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## UNSTEADY/STEADY FLOW OF VISCOUS REACTIVE, HEAT GENERATION/ABSORBING FLUID WITH SORET, DUFOUR AND VARIABLE VISCOSITY EFFECTS

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**Abstract:** This study investigates the unsteady/steady natural convection and mass transfer flow of viscous reactive, heat generating/absorbing fluid in the presence of thermal diffusion, diffusion-thermo and temperature dependent variable viscosity in a vertical channel formed by two infinite parallel plates. The motion of the fluid is induced due to natural convection caused by the reactive property as well as the heat generating/absorbing nature of the fluid. The nonlinear governing equations are solved numerically using unconditionally stable and convergent semi-implicit finite difference scheme. Results of the computations for velocity, temperature, concentration, skin friction, rate of heat and mass transfer are presented graphically and discussed quantitatively for various controlling parameters embedded in the problem.

**Keywords:** Soret, Dufour, heat generation/absorbing, variable viscosity, reactive fluid

### 1. INTRODUCTION

Heat and mass transfer processes by natural convection in the presence of chemical reaction is important in many chemical engineering and petroleum chemical industries. Examples of such area of applications include tubular laboratory reactors, chemical vapor deposition systems, the oxidation of solid materials in large containers, the synthesis of ceramic materials by a self-propagating reaction, combustion in underground reservoirs for enhanced oil recovery and the reduction of hazardous combustion product using catalytic porous beds and many others. Campbell et al [1] concluded that the transport of heat and mass transfer within the reactors is controlled by diffusion or natural convection. When heat and mass transfer occur concurrently between the fluxes, the deriving potentials are of more complicated nature. Mass flux produced by a temperature gradient is refers to thermal diffusion (Soret effect) and heat flux generated by a concentration gradient is term as diffusion-thermo (Dufour effect). The Soret effect has been utilized for isotope separation, and in mixture between gases with very light molecular weight ( $\text{He}$ ,  $\text{H}_2$ ) and of medium molecular weight ( $\text{N}_2$ , air). The Dufour effect was found to be of considerable magnitude such that it cannot be neglected [2]. Many outstanding studies have considered combined thermal diffusion and diffusion-thermo effects in natural convection and mass transfer flow. Sparrow et al [3] studied the effects of Soret and Dufour for helium-air boundary layer in stagnation flow. Combined effects of Soret and Dufour on steady mixed convection flow are to be found in Hayat et al [4], and Alam and Rahman [5]. Cheng [6] analyzed the effects of Soret and Dufour over a vertical cone in a porous medium. In nutshell, there has been considerable published work dealing with steady flow with Soret and Dufour effect, some of them are Tsai and Huang [7, 8], Eugen and Postelnicu [9], Postelnicu [10], Shyam et al [11], Olanrewaju et al [12], Reddy and Reddy [13], Afify [14], Mahdy [15], Osalusi et al [16], Alam et al [17], and Chamkha and Ben-Nakhi [18]. Steady flows with chemical reaction considering Soret and Dufour effect are to be found in Postelnicu [19], Mansour et al [20], Beg et al [21], El-Kabeir et al [22], and

Gangadhar [23]. Unsteady flow problems in the presence of Soret and Dufour effects with chemical reaction can be found in Bhargava et al [24], and Pal and Mondal [25]. Vempati and Laxmi-Narayana-Gari [26] investigated Soret and Dufour effects on MHD flow in a vertical porous plate. Recently Al-Odat and Al-Ghamdi [27] examined numerically the effects of Soret and Dufour on unsteady MHD natural convection flow in a non-Darcy porous medium.

All the above-mentioned studies, assumed the viscosity of the fluid to be constant. However, it is known that the fluid physical properties may change significantly with temperature changes. To accurately predict flow behavior and heat transfer rate it is necessary to take into account the variation of viscosity with temperature Hossain et al [28]. In this direction, Kafoussias and Willians [29], Seddeek [30], and Moorthy and Senthilvadivu [31] studied the effects of thermal diffusion and diffusion-thermo in the presence of temperature dependent variable viscosity. On the other hand, the study of heat generation/absorption effects is also important in view of several physical problems, such as fluids undergoing endothermic or exothermic chemical reactions, fire and combustion modelling, and the development of metal waste from nuclear fuel. All the features for heat generating/absorbing fluids has been reported by Jha and Ajibade [32] in their studies on the free convective flow between vertical porous plates with periodic input. Abdullah [33] analyzed analytically heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall Effect.

Majority of the above-mentioned studies focused on steady flow and considered first order homogeneous chemical reaction. In this paper, an exothermic chemical reaction of Arrhenius kinetic is employed in the investigation of unsteady as well as steady flow of viscous reactive, heat generating/absorbing fluid in the presence of thermal diffusion, diffusion-thermo and temperature dependent variable viscosity in a channel formed by two infinite vertical parallel plates.

## 2. MATHEMATICAL ANALYSIS

Consider the unsteady natural convection and mass transfer flow of viscous reactive, heat generating/absorbing fluid between two infinite vertical parallel plates (see fig.1). The convective current is as a result of the reactive property of the fluid as well as heat generating/absorbing nature of the fluid. Initially, it is assumed that both the fluid and the plates are at rest and at same temperature and concentration  $T_0'$  and  $C_0'$  respectively. At time  $t' > 0$  the temperature and concentration of the plate  $y' = 0$  is raised to  $T_\omega'$  and  $C_\omega'$ , and thereafter remains constant and that of  $y' = H$  is lowered to  $T_0'$  and  $C_0'$ , where  $T_\omega' > T_0'$  and  $C_\omega' > C_0'$ . The Soret and Dufour effects are taken into account. The fluid temperature dependent variable viscosity is of type given by Kafoussias and Willians [29]  $\bar{\mu} = \frac{\mu_0}{[1 + \gamma(T' - T_0')]}$ , where  $\mu_0$  is the initial fluid dynamic viscosity at

the temperature,  $T_0'$ . Under these assumptions along with the Boussinesq's approximation, the governing equations can be written as

$$\frac{\partial u'}{\partial t'} = \frac{\partial}{\partial y'} \left( \bar{\mu} \frac{\partial u'}{\partial y'} \right) + g \beta (T' - T_0') + g \beta^* (T' - T_0') \quad (1)$$

$$\frac{\partial C'}{\partial t'} = D_m \frac{\partial^2 C'}{\partial y'^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T'}{\partial y'^2} \quad (2)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 u'}{\partial y'^2} + \frac{QC_0^* A}{\rho C_p} e^{\left(\frac{-E}{RT'}\right)} + \frac{D_m k_T}{c_s C_p} \frac{\partial^2 C'}{\partial y'^2} + \frac{Q_0 (T_0' - T')}{\rho C_p} \quad (3)$$

The initial and boundary conditions for the present problem are

$$\left. \begin{aligned} t' \leq 0 : u' = 0, T' \rightarrow T_0', C' \rightarrow C_0', 0 \leq y' \leq H \\ t' > 0 : u' = 0, T' = T_\omega', C' = C_\omega' \text{ at } y' = 0 \\ u' = 0, T' = T_0', C' = C_0' \text{ as } y' \rightarrow H \end{aligned} \right\} \quad (4)$$

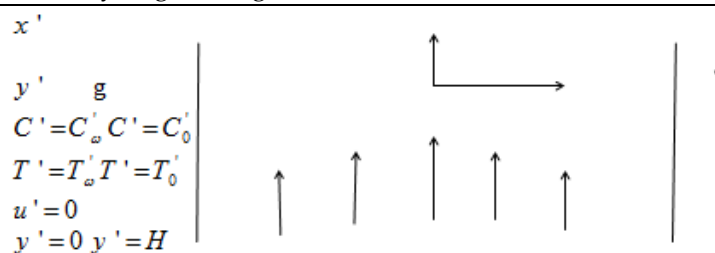


Fig.1. Schematic diagram of the physical model and coordinate system

where  $\beta$  is the coefficient of thermal expansion,  $\beta^*$  is the coefficient of concentration expansion,  $Q$  is the heat of reaction,  $Q_0$  is the dimensional heat generating/absorbing fluid parameter,  $A$  is the rate constant,  $E$  is the activation energy,  $R$  is the universal gas constant,  $\nu$  is the kinematic viscosity,  $C_0^*$  is the initial concentration of the reactant species,  $g$  is the gravitational force,  $C_p$  is the specific heat at constant pressure,  $k$  is the thermal conductivity of the fluid,  $\rho$  is the density of the fluid,  $D_m$  is the coefficient of mass diffusivity,  $T_m$  is the mean fluid temperature,  $k_T$  is the thermal diffusion ratio and  $c_s$  is the concentration susceptibility.

To solve equations (1) to (4), we employ the following dimensionless variables and parameters

$$\left. \begin{aligned} y &= \frac{y'}{H}, \quad t = \frac{t' \mu_0}{H^2}, \quad U = \frac{u'}{\mu_0}, \quad \text{Pr} = \frac{\mu_0 \rho C_p}{k}, \quad Gc = \frac{g \beta^* R C_0^2 H^2}{E \mu_0^2}, \quad Gr = \frac{g \beta R T_0^2 H^2}{E \mu_0^2} \\ \theta &= \frac{E(T' - T_0)}{R T_0^2}, \quad \lambda = \frac{Q C_0^* A E H^2}{R T_0^2} e^{\left(\frac{-E}{R T_0}\right)}, \quad Sc = \frac{\nu}{D_m}, \quad Sr = \frac{k_T T_0^2}{T_m C_0^2}, \quad C = \frac{E(C' - C_0)}{R C_0^2} \\ \varepsilon &= \frac{R T_0}{E}, \quad \theta_a = \frac{E(T_\infty - T_0)}{R T_0^2}, \quad C_a = \frac{E(C_\infty - C_0)}{R C_0^2}, \quad S = \frac{Q_0 H^2}{k}, \quad Du = \frac{D_m k_T C_0^2}{\mu_0 c_s C_p T_0^2}, \quad X = \frac{b R T_0^2}{E} \end{aligned} \right\} \quad (5)$$

Using (5), the equations (1) to (5) take the following form:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial y} \left( (1 - X \theta) \frac{\partial U}{\partial y} \right) + Gr \theta + Gc C \quad (6)$$

$$Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2} \quad (7)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + \frac{\lambda}{\text{Pr}} e^{\left(\frac{\theta}{1 + \varepsilon \theta}\right)} - \frac{S \theta}{\text{Pr}} + Du \frac{\partial^2 C}{\partial y^2} \quad (8)$$

The initial and boundary conditions in dimensionless form are

$$\left. \begin{aligned} u &= 0, \quad \theta = 0, \quad 0 \leq y \leq 1, \quad t \leq 0 \\ t > 0: u &= 0, \quad \theta = \theta_a, \quad C = C_a \quad \text{at} \quad y = 0 \\ u &= 0, \quad \theta = 0, \quad C = 0, \quad \text{as} \quad y = 1 \end{aligned} \right\} \quad (9)$$

where  $X$ ,  $Gr$ ,  $Gc$ ,  $S$ ,  $Sc$ ,  $Sr$ ,  $\text{Pr}$ ,  $\lambda$ ,  $\varepsilon$ ,  $Du$ ,  $\theta_a$ , and  $C_a$  are variable viscosity parameter, thermal Grashof number, mass Grashof number, heat generating/absorbing parameter, Schmidt number, Soret number, Prandtl number, Frank-Kamenetskii parameter, activation energy parameter, Dufour number, ambient temperature, and ambient concentration respectively. The other non-dimensional quantities are the skin friction ( $C_f$ ), the heat transfer rate ( $Nu$ ), and the rate of mass transfer ( $Sh$ ) given as

$$C_f = \frac{\partial U}{\partial y} \Big|_{y=0,1}, \quad Nu = -\frac{\partial \theta}{\partial y} \Big|_{y=0,1}, \quad Sh = -\frac{\partial C}{\partial y} \Big|_{y=0,1}, \quad \text{where } C_f \text{ is the skin friction, } Nu \text{ is the Nusselt}$$

number and  $Sh$  is the Sherwood number.

### 3. NUMERICAL SOLUTIONS

The non-dimensional governing equations (6) to (8) with the boundary conditions (9) are solved numerically using semi-implicit finite difference scheme given in [34]. We used forward difference formulas for all time derivatives and approximate both the second and first derivatives with

second order central differences. The semi-implicit finite difference equation corresponding to equations (6) to (8) are as follows

$$-r_1 \mu U_{j-1}^{(N+1)} + (1 + 2r_1 \mu) U_j^{(N+1)} - r_1 \mu U_{j+1}^{(N+1)} = r_2 \mu U_{j-1}^{(N)} + (1 - 2r_2 \mu) U_j^{(N)} + r_2 \mu U_{j+1}^{(N)} - r_3 (\theta_{j+1}^N - \theta_{j-1}^N) (U_{j+1}^N - U_{j-1}^N) + \Delta t Gr \theta_j^N + \Delta t Gc C_j^N \quad (10)$$

$$-r_1 C_{j-1}^{(N+1)} + (Sc + 2r_1) C_j^{(N+1)} - r_1 C_{j+1}^{(N+1)} = r_2 C_{j-1}^{(N)} + (Sc - 2r_2) C_j^{(N)} + r_2 C_{j+1}^{(N)} + r_5 (\theta_{j-1}^N - 2\theta_j^N + \theta_{j+1}^N) \quad (11)$$

$$-r_1 \theta_{j-1}^{(N+1)} + (Pr + 2r_1) \theta_j^{(N+1)} - r_1 \theta_{j+1}^{(N+1)} = r_2 \theta_{j-1}^{(N)} + (Pr - 2r_2 - S \Delta t) \theta_j^{(N)} + r_2 \theta_{j+1}^{(N)} + \lambda \Delta t \exp\left(\frac{\theta_j^{(N)}}{1 + \varepsilon \theta_j^{(N)}}\right) + r_4 (C_{j-1}^N - 2C_j^N + C_{j+1}^N) \quad (12)$$

where  $r_1 = \xi \Delta t / \Delta y^2$ ,  $r_2 = (1 - \xi) \Delta t / \Delta y^2$ ,  $r_3 = \mu X \Delta t / 4 \Delta y^2$ ,  $r_4 = Du \Pr \Delta t / \Delta y^2$ ,  $r_5 = Sr \Delta t / \Delta y^2$ ,  $\mu = 1 - X \theta$ , and  $0 \leq \xi \leq 1$ . We chose  $\xi = 1$  the detailed reasons to this particular selection is documented in [34].

#### 4. RESULTS AND DISCUSSION

To study the behavior of the solutions, numerical calculations for some values of Frank-Kamenetskii parameter ( $\lambda$ ), Soret number (Sr), Dufour number (Du), variable viscosity parameter ( $X$ ), heat generating/absorbing parameter (S), Schmidt number (Sc), Prandtl number (Pr), thermal Grashof number (Gr), mass Grashof number (Gc), and non-dimensional time (t) have been carried out. Unless otherwise stated, the values:  $\lambda = 0.1$ ,  $X = 0.1$ ,  $Du = 0.01$ ,  $Gr = 0.1$ ,  $S = 0.1$ ,  $Gc = 0.1$ ,  $Sr = 0.1$ ,  $\theta_a = 1$ ,  $C_a = 1$ ,  $Pr = 0.71$ ,  $Sc = 0.62$ ,  $t = 0.1$  and  $\varepsilon = 0.01$  are used for the investigation. Results obtained are displayed graphically for velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number for various flow parameters.

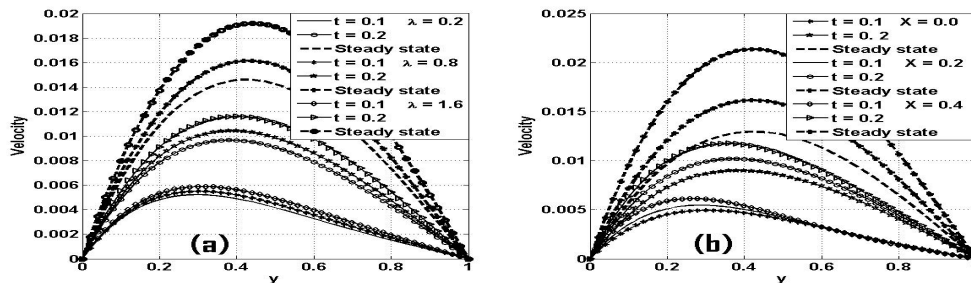


Fig.2. Variation of unsteady and steady-state velocity with  $\lambda$  and  $X$

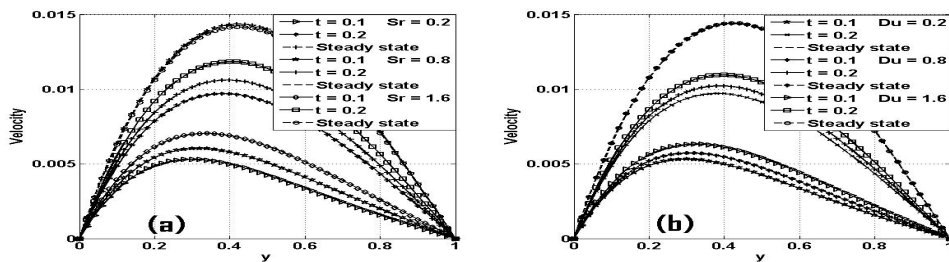


Fig.3. Variation of unsteady and steady-state velocity with  $Sr$  and  $Du$

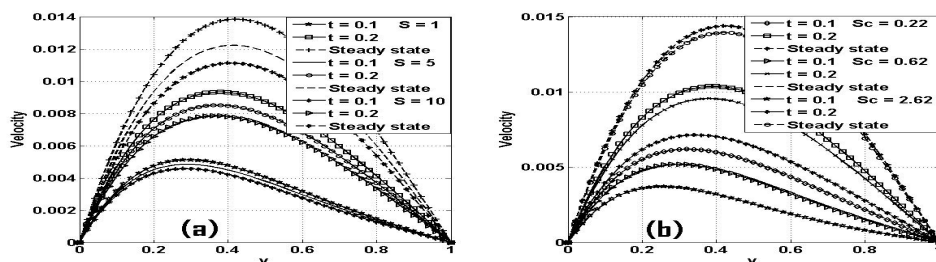
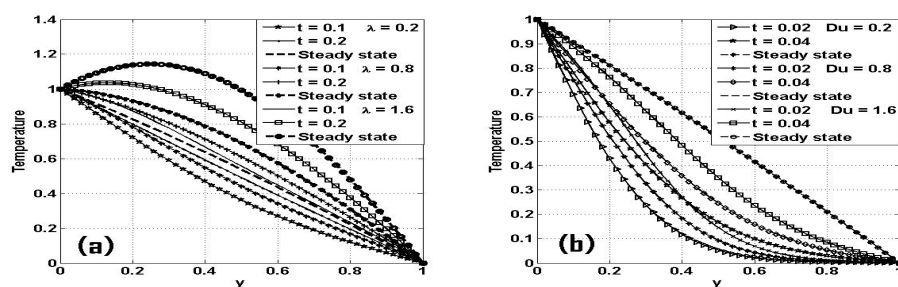
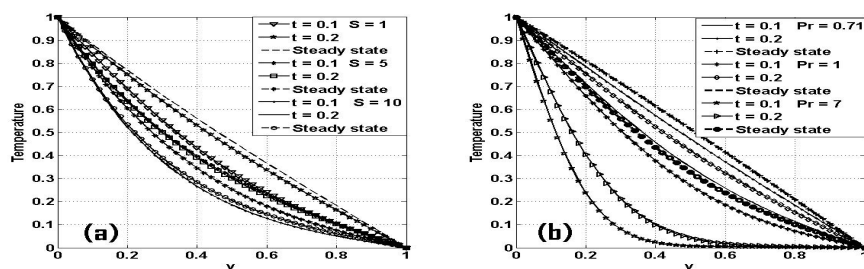
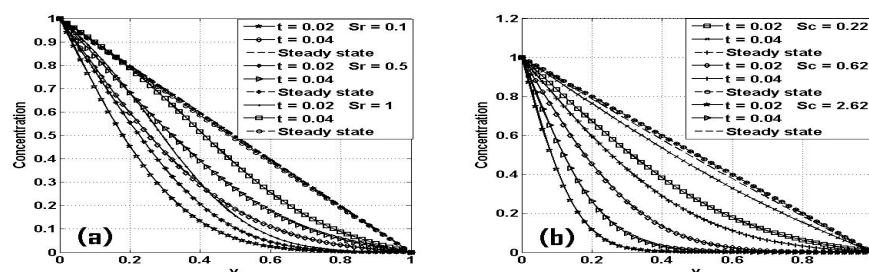
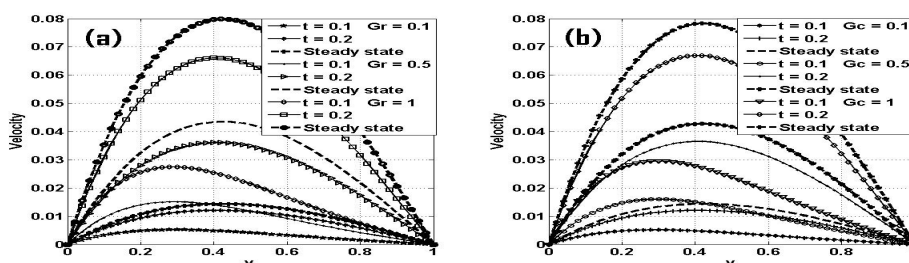


Fig.4. Variation of unsteady and steady-state velocity with  $S$  and  $Sc$



Fig.5. Variation of unsteady and steady-state temperature with  $\lambda$  and  $Du$ Fig.6. Variation of unsteady and steady-state temperature with  $S$  and  $Pr$ Fig.7. Variation of unsteady and steady-state concentration with  $Sr$  and  $Sc$ Fig.8. Variation of unsteady and steady-state velocity with  $Gr$  and  $Gc$ 

In fig.2a, the effect of the Frank-Kamenetskii parameter ( $\lambda$ ) and non-dimensional time ( $t$ ) on velocity profiles is analyzed. From this figure it is observed that both unsteady and steady-state velocity of the fluid increases with increasing values of  $\lambda$ . This is physically true since an increase in  $\lambda$  lead to significant increases in the reaction and viscous heating source terms and hence considerably increases the fluid velocity. The influence of the variable viscosity parameter ( $X$ ) and time ( $t$ ) on the velocity profiles is illustrated in fig.2b. The graph shows that as  $X$  increases, the fluid becomes less viscous and hence their molecular resistance to motion decreases, and thereby increases in the fluid flow. It is also seen from fig.2b that the velocity of the fluid is maximum in the vicinity of the hot wall ( $y = 0$ ) and then gradually decreases as it moves towards the cooled wall ( $y = 1$ ). The response of the velocity variations in the thermal diffusion ( $Sr$ ) and diffusion-thermo ( $Du$ ) is illustrated in fig.3a and 3b respectively. Fig.3a and 3b revealed that an increase in  $Sr$  and  $Du$  accelerate the velocity of the fluid. In addition, as time increases, the velocity of the fluid also increases until a steady-state value is achieved. Fig.4a and 4b represent the velocity profiles for different values of heat sinks parameter ( $S$ ) and Schmidt number ( $Sc$ ) respectively. From fig.4a it is noted that velocity of the fluid decreases with increasing values of  $S$ . As expected, since increasing values of  $S$  results to the cooling of the fluid which leads to the decrease in the fluid

velocity. The values of Schmidt number ( $Sc$ ) are chosen for hydrogen ( $Sc = 0.22$ ), water vapor ( $Sc = 0.62$ ) at temperature  $25^{\circ}\text{C}$  and propylbenzene ( $Sc = 2.62$ ). In fig.4b, it is seen that small value of  $Sc$  increases both unsteady and steady-state velocity, while higher values of  $Sc$  decreases the velocity of the fluid.

The effect of the Frank-Kamenetskii parameter ( $\lambda$ ) and diffusion-thermo ( $Du$ ) on the temperature field is displayed in fig.5a and 5b respectively. An increase in  $\lambda$  cause an increase in the reaction and viscous heating source terms and in turn increase the fluid temperature see fig.5a. The diffusion-thermo ( $Du$ ) represents the thermal energy flux in the flow. It is clear from fig.5b that an increase in  $Du$  causes a rise in the temperature of the fluid. Fig.6a and 6b show the influence of the heat sinks parameter ( $S$ ) and Prandtl number ( $Pr$ ) on the temperature field respectively. From fig.6a it is observed that temperature of the fluid decreases with increasing  $S$ . This is true since increasing  $S$  results to the cooling of the fluid which leads to the decrease in the temperature of the fluid. Larger values of the Prandtl number ( $Pr$ ) correspondingly decrease the strength of the source terms in the temperature equation and hence in turn reduce the temperature as clearly illustrated in fig.6b. Fig.7a and 7b represent the concentration profiles for different values of  $Sr$  and  $Sc$  respectively. It is evident from fig.7a that increasing values of  $Sr$  increases the concentration of the fluid. Fig.7b revealed that small value of  $Sc$  increases fluid concentration while larger value of  $Sc$  reduces the concentration of the fluid. Fig.8a and 8b represent the velocity profiles for different values of the thermal Grashof number ( $Gr$ ) and mass Grashof number ( $Gc$ ) respectively. These plots fig.8a and 8b indicate that the momentum boundary layer thickness increases with increasing values of  $Gr$  and  $Gc$ . The consequence of the increased buoyancy source terms due to higher values of  $Gr$  and  $Gc$  increases both unsteady and steady-state velocity of the fluid.

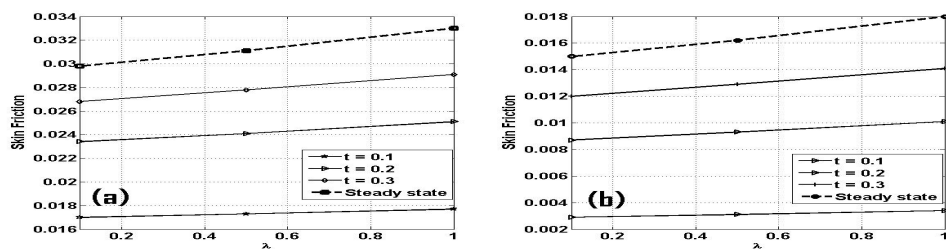


Fig.9. Variation of unsteady and steady state skin friction at  $y = 0$  and  $y = 1$  with  $\lambda$

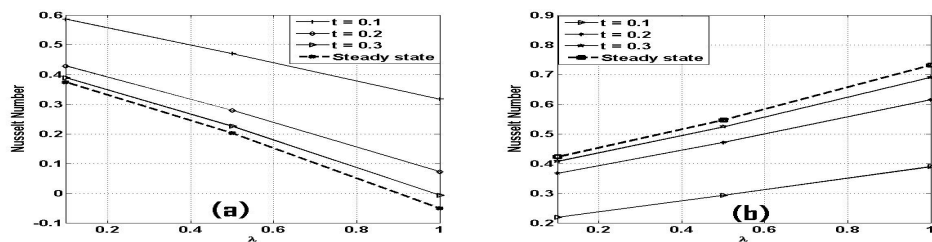


Fig.10. Variation of unsteady and steady state Nusselt number at  $y = 0$  and  $y = 1$  with  $\lambda$

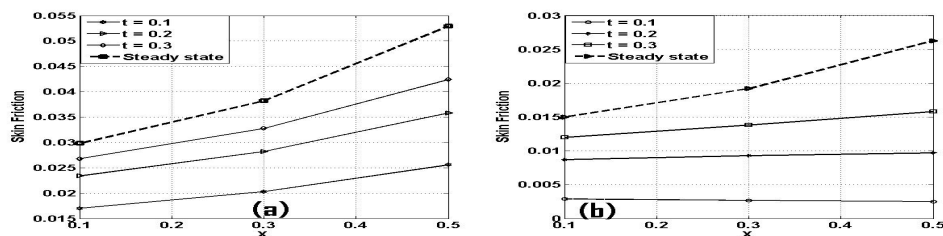


Fig.11. Variation of unsteady and steady state skin friction at  $y = 0$  and  $y = 1$  with  $X$

The wall shear stress dependence on Frank-Kamenetskii parameter ( $\lambda$ ) for varying values of time ( $t$ ) at the plate  $y = 0$  and  $y = 1$  is illustrated in fig.9a and 9b respectively. From fig.9a and 9b it can be noticed that skin friction increases with increasing both  $\lambda$  and  $t$  until a steady-state condition is achieved. Fig.10a and 10b are plotted to see the effect of  $\lambda$  and  $t$  on the rate of heat

transfer (Nusselt number) at the plate  $y = 0$  and  $y = 1$  respectively. Fig.10b reveals that the rate of heat transfer increases as  $\lambda$  and  $t$  increases until a steady-state condition is attained. A reverse effect is observed at the plate  $y = 0$  see fig.10a.

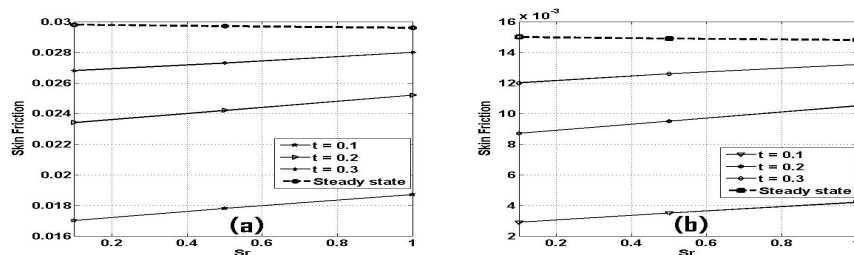


Fig.12. Variation of unsteady and steady state skin friction at  $y = 0$  and  $y = 1$  with  $Sr$

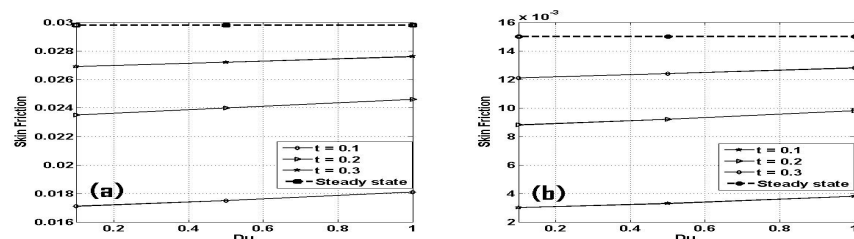


Fig.13. Variation of unsteady and steady state skin friction at  $y = 0$  and  $y = 1$  with  $Du$

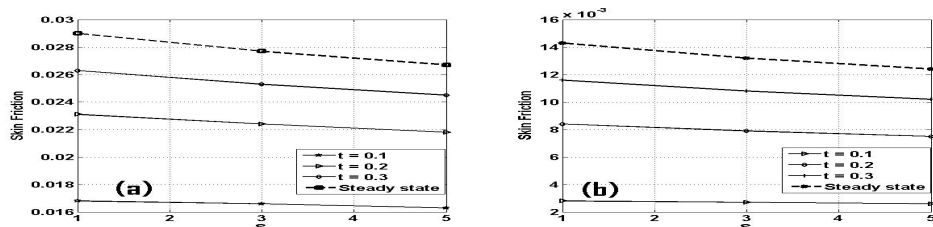


Fig.14. Variation of unsteady and steady state skin friction at  $y = 0$  and  $y = 1$  with  $S$

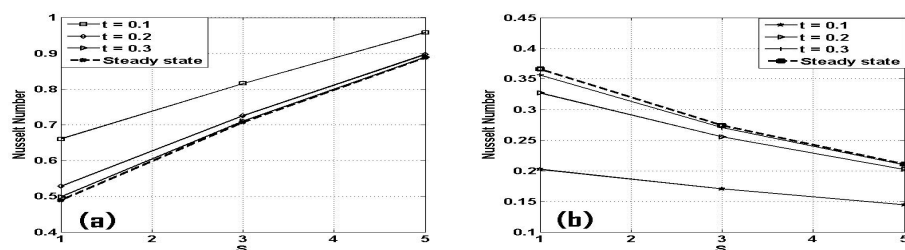


Fig.15. Variation of unsteady and steady state Nusselt number at  $y = 0$  and  $y = 1$  with  $S$

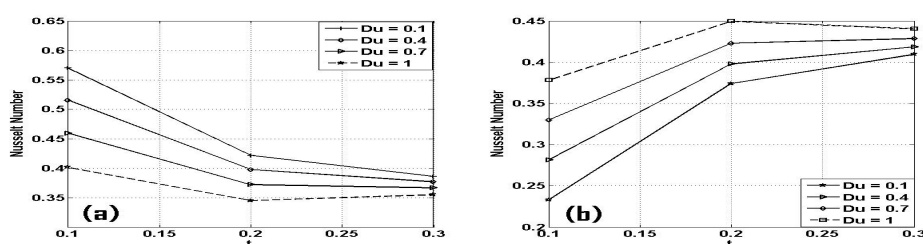


Fig.16. Variation of Nusselt number at  $y = 0$  and  $y = 1$  with  $Du$  and  $t$

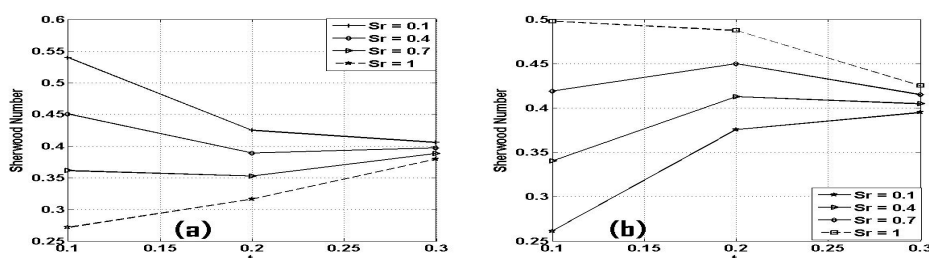


Fig.17. Variation of Sherwood number at  $y = 0$  and  $y = 1$  with  $Sr$  and  $t$

The skin friction dependence on variable viscosity parameter ( $X$ ) for varying values of time ( $t$ ) at the plate  $y = 0$  and  $y = 1$  is illustrated in fig.11a and 11b respectively. These figures reflected that skin friction increases with increasing  $X$  and  $t$  until a steady-state condition is reached. Figs.12 and 13 represent the skin friction dependence on  $Sr$  and  $Du$  for varying values of time respectively. It is clear from these plots that skin friction increases as  $Sr$  and  $Du$  increases. It is also noticed that as  $t$  increases, the skin friction increases until a steady-state value is achieved.

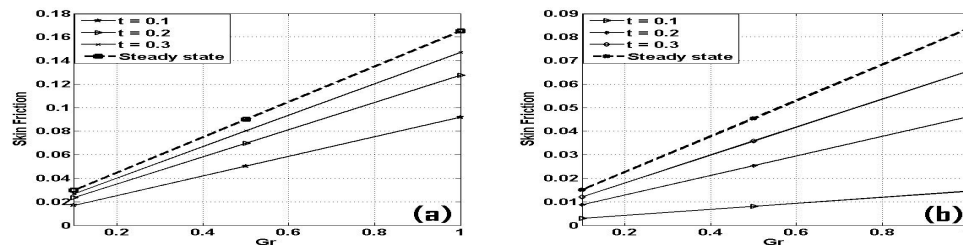


Fig.18. Variation of unsteady and steady state skin friction at  $y = 0$  and  $y = 1$  with  $Gr$

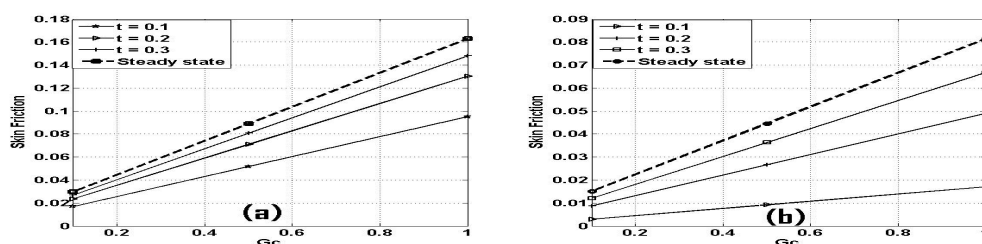


Fig.19. Variation of unsteady and steady state skin friction at  $y = 0$  and  $y = 1$  with  $Gc$

Figs.14 and 15 illustrated the wall shear stress and Nusselt number dependence of heat sinks parameter ( $S$ ) for varying values of time ( $t$ ). It is seen from fig.14 that skin friction slightly decreases with increasing values of  $S$  and  $t$  at the plate  $y = 0$  and  $y = 1$  until a steady-state is reached. In fig.15b it is observed that rate of heat transfer at the wall  $y = 1$  decreases with increasing values of  $S$  and  $t$ . A similar effect is observed but in opposite direction at the plate  $y = 0$  sees fig.15a.

The rate of heat transfer (Nusselt number) dependence on time ( $t$ ) at the plate  $y = 0$  and  $y = 1$  is depicted in fig.16a and 16b respectively for varying values of  $Du$ . It is clearly seen from fig.16a that Nusselt number decreases as  $Du$  and  $t$  increase. Similar trend is observed but in opposite direction at the plate  $y = 1$  sees fig.16a. The rate of mass transfer (Sherwood number) dependence on  $t$  at the plate  $y = 0$  and  $y = 1$  is displayed in fig.17a and 17b respectively for varying values of  $Sr$ . Fig.17a reveals that the rate of mass transfer decreases due to the increase in  $Sr$  and time. A similar trend is noticed but in opposite direction at the plate  $y = 1$  sees fig.17b. Figs.18 and 19 show the wall shear stress dependence on  $Gr$  and  $Gc$  for varying values of  $t$  respectively. From these figures it is noticed that as time increase, the frictional force due to the motion of the fluid also increases until a steady-state condition is attained. Figs.18 and 19 reflected that values of  $Gr$  and  $Gc$  increase the skin friction.

## 5. CONCLUSION

In this paper, the problem of unsteady as well as steady flow of viscous reactive, heat generating/absorbing fluid between two infinite vertical walls in the presence of thermal diffusion, diffusion-thermo and temperature dependent variable viscosity was investigated numerically using unconditionally stable and convergent semi-implicit finite difference scheme. It is found that thermal diffusion, diffusion-thermo, non-dimensional time and temperature dependent variable viscosity accelerate fluid flow while heat sinks reduce the motion of the fluid.

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