGREE-OF-FREEDOM TRANSLATIONAL OSCILLATING SYSTEMS USING SIMPLE NUMERICAL METHOD

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Abstract: Mechanical vibration is an important field in the education of mechanical engineers. In case of special machines the vibration at demanded amplitude and frequency is condition of operation (for example vibrating screens, compacting machines, vibration transport etc.). In general engineering the main task is the vibration damping. The role of mechanical vibration as professional field was revalued during last decades. There is a fundamental point of view for constructors the reduction of mass. The machine speed increases and in consequence of this the vibration is getting higher and higher. A simple numerical method will be demonstrated by author by the aid of several examples. Some parts of results obtained by using the numerical method were checked by analytical way. The published method can be used in the technical higher education.
Key words: translational oscillation, multi degree-of-freedom system, numerical solution procedure.

1. INTRODUCTION

In case of translational oscillation the position of mass points moving along an axis are determined by scalar variables. When the number of independent variables is more than one it is about multi degree-of-freedom system. Different analyzed excited models consist of mass points and between them springs and viscous damping. The motion of mass points can be described by second order differential equation systems. Numerical methods can be applied to solve such problems.

2. TWO DEGREE-OF-FREEDOM DAMPED EXCITED TRANSLATIONAL OSCILLATING SYSTEM

In Fig 1 the model of two mass translational damped excited oscillating system can be seen. Mass points m_1 and m_2 can translate along straight line. They are connected by massless springs. Their spring stiffness is denoted by s. Such oscillations, independently from number of mass points, are called translational oscillations. Mass points can move independently from each other for this reason the number of degree-of-freedom is two. In case of state of continuous rest the values of displacements are zero.

Kinematical functions of oscillation can be determined on numerical way applying of Lagrange equations of the second kind.

Data (Fig 1): $m_1=2$ kg, $m_2=4$ kg, $s_1=3000$ N/m, $s_2=1000$ N/m, $k_1=10$ Ns/m, $k_2=20$ Ns/m, $F_{g1o}=0$ N, $\omega_{g1}=0$ s^{-1}, $F_{g2o}=100$ N, $\omega_{g2}=30$ s^{-1}.

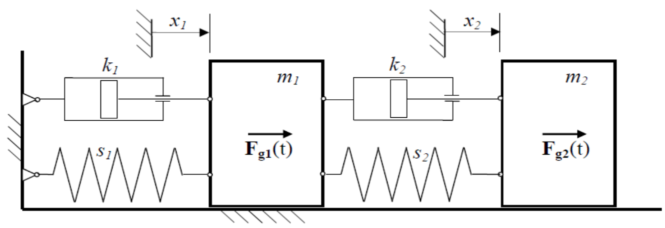


Figure 1. Sketch of two degree-of-freedom damped excited translational oscillating system

Generalized coordinates are displacements of mass points x_1 and x_2 furthermore Lagrange equations concerning chosen generalized coordinates are

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{x}_1}\right)-\frac{\partial E}{\partial x_1}+\frac{\partial U}{\partial x_1}+\frac{\partial D}{\partial x_1}=F_{g1}(t) \text{ and } \frac{d}{dt}\left(\frac{\partial E}{\partial \dot{x}_2}\right)-\frac{\partial E}{\partial x_2}+\frac{\partial U}{\partial x_2}+\frac{\partial D}{\partial x_2}=F_{g2}(t).$$

Kinetic energy of oscillating system and terms of Lagrange equations are

$$\frac{\partial E}{\partial \dot{x}_1}=m_1\dot{x}_1, \quad \frac{d}{dt}\left(\frac{\partial E}{\partial \dot{x}_1}\right)=m_1\ddot{x}_1, \quad \frac{\partial E}{\partial x_1}=0,$$

$$\frac{\partial E}{\partial \dot{x}_2}=m_2\dot{x}_2, \quad \frac{d}{dt}\left(\frac{\partial E}{\partial \dot{x}_2}\right)=m_2\ddot{x}_2, \quad \frac{\partial E}{\partial x_2}=0.$$

Potential energy stored by applied springs and its derivatives are

$$U=\frac{1}{2}\left(s_1x_1^{\,2}+s_2(x_2-x_1)^2\right), \quad \frac{\partial U}{\partial x_1}=s_1x_1+s_2(x_2-x_1), \quad \frac{\partial U}{\partial x_2}=-s_2(x_2-x_1).$$

Dissipative function is

$$D=\frac{1}{2}\left(k_1\dot{x}_1^{\,2}+k_2(\dot{x}_2-\dot{x}_1)^2\right),$$

from which

$$\frac{\partial D}{\partial \dot{x}_1}=k_1\dot{x}_1-k_2(\dot{x}_2-\dot{x}_1), \qquad \frac{\partial U}{\partial \dot{x}_2}=k_2(\dot{x}_2-\dot{x}_1).$$

Lagrange equations with above terms are

$$m_1\ddot{x}_1+k_1\dot{x}_1-k_2(\dot{x}_2-\dot{x}_1)+s_1x_1+s_2(x_2-x_1)=F_{g1}(t),$$

$$m_2\ddot{x}_2+k_2(\dot{x}_2-\dot{x}_1)-s_2(x_2-x_1)=F_{g2}(t)$$

differential-equations can be obtained. Above differential equation-system can be ordered in matrix form as well, i.e.

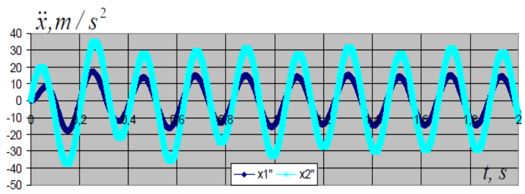
$$\mathbf{M\ddot{q}}+\mathbf{K\dot{q}}+\mathbf{Sq}=\mathbf{f},$$

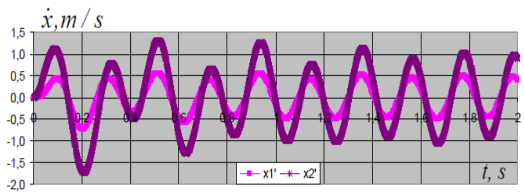
where **M** mass matrix, **K** damping matrix, **S** spring stiffness matrix and finally **f** vector of generalized forces as exciting forces. In details,

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}+\begin{bmatrix} s_1+s_2 & -s_2 \\ -s_2 & s_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} F_{g1}(t) \\ F_{g2}(t) \end{bmatrix}.$$

Using above data and the following initial conditions, are $x_{1o}=0m$, $\dot{x}_{1o}=0\frac{m}{s}$, $x_{2o}=0m$, $\dot{x}_{2o}=0\frac{m}{s}$.

Results of numerical solution of differential equation-system can be shown in Fig 2.





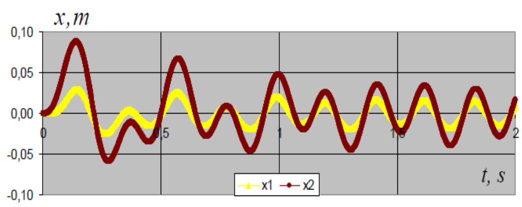


Figure 2. Kinematical functions of two degree-of-freedom damped excited translational oscillation
($0\ s \le t \le 2\ s$)

Applied simple numerical method: the mechanical system can be described by q=$q(x_1,x_2)$ generalized coordinates. Physical quantities $\dot{x}_1,\dot{x}_2,x_1,x_2$ describe the initial condition of the system. Time step: $t_{i+1}-t_i$. Applied algorithms can be seen in table below.

Table 1. Applied algorithms of numerical solution

t	$\ddot{x}_1(\dot{x}_1, x_1, \dot{x}_2, x_2)$	$\dot{x}_1$	x_1	$\ddot{x}_2(\dot{x}_1, x_1, \dot{x}_2, x_2)$	$\dot{x}_2$	x_2
t_o	$\ddot{x}_{1o}(\dot{x}_{1o}, x_{1o}, \dot{x}_{2o}, x_{2o})$	$\dot{x}_{1o}$	x_{1o}	$\ddot{x}_{2o}(\dot{x}_{1o}, x_{1o}, \dot{x}_{2o}, x_{2o})$	$\dot{x}_{2o}$	x_{2o}
t_1	$\ddot{x}_{11}(\dot{x}_{11}, x_{11}, \dot{x}_{21}, x_{21})$	$\dot{x}_{11} = \dot{x}_{1o} + \ddot{x}_{1o}(t_1 - t_o)$	$x_{11} = x_{1o} + \dot{x}_{1o}(t_1 - t_o)$	$\ddot{x}_{21}(\dot{x}_{11}, x_{11}, \dot{x}_{21}, x_{21})$	$\dot{x}_{21} = \dot{x}_{2o} + \ddot{x}_{2o}(t_1 - t_o)$	$x_{21} = x_{2o} + \dot{x}_{2o}(t_1 - t_o)$
t_2	$\ddot{x}_{12}(\dot{x}_{12}, x_{12}, \dot{x}_{22}, x_{22})$	$\dot{x}_{12} = \dot{x}_{11} + \ddot{x}_{11}(t_2 - t_1)$	$x_{12} = x_{11} + \dot{x}_{11}(t_2 - t_1)$	$\ddot{x}_{22}(\dot{x}_{12}, x_{12}, \dot{x}_{22}, x_{22})$	$\dot{x}_{22} = \dot{x}_{21} + \ddot{x}_{21}(t_2 - t_1)$	$x_{22} = x_{21} + \dot{x}_{21}(t_2 - t_1)$
t_3	....	....	....	....	....	....
t_4	....	....	....	....	....	....

For calculation of natural frequencies of oscillation let us solve the

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{S}\mathbf{q} = 0$$

homogeneous matrix differential-equation. Looking for the solution in form of $x_1 = x_{1o}\sin(\omega t + \varphi_1)$, $x_2 = x_{2o}\sin(\omega t + \varphi_2)$, from which

$$\dot{x}_1 = \omega x_{1o}\cos(\omega t + \varphi_1),\ \dot{x}_2 = \omega x_{2o}\cos(\omega t + \varphi_2),$$

$$\ddot{x}_1 = -\omega^2 x_{1o}\sin(\omega t + \varphi_1),\ \ddot{x}_2 = -\omega^2 x_{2o}\sin(\omega t + \varphi_2).$$

Above equations substituted into homogeneous matrix differential-equation after rearrangement the following equation system can be written down:

$$\begin{bmatrix} -m_1\omega^2 + s_1 + s_2 & -s_2 \\ -s_2 & -m_2\omega^2 + s_2 \end{bmatrix}\begin{bmatrix} x_{1o} \\ x_{2o} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solution different from trivial one can be obtained in case of determinant of matrix equals to zero, i.e.

$$\begin{vmatrix} -m_1\omega^2 + s_1 + s_2 & -s_2 \\ -s_2 & -m_2\omega^2 + s_2 \end{vmatrix} = 0\ .$$

After development of determinant and rearrangement,

$$m_1 m_2\omega^4 - (s_2 m_1 + (s_1 + s_2)m_2)\omega^2 + (s_1 + s_2)s_2 - s_2^{\ 2} = 0,$$

from where

$$\omega_{12}^{\ 2} = \frac{s_2 m_1 + (s_1 + s_2)m_2 \pm \sqrt{(s_2 m_1 + (s_1 + s_2)m_2)^2 - 4m_1 m_2 s_1 s_2}}{2m_1 m_2}.$$

Natural angle frequencies of oscillating system using data of this example, $\omega_1 = 13.46\ s^{-1}$, $\omega_2 = 45.48\ s^{-1}$, respectively natural frequencies, $f_1 = 2.14\ s^{-1}$, $f_2 = 7.24\ s^{-1}$.

3. THREE DEGREE-OF-FREEDOM DAMPED EXCITED TRANSLATIONAL OSCILLATING SYSTEM

In Fig 3 sketch of three degree-of-freedom damped excited closed translational oscillating system can be seen. Friction coefficient between mass points and horizontal surface is zero. Write down the motion equations of oscillating system on synthetic way and determine the natural angle frequencies concerning this example as well and plot the kinematical functions of the motion!

Data (see Fig 3): $m_1 = 2\ kg$, $m_2 = 1.5\ kg$, $m_3 = 1\ kg$, $s_1 = 150\ N/m$, $s_2 = 180\ N/m$, $s_3 = 210\ N/m$, $k_1 = 10\ Ns/m$, $k_2 = 20\ Ns/m$, $k_2 = 30\ Ns/m$.

Motion equations of mass points can be written by forces acting on each one of them (forces denoted in Fig 3),

$$m_1\ddot{x}_1 + (k_1 + k_2)\dot{x}_1 - k_2\dot{x}_2 + (s_1 + s_2)x_1 - s_2 x_2 = F_{g1}(t),$$

$$m_2\ddot{x}_2 - k_2\dot{x}_1 + (k_2 + k_3)\dot{x}_2 - k_3\dot{x}_3 - s_2 x_1 + (s_2 + s_3)x_2 - s_3 x_3 = F_{g2}(t)$$

$$m_3\ddot{x}_3 - k_3\dot{x}_2 + (k_3 + k_4)\dot{x}_3 - s_3 x_2 + (s_3 + s_4)x_3 = F_{g3}(t)$$

Using above data and $F_{g1o} = 0\ N$, $\omega_{g1} = 0\ s^{-1}$, $F_{g2o} = 0\ N$, $\omega_{g2} = 0\ s^{-1}$, $F_{g3o} = 100\ N$, $\omega_{g3} = 30\ s^{-1}$ furthermore initial conditions are $x_{1o} = 0m$, $\dot{x}_{1o} = 0\frac{m}{s}$, $x_{2o} = 0m$, $\dot{x}_{2o} = 0\frac{m}{s}$, $x_{3o} = 0m$, $\dot{x}_{3o} = 0\frac{m}{s}$ the results of numerical solution (as kinematical functions) in Fig 4 can be studied.

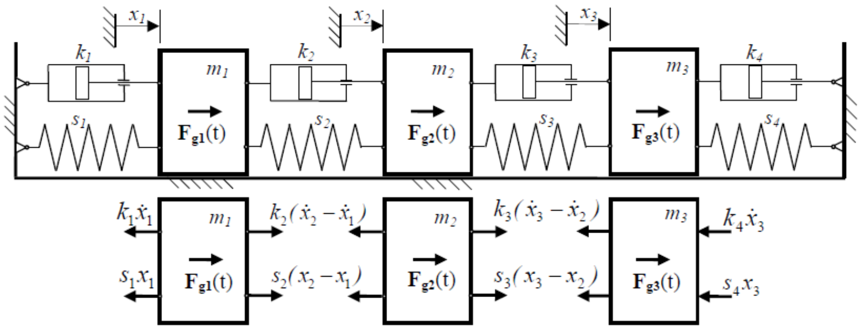
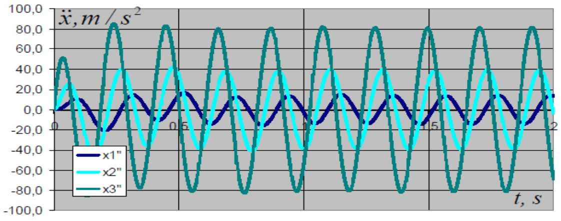
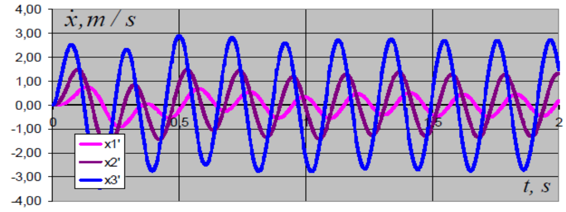


Figure 3. Sketch of three degree-of-freedom damped excited closed translational oscillating system





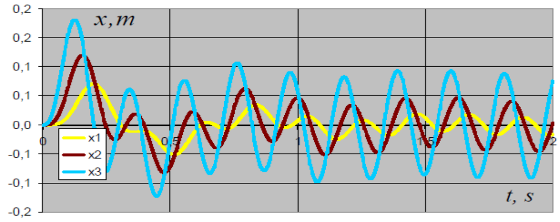


Figure 4. Kinematical functions of three degree-of-freedom damped excited closed translational oscillating system $(0\ s \le t \le 2\ s)$

Let us solve the

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{S}\mathbf{x} = 0$$

homogenous matrix differential-equation in order to calculate the natural frequencies of the oscillating system. In details

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} s_1+s_2 & -s_2 & 0 \\ -s_2 & s_2+s_3 & -s_3 \\ 0 & -s_3 & s_3+s_4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} F_{g1}(t) \\ F_{g1}(t) \\ F_{g1}(t) \end{bmatrix}.$$

Solution can be looked for in form of

$$x_1 = x_{1o}\, sin(\omega t + \varphi_1),\ x_2 = x_{2o}\, sin(\omega t + \varphi_2) \text{ and } x_3 = x_{3o}\, sin(\omega t + \varphi_3).$$

Using their derivatives after rearrangement the following equation-system can be obtained,

$$\begin{bmatrix} -m_1\omega^2 + s_1 + s_2 & -s_2 & 0 \\ -s_2 & -m_1\omega^2 + s_2 + s_3 & -s_3 \\ 0 & -s_3 & -m_1\omega^2 + s_3 + s_4 \end{bmatrix}\begin{bmatrix} x_{1o} \\ x_{2o} \\ x_{3o} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solution different from trivial one can be achieved when determinant of matrix equals to zero, i.e.

$$\begin{vmatrix} -m_1\omega^2 + s_1 + s_2 & -s_2 & 0 \\ -s_2 & -m_1\omega^2 + s_2 + s_3 & -s_3 \\ 0 & -s_3 & -m_1\omega^2 + s_3 + s_4 \end{vmatrix} = 0\,.$$

After development of determinant and rearrangement the characteristic equation of the system,

$$-m_1m_2m_3\omega^6 - \left[m_1m_2(s_3+s_4) + m_1m_3(s_2+s_3) + m_2m_3(s_1+s_2)\right]\omega^4 +$$
$$+\left[m_1(s_2+s_3)(s_3+s_4) + m_2(s_1+s_2)(s_3+s_4) + m_3(s_1+s_2)(s_2+s_3) - m_1s_3^{\,2} - m_3s_2^{\,2}\right]\omega^2 +$$
$$+(s_1+s_2)(s_2+s_3)(s_3+s_4) - (s_1+s_2)s_3^{\,2} - (s_3+s_4)s_2^{\,2} = 0.$$

Above equation is third-degree of ω^2. Plotting it the roots of equation can be determined by zeros of function (Fig 5). Using data of this example natural angle frequencies are $\omega_1=8.44\ s^{-1}$, $\omega_2=15.84\ s^{-1}$, $\omega_3=23.64\ s^{-1}$ and natural frequencies $f_1=1.34\ s^{-1}$, $f_2=2.52\ s^{-1}$, $f_3=376\ s^{-1}$.

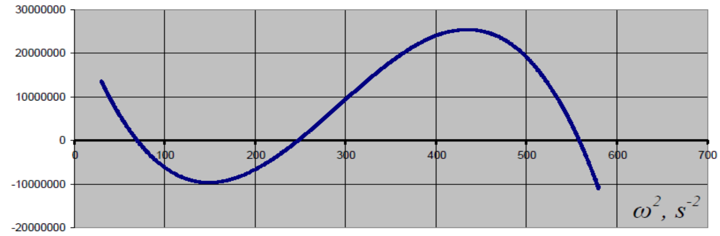


Figure 5. Characteristic equation of three degree-of-freedom translational oscillating system

Data of excitation on resonance angle frequency (Fig. 6): $m_1=2\ kg$, $m_2=1.5\ kg$, $m_3=1\ kg$, $s_1=150\ N/m$, $s_2=180\ N/m$, $s_3=210\ N/m$, $k_1=k_2=k_3=0\ Ns/m$, $F_{g1o}=0\ N$, $\omega_{g1}=0\ s^{-1}$, $F_{g2o}=0\ N$, $\omega_{g2}=0\ s^{-1}$, $F_{g3o}=100\ N$, $\omega_{g3}=23.64\ s^{-1}$. Initial conditions are $x_{1_o}=0m$, $\dot{x}_{1_o}=0\frac{m}{s}$, $x_{2_o}=0m$, $\dot{x}_{2_o}=0\frac{m}{s}$, $x_{3_o}=0m$, $\dot{x}_{3_o}=0\frac{m}{s}$.

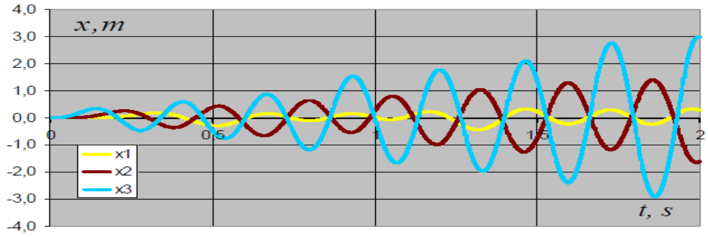


Figure 6. Excitation of translational oscillating system on resonance angle frequency

4. CONCLUSION

It can be concluded: The above demonstrated method can be applied easily for engineer students in the higher education. The method is suitable for investigation of similar oscillating systems having multi-degrees-of freedom. By consequent modification of data (physical quantities) of systems a wide range of possible structures and their kinematical behavior can be analyzed. For this reason the application of this method can be advantageous for engineer students.

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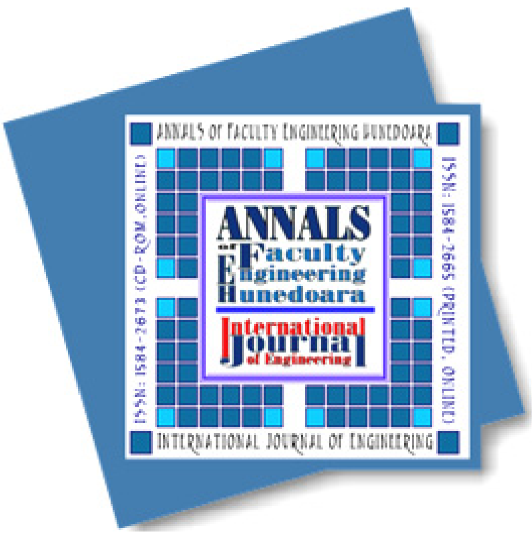

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