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THERMO CREEP TRANSITION IN FUNCTIONALLY GRADED THICK-WALLED CIRCULAR CYLINDER UNDER EXTERNAL PRESSURE

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Abstract: Thermal creep stresses have been obtained for thick-walled circular cylinder under external pressure using transition theory which is based on the concept of generalized principal strain measure that simplifies the constitutive equations by prescribing a priory the order of the measure of deformation and helps to achieve better agreement between the theoretical and experimental results. Results obtained have been analyzed and discussed numerically as well as graphically. From our analysis we can conclude that by introducing a suitably chosen temperature gradient, the compressible stresses due to pressure may be increased, which leads to the idea of 'Stress Saving'. It means that introduction of thermal effects minimizes the possibility of fracture at the internal surface for N = 5.
Keywords: Creep, Cylinder, Pressure, Non-homogeneous, Thermal

1. INTRODUCTION

Thick-walled cylinder is a common structure type which can be used in applications involving aerospace, submarine structures, nuclear reactors as well as chemical pipes. The broad use of cylindrical structural elements in mechanical engineering has stimulated great interest of their characteristics. Due to recent aerospace and commercial applications, non-homogeneous materials are effectively utilized. Some degree of non-homogeneity is present in wide class of materials such as hot rolled metals, aluminium and magnesium alloys. Non-homogeneity can also be generated by certain external field that is a thermal field, as the elastic module of the material varies with temperature or co-ordinates, etc. The effect of induced non-homogeneity on the stress distribution caused by external fields is much more pronounced and of larger duration than the effect of thermal stresses themselves. The mathematical theory of elasticity and creep has been analyzed by many authors [2, 6-7]. Bhatnagar and Arya [3] shows that the anisotropy of the material has a significant effect on the creep behaviour of a thick-walled cylinder and these stresses, strains and strain rates are found to be dependent on material anisotropy. Vasilenko [12] proposed an approach to determining the stress state of anisotropic cylinders with arbitrary inhomogeneity of the elastic properties over the thickness, under the action of centrifugal forces due to rotation at constant angular velocity and the length of the cylinders is assumed to be such that the influence of the ends may be neglected. The effect of material parameters on steady state creep in a thick composite Al-SiC_P cylinder subjected to internal pressure is studied by Singh and Gupta [11]. Deepak et al. [5] have investigated creep behaviour of rotating discs made of functionally graded materials with linearly varying thickness. Yoo et. al. [13] investigated the collapse behaviour of cylinders subjected to pressure for elastic - perfectly plastic cylinder. Safety analysis has been done for thick-walled circular cylinder under internal and external pressure using transition theory by Aggarwal et.al [1] which is based on the concept of generalized principal Lebesgue strain measure.

They concluded that circular cylinder made of functionally graded material is on the safer side of the design as compared to homogeneous cylinder with internal and external pressure that minimizes the possibility of fracture of cylinder. Sharma and Sahni [10] investigated creep stresses for a thin rotating disc having variable thickness and variable density with edge load and concluded that a rotating disc whose density and thickness decreases radially with edge load is much safer for a design in comparison to a flat rotating disc with variable density. It is observed by Sharma et al. [9] that the rotating disc made of incompressible material with inclusion require higher angular speed to yield at the internal surface as compared to disc made of compressible material. In the present work, we propose an approach of transition theory to determining the thermal stresses for non-homogeneous thick walled cylinder under external pressure. Taking the non-homogeneity as the compressibility of the material in the cylinder as,

$$C = C_0\, r^{-k}, \tag{1}$$

where $a \le r \le b$, C_0 and k are constants.

Results obtained have been discussed numerically and depicted graphically. The generalized principal strain measure [4, 8] is defined as

$$e_{ii} = \int_0^{e_{ii}^A} \left[1 - 2e_{ii}^A\right]^{\frac{n}{2}-1} de_{ii}^A = \frac{1}{n}\left[1 - \left(1 - 2e_{ii}^A\right)^{\frac{n}{2}}\right]. \tag{2}$$

2. GOVERNING EQUATIONS

Consider a non-homogeneous thick-walled circular cylinder of internal and external radii a and b respectively, subjected to external pressure p and temperature θ_0 on the inner surface $r = a$. The non-homogeneity in the cylinder is due to variation of compressibility C. The cylinder is taken so large that plane transverse sections remains plane during the expansion, and hence the longitudinal strain is the same for all elements at each stage of the expansion.

In cylindrical polar co-ordinates the displacements are given by [1, 4, 8-11],

$$u = r(1-\beta)\ , \quad v = 0 \quad \text{and} \quad w = d\, z, \tag{3}$$

where β is a function of $r = \sqrt{x^2 + y^2}$ and d is a constant.

The generalized components of strain are,

$$e_{rr} = \frac{1}{n}\left[1 - (r\beta' + \beta)^n\right],\ e_{\theta\theta} = \frac{1}{n}\left[1 - \beta^n\right],\ e_{zz} = \frac{1}{n}\left[1 - (1-d)^n\right],\ e_{r\theta} = e_{\theta z} = e_{zr} = 0, \tag{4}$$

where n is the measure and $\beta' = d\beta/dr$.

The stress strain relation for thermo elastic isotropic material is

$$T_{ij} = \lambda\delta_{ij} I_1 + 2\mu e_{ij} - \xi\theta\delta_{ij},\ (i, j = 1, 2, 3) \tag{5}$$

where $I_1 = e_{kk}$; T_{ij}, e_{ij} are stress and strain tensors respectively, $\xi = \alpha(3\lambda + 2\mu)$, λ, μ are Lame's constant, δ_{ij} is Kronecker delta, α being coefficient of thermal expansion and θ is temperature.

The temperature θ has to satisfy $\theta_{ii} = 0$. (6)

The temperature field satisfying equation (5) is $\theta = \theta_0$ at $r = a$; $\theta = 0$ at $r = b$, where θ_0 is constant, is given by $\theta = \left(\theta_0 \log\frac{r}{b}\right)\Big/\left(\log\frac{a}{b}\right)$

Equations of equilibrium are all satisfied except

$$\frac{d}{dr}(T_{rr}) + \frac{(T_{rr} - T_{\theta\theta})}{r} = 0. \tag{7}$$

Using equation (5) in equation (7), one gets a non linear equation in β as

$$nP\beta(P+1)^{n-1}\frac{dP}{d\beta} = \begin{bmatrix} r\left(\frac{\mu'}{\mu} - \frac{C'}{C}\right)[\{(3-2C) - (1-C)(1-d)^n\}\frac{1}{\beta^n} - (1-C) - \\ (P+1)^n] + C[1 - (P+1)^n] + r\ C'[1 - \{2 - (1-d)^n\}\frac{1}{\beta^n}] - \\ nP[(1-C) + (P+1)^n] - \frac{Cn\bar{\theta}_0}{2\mu\beta^n}\left(\xi + r\xi'\log\frac{r}{b}\right) \end{bmatrix}, \tag{8}$$

where $r\beta' = \beta P$; $C = \frac{2\mu}{(\lambda+2\mu)}$; and $\bar{\theta}_0 = \frac{\theta_0}{\log\frac{a}{b}}$.

The transition point of β in equation (8) are $P \to$ -1 and $P \to \pm\infty$.

The boundary conditions are $T_{rr} = 0$ at $r = a$; $T_{rr} = -p$ at $r = b$. (9)

The resultant axial force in the cylinder is given by

$$L = 2\pi\int_a^b rT_{zz}dr \cdot \tag{10}$$

3. SOLUTION THROUGH THE PRINCIPAL STRESS DIFFERENCE

When a deformable solid is subjected to an external loading then the solid first deforms elastically and if this loading is continued then plastic flow may start. If this loading is continued further, it gives rise to time dependent deformation which is known as creep deformation. In this problem, we only considered the principal stresses σ_{rr}, $\sigma_{\theta\theta}$ and σ_{zz} and therefore, the transition can take place either through the principal stresses σ_{rr} or $\sigma_{\theta\theta}$ becoming critical or through the principal stress difference $\sigma_{rr}-\sigma_{\theta\theta}$ becoming critical. It has been shown by many authors [3, 10-11] that transition through principal stress difference leads to creep for critical point $P \to -1$. Thus, the transition function R is defined as

$$R = T_{rr} - T_{\theta\theta} = \frac{2\mu\beta^n}{n}\left[1-(P+1)^n\right], \tag{11}$$

Taking logarithmic differentiation of the equation (11) with respect to 'r' and taking asymptotic value of β as $P \to -1$ and integrating, one gets

$$R = A\frac{\mu^2}{C\,r^{2n}}\exp f , \tag{12}$$

where D being a constant and $f = \int\left\{\begin{array}{l}\frac{(n-1)}{r}C - C\frac{\mu'}{\mu} + \frac{C}{\beta^n}\left[2-(1-d)^n\right] - \left(\frac{\mu'}{\mu} - \frac{C'}{C}\right)\\ \left[(3-2C)-(1-C)(1-d)^n\right]\frac{1}{\beta^n} + \frac{Cn\,\bar{\theta}_0}{2\mu\,r\,\beta^n}\left[\xi + r\xi'\log\frac{r}{b}\right]\end{array}\right\}dr$.

From equations (11) and (12), it is found that

$$T_{rr} - T_{\theta\theta} = A\,rF \text{, where } F = \frac{\mu^2}{C\,r^{2n+1}}\exp f \cdot \tag{13}$$

Substituting the value of $T_{rr} - T_{\theta\theta}$ from equation (13) in equation (7) and integrating, we get $T_{rr} = B - A\int F dr$, where B is constant of integration. (14)

The constants A and B are obtained by using the boundary conditions (9) as

$$A = \frac{p}{\int_a^b F dr}, \quad B = \frac{p}{\int_a^b F dr}\left[\int F dr\right]_{r=a}$$

As the non homogeneity in the cylinder is due to variable compressibility C given by equation (1), the creep stresses in a non homogeneous cylinder under external pressure have been obtained as

$$T_{rr} = A_2\int_r^a F_1 dr,\ T_{\theta\theta} = A_2\left[\int_r^a F_1 dr - rF\right], T_{zz} = A_2\left(\frac{1-C_0r^{-k}}{2-C_0r^{-k}}\right)\left[2\int_r^a F_1 dr - rF_1\right] + Ee_{zz} - E\alpha\theta, \tag{15}$$

where $e_{zz} = \frac{\left[\frac{L}{2\pi} - \int_a^b \frac{r(1-C_0r^{-k})}{(2-C_0r^{-k})}\left[T_{rr}+T_{\theta\theta}\right]dr + E\alpha\int_a^b r\theta dr\right]}{E\left(\frac{b^2-a^2}{2}\right)}$, $A_2 = \frac{p}{\int_a^b F_1 dr}$, $F_1 = \frac{r^{-(2n+k+1)}E^2(2-C_0r^{-k})^2}{4C_0(3-2C_0r^{-k})^2}\exp f_1$

$$f_1 = -\frac{(n-1)}{k}C_0r^{-k} - \frac{2kC_0r^{n-k}}{D^n(n-k)} + \frac{kC_0}{D^n}\left[\int\frac{r^{n-k-1}(3-2C_0r^{-k})}{(1-C_0r^{-k})}dr\right] + \log(1-C_0r^{-k}) + $$
$$+\frac{n\bar{\theta}_0C_0}{ED^n}\int\frac{(3-2C_0r^{-k})r^{k-2n-1}}{(2-C_0r^{-k})}\left[\xi + r\xi'\log\frac{r}{b}\right]dr,$$

Equation (15) give thermal creep stresses for a thick walled circular cylinder having variable compressibility.

Let us now introduce the following non-dimensional components as

$$R_0 = \frac{a}{b},\ R = \frac{r}{b},\ \sigma_r = \frac{T_{rr}}{E},\ \sigma_\theta = \frac{T_{\theta\theta}}{E},\ \sigma_z = \frac{T_{zz}}{E}.$$

The equations (15) in non dimensional form can be written as

$$\sigma_r = \frac{P^* \int_R^{R_0} F_1^* dr}{\int_{R_0}^{1} F_1^* dr},\ \sigma_\theta = \sigma_r - RA_2F_1^*,\ \sigma_z = \left(\frac{1-C_0R^{-k}b^{-k}}{2-C_0R^{-k}b^{-k}}\right)(\sigma_\theta+\sigma_r)+e_{zz}-\alpha\theta, \tag{16}$$

where $e_{zz} = \dfrac{\dfrac{L}{2\pi} - \int_{R_0}^{1} Rb\left(\dfrac{1-C_0R^{-k}b^{-k}}{2-C_0R^{-k}b^{-k}}\right)(\sigma_\theta+\sigma_r)dR + \int_{R_0}^{1} b^2R\theta dR}{\frac{1}{2}\left(1-R_0^2\right)}$, $A_2 = \dfrac{P^*}{\int_{R_0}^{1} bF_1^* dr}$, $F_1^* = \dfrac{E^2}{4C_0}\left(\dfrac{2-C_0R^{-k}b^{-k}}{3-2C_0R^{-k}b^{-k}}\right)^2 b^{k-2n-1}R^{k-2n-1}\exp f_1^*$,

$$f_1^* = \frac{-(n-1)C_0R^{-k}b^{-k}}{k} - \frac{2kC_0R^{n-k}b^{n-k}}{D^n(n-k)} + \frac{kC_0}{D^n}\int\left(\frac{3-2C_0R^{-k}b^{-k}}{1-C_0R^{-k}b^{-k}}\right)R^{n-k-1}b^{n-k}dR$$

$$+\log\left(1-C_0R^{-k}b^{-k}\right) + \frac{n\theta_1}{\log R_0 D^n}\int\left(\frac{3-2C_0R^{-k}b^{-k}}{1-C_0R^{-k}b^{-k}}\right)R^{n-k-1}b^{n-k}\left[\frac{2-C_0R^{-k}b^{-k}}{C_0R^{-k}b^{-k}} - \frac{2k}{C_0R^{-k}b^{-k}}\log r\right]dR.$$

4. NUMERICAL DISCUSSION:

The definite integrals in equations (16) have been evaluated by using Simpson's rule by taking $D = 1$. Curves have been drawn for radial and circumferential stresses for measures N = 1, 2 and 5 with temperature $\alpha\theta_0 = \theta_1$ = 0, 3 and 5 with respect to the radii ratio R_0. In classical theory, the measure N is equal to (1/n).

Figures 1-8 have been drawn between thermal stresses and radii ratio to show the effect of external pressure with nonlinear measure on a cylinder made of homogeneous and non-homogeneous material.

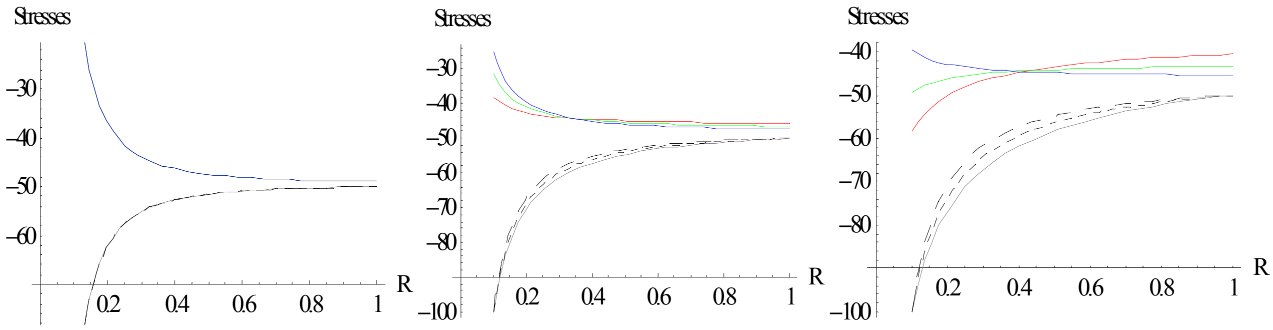
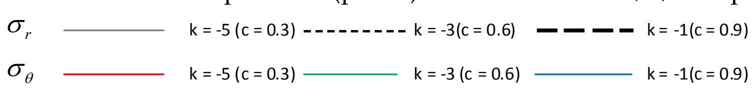


Figure 1. Creep Stresses without temperature for a homogeneous thick-walled circular cylinder under external pressure (p = 50) for measure N = 1, 2, 5 resp.



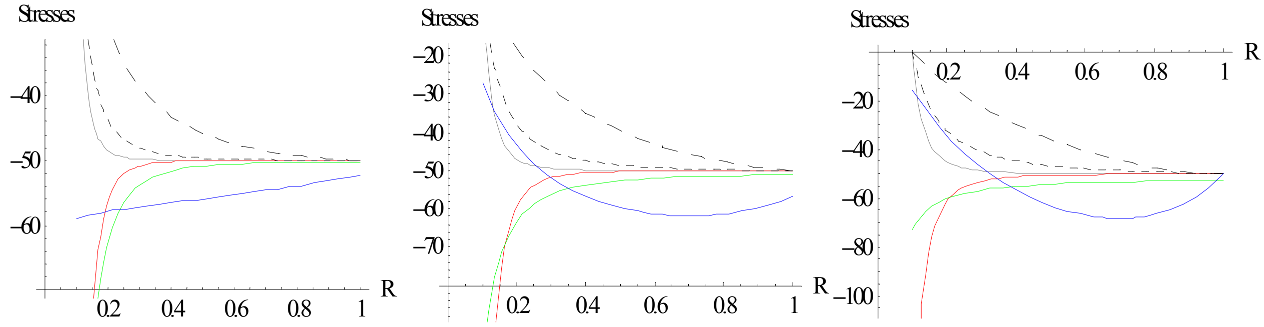


Figure 2. Creep Stresses without temperature for a non-homogeneous thick-walled circular cylinder under external pressure (p = 50) for measure N = 1, 2, 5 resp.

It has been observed from figure 1 that circumferential stresses for linear measure are compressible in nature and are maximum at external surface for homogeneous cylinder. With the change in measure from linear to nonlinear (N = 1 to 2), it has been observed that these stresses are again maximum at external surface. With the further increase in measure it has been observed that for less compressible cylinder, circumferential stress is maximum at internal surface while at external surface for others. Also with the increase in measure these stresses increases significantly.

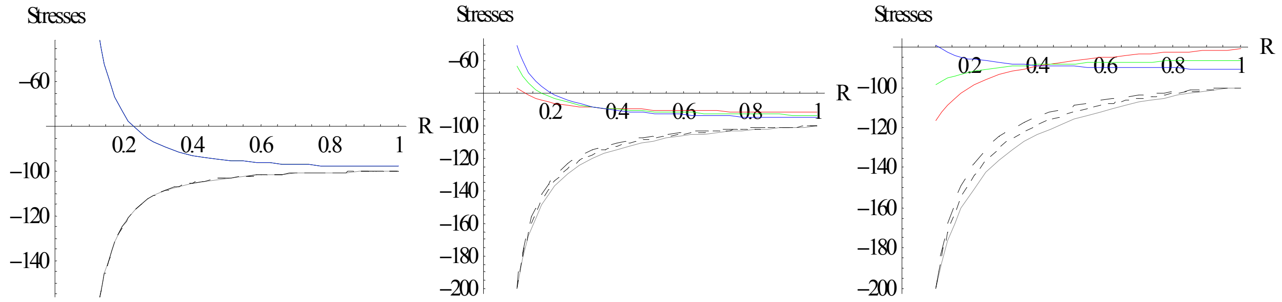


Figure 3. Creep Stresses without temperature for a homogeneous thick-walled circular cylinder under external pressure (p = 100) for measure N = 1, 2, 5 resp.

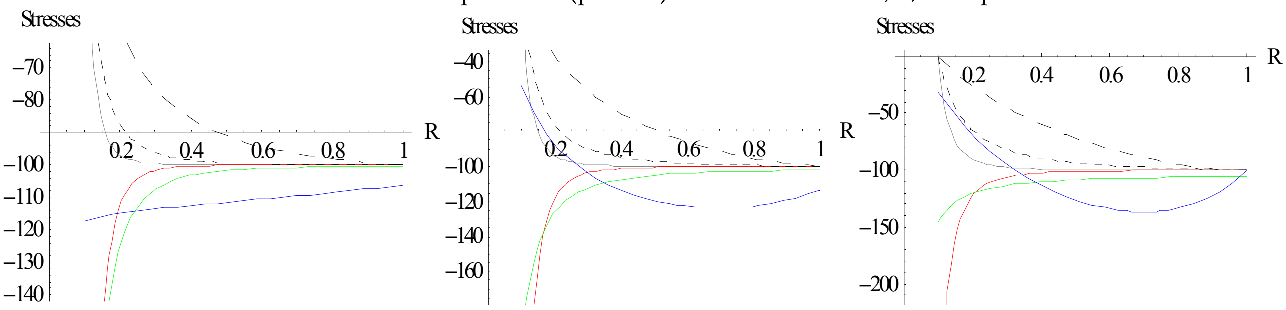


Figure 4. Creep Stresses without temperature for a non-homogeneous thick-walled circular cylinder under external pressure (p = 100) for measure N = 1, 2, 5 resp.

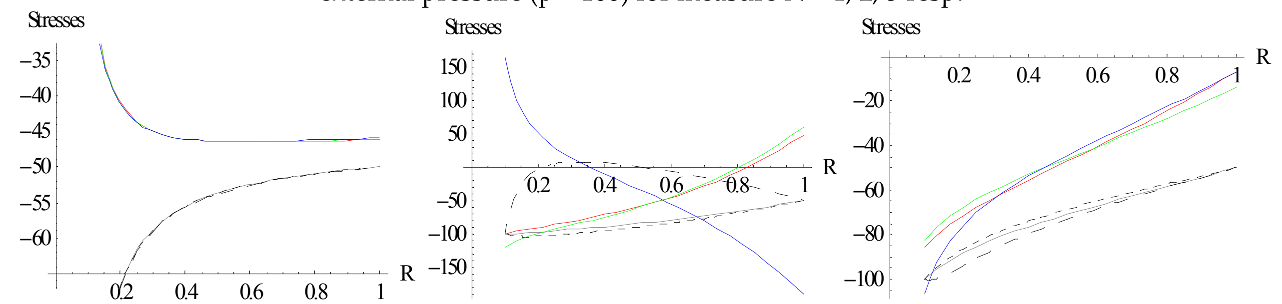


Figure 5. Thermal creep Stresses (θ1 = 3) for a homogeneous thick-walled circular cylinder under external pressure (p = 50) for measure N = 1, 2, 5 resp.

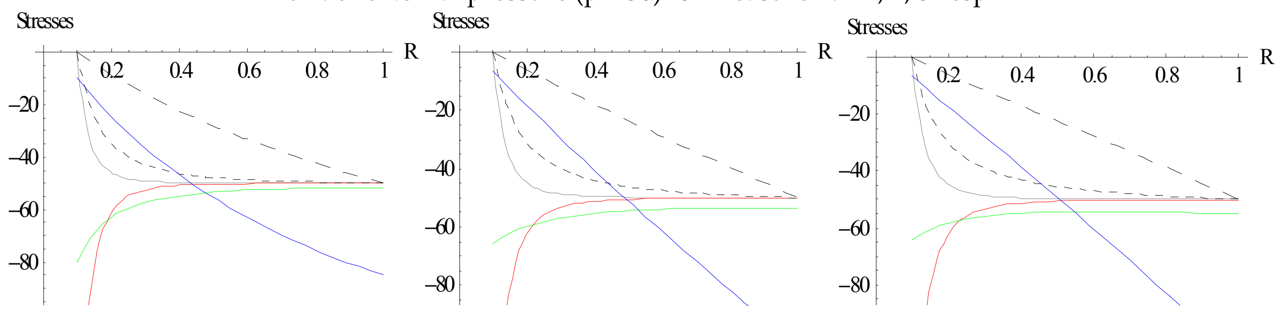


Figure 6. Thermal creep Stresses (θ1 = 3) for a non-homogeneous thick-walled circular cylinder under external pressure (p = 50) for measure N = 1, 2, 5 resp.

From figure 2, it has been noticed that circumferential stresses are compressible in nature and are maximum at internal surface for cylinder made of non-homogeneous material with linear measure. Also, for non-homogeneous cylinder (k = -5) whose compressibility varying radially, compressible circumferential stresses are high as compared to other non-homogeneous cylinders (i.e. k = -3, -1). With the change in measure from linear to nonlinear these stresses increases significantly.

Circumferential stresses are maximum in between R for non-homogeneous cylinder (k = -1) while stresses are maximum at internal surface for other non-homogeneous cylinders as can be seen from figure 2. It has also been observed that circumferential stresses increases significantly for non-homogeneous cylinders as compared to homogeneous cylinder. With the increase in external pressure these circumferential stresses increases significantly for homogeneous and non-homogeneous cylinder as can be seen for figures 3 and 4. With the introduction of thermal effects these circumferential stress increases but are maximum at external surface for homogeneous and non-homogeneous cylinder (k = -1) while at internal surface for non-homogeneous cylinder (k = -3, -5) with linear measure as can be seen from figures 5 and 6. With the change in measure from linear to nonlinear (N = 1 to 2), circumferential stresses are maximum at external surface for highly compressible homogeneous cylinder and are maximum at internal surface for less compressible homogeneous cylinders and non-homogeneous cylinder (k = -1) while at internal surface for less compressible homogeneous and non-homogeneous cylinder (k = -3, -5). With the increase in external pressure and temperature, circumferential stresses increased significantly for homogeneous and non-homogeneous cylinder as can be seen from figures 7 and 8.

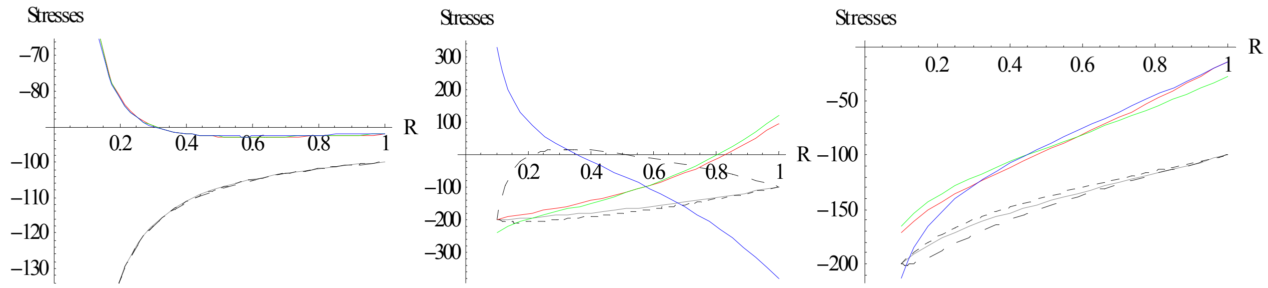


Figure 7. Thermal creep Stresses (θ1 = 3) for a homogeneous thick-walled circular cylinder under external pressure (p = 100) for measure N = 1, 2, 5 resp.

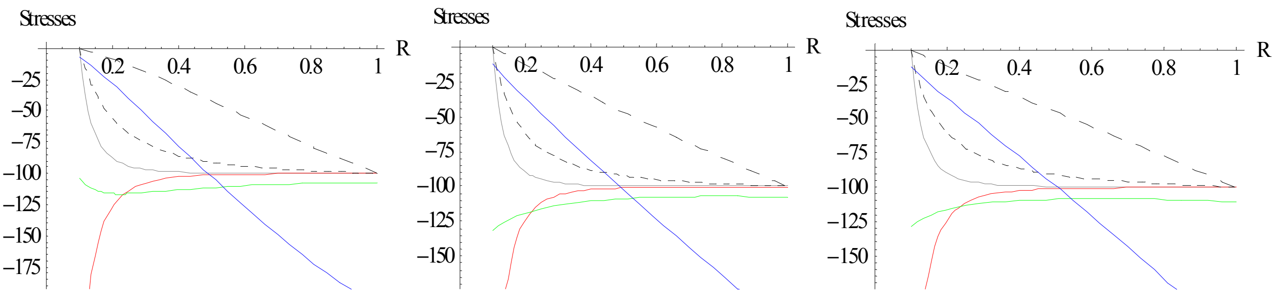


Figure 8. Thermal creep Stresses (θ1 = 3) for a non-homogeneous thick-walled circular cylinder under external pressure (p = 100) for measure N = 1, 2, 5 resp.

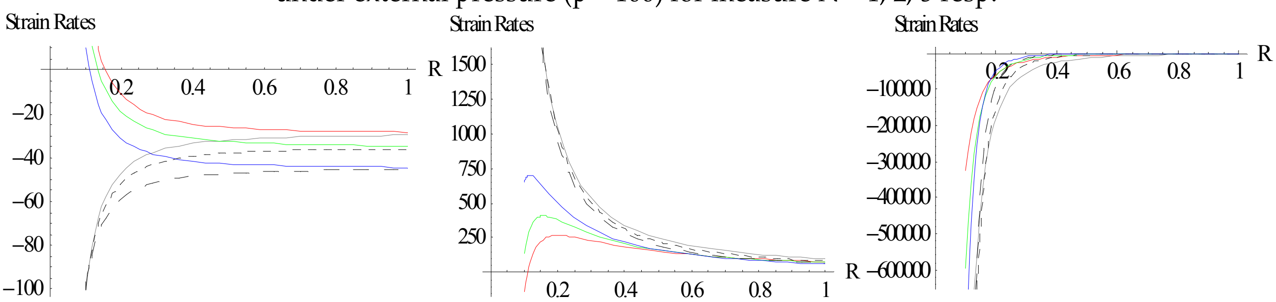
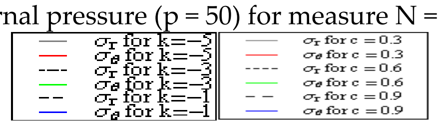


Figure 9. Creep strain rates without temperature for a homogeneous thick-walled circular cylinder under external pressure (p = 50) for measure N = 1, 2, 5 resp.



Figures 9-16 have been drawn between strain rates and radii ratio to show the effect of external pressure with nonlinear measure on a cylinder made of homogeneous and non-homogeneous material with thermal effects. It has been observed from figures 9 and 10 that circumferential strain rates are maximum at the external surface for homogeneous and non-homogeneous cylinder (k = -

5) while at internal surface for non-homogeneous cylinder (k = -3, -1) with linear measure. As measure changes from linear to nonlinear and increase in external pressure these strain rates increases significantly as can be seen from figures 11 and 12. With the introduction of thermal effects, strain rates decreases significantly for homogeneous as well as non-homogeneous cylinder which can be seen from figures 13 to 16.

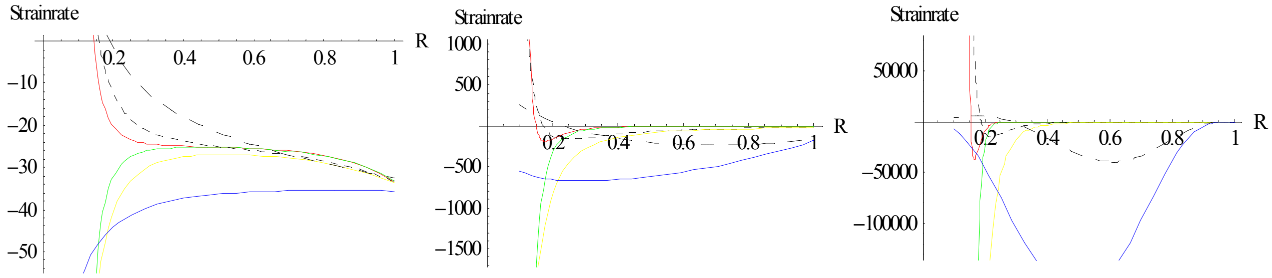


Figure 10. Creep strain rates without temperature for a non-homogeneous thick-walled circular cylinder under external pressure (p = 50) for measure N = 1, 2, 5 resp.

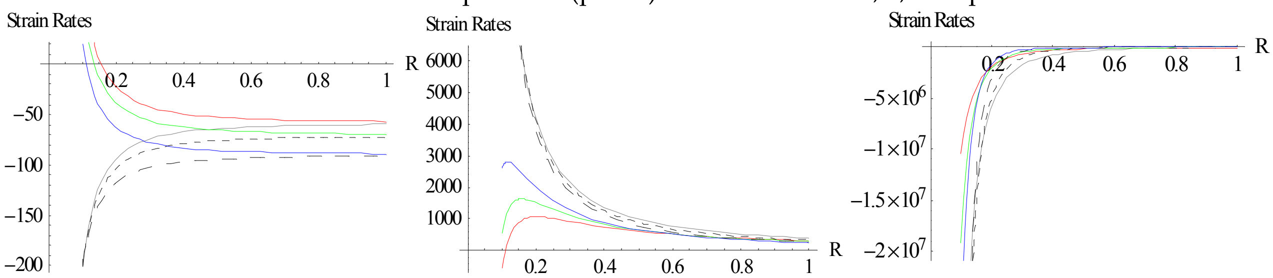


Figure 11. Creep strain rates without temperature for a homogeneous thick-walled circular cylinder under external pressure (p = 100) for measure N = 1, 2, 5 resp.

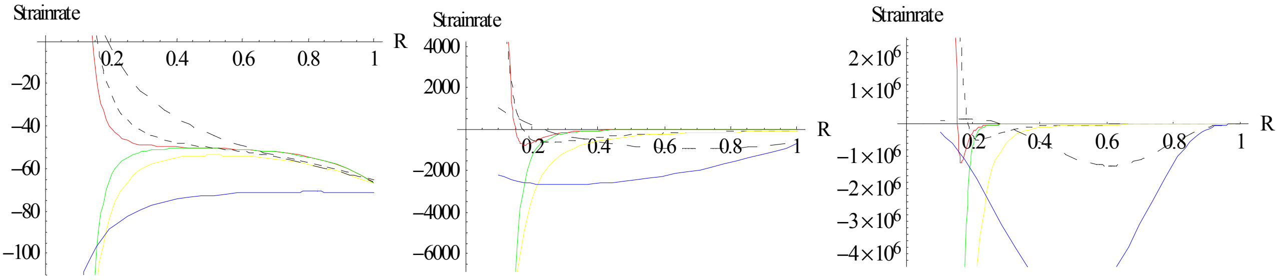


Figure 12. Creep strain rates without temperature for a non-homogeneous thick-walled circular cylinder under external pressure (p = 100) for measure N = 1, 2, 5 resp.

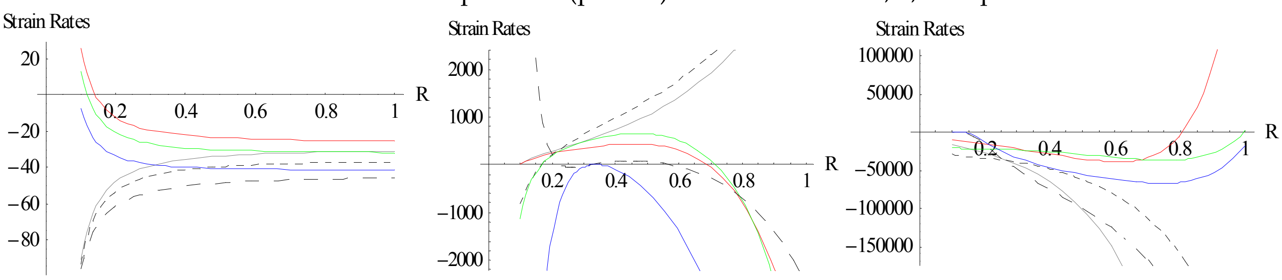


Figure 13. Thermal creep strain rates (θ1 = 3) for a homogeneous thick-walled circular cylinder under external pressure (p = 50) for measure N = 1, 2, 5 resp.

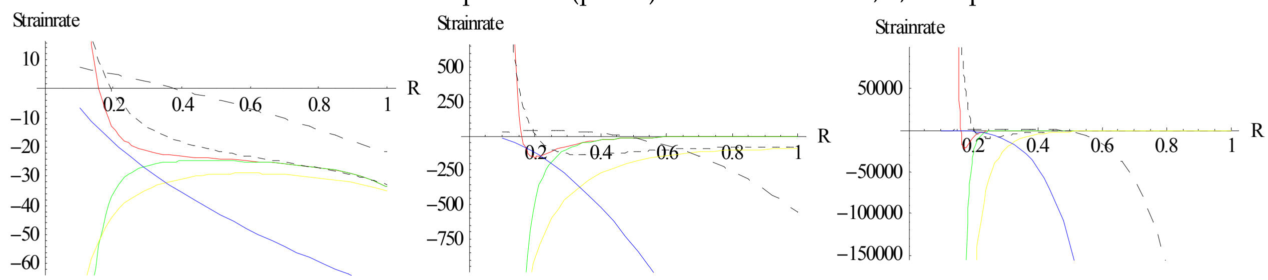


Figure 14. Thermal creep strain rates (θ1 = 3) for a non-homogeneous thick-walled circular cylinder under external pressure (p = 50) for measure N = 1, 2, 5 resp.

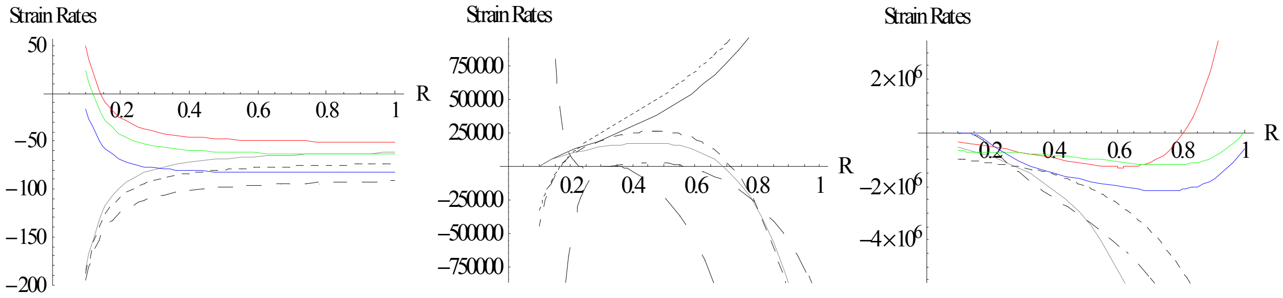


Figure 15. Thermal creep strain rates (θ1 = 3) for a homogeneous thick-walled circular cylinder under external pressure (p = 100) for measure N = 1, 2, 5 resp.

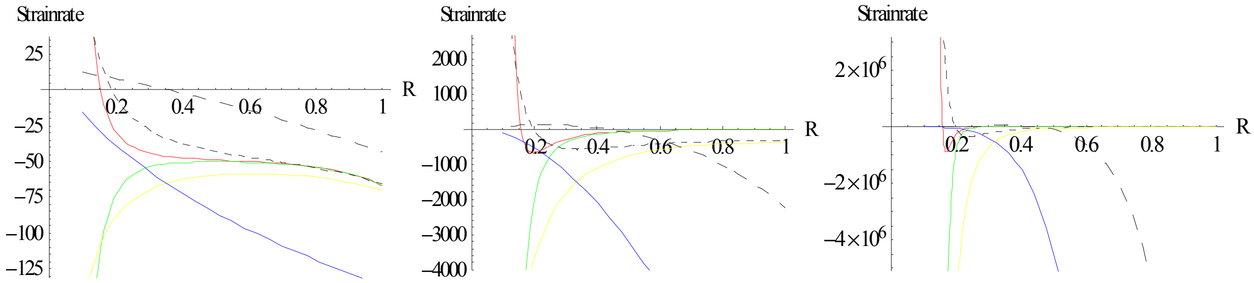


Figure 16. Thermal creep strain rates (θ1 = 3) for a non-homogeneous thick-walled circular cylinder under external pressure (p = 100) for measure N = 1, 2, 5 resp.

5. CONCLUSION

Non-homogeneous thick-walled circular cylinder (k = -5) with thermal effects for nonlinear measure N = 5 is on the safer side of the design as compared to non-homogeneous cylinder (k = -3, -1) as well as homogeneous cylinder without temperature because compressible circumferential stresses are high for non-homogeneous (k = -5) thick-walled circular cylinder under external pressure as compared to non-homogeneous cylinder (k = -3, -1) as well as homogeneous cylinder. Thus we can conclude that by introducing a suitably chosen temperature gradient, the compressible stresses due to pressure may be increased, which leads to the idea of 'Stress Saving'. It means that introduction of thermal effects minimizes the possibility of fracture at the internal surface for N = 5.

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References:

[1.] Aggarwal AK; Sharma R and Sharma S, Safety Analysis using Lebesgue Strain Measure of Thick-Walled Cylinder for Functionally Graded Material under Internal and External Pressure, The Scientific World Journal, 2013, dx.doi.org/10.1155/2013/676190, 1-10, 2013.
[2.] Altenbach H and Skrzypek JJ, Creep and Damage in Materials and Structures, Springer Verlag, Berlin, 1999.
[3.] Bhatnagar NS and Arya VK, Large Strain Creep Analysis of Thick-Walled Cylinders, Int. J. Non-Linear Mechanics, 9, 127–140, 1974.
[4.] Borah BN, Thermo Elastic-Plastic Transition, Contemporary Mathematics, 379, 93-111, 2005.
[5.] Deepak D; Gupta VK and Dham AK, Creep Modelling in Functionally Graded Rotating Disc of Variable Thickness, Journal of Mechanical Science and Technology, 24 (11), 2221–2232, 2010.
[6.] Hetnarski RB and Ignaczak J, Mathematical Theory of Elasticity, Taylor & Francis, 2003.
[7.] Kraus H, Creep Analysis, Wiley, New York, Toronto, 1980.
[8.] Singh T and Gupta VK, Effect of Material Parameters on Steady State Creep in a Thick Composite Cylinder Subjected to Internal Pressure, The Journal of Engineering Research, 6(2), 20-32, 2009.
[9.] Seth BR, Transition Theory of Elastic-Plastic Deformation Creep and Relaxation, NATURE, 195(4844), 896-897, 1962.
[10.] Sharma S, Thakur P and Sahni M, Elastic-plastic Analysis for Finite Deformation of a Rotating Disk of Exponentially Varying Thickness with Edge Load and Inclusion, Annals of Faculty Engineering Hunedoara- International Journal of Engineering, Tome X- Fascicule, 225-232, 2012.
[11.] Sharma S and Sahni M, Creep Analysis of Thin Rotating Disc Having Variable Thickness and Variable Density with Edge Loading, Annals of Faculty Engineering Hunedoara- International Journal of Engineering, Tome XI-Fascicule-3, 289-296, 2013.
[12.] Vasilenko AT and Klimenko NI, Stress State of Rotating Inhomogeneous Anisotropic Cylinders, International Applied Mechanics, 35(8), 778-783, 1999.
[13.] Yoo, YS; Huh S, Choi, S, Kim,TW and Kim, JI, Collapse Pressure Estimates and the Application of a Partial Safety Factor to Cylinders Subjected to External Pressure, Nuclear Engineering And Technology, 42(4), 450-459, 2010