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MODELLING THE EHL LINE CONTACTS THROUGH DIFFERENT PRESSURE–VISCOSITY RELATIONSHIPS

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Abstract: As well known the properties of the lubricant play a significant role in the build up of the lubricating film between the contacting surfaces. In this regard, the influence of such properties on the pressure profiles and film shapes is showed along with the minimum film thickness. The influences of pressure and temperature on viscosity, limiting shear stress and density, have been considered in the numerical calculation of film thickness and friction. The effects of different pressure–viscosity relationships, including the exponential model, the Roelands model and the free-volume model, are investigated to analyse the literature different approaches to describe the piezo-viscous behaviour at pressures as high as the typical EHL values.
Keywords: elastohydrodynamic lubrication, piezo-viscosity model, free volume, Reynolds equation, oil film thickness

1. INTRODUCTION

The elastohydrodynamic regime is based on two primary aspects: the strong increase of viscosity with pressure, and the magnitude of the elastic deformation caused by high pressure which is comparable to that of the film thickness. Due to the high pressure and the limited contact area elastic deformation of the surfaces will occur and it is not negligible, as well as the pressure dependence of viscosity play a crucial role in EHL simulation because the viscosity at the inlet has crucial influence on film formation.

The rheology of liquid lubricants in the Hertzian zone of concentrated contacts was one of the main research topic for the tribologists for many years. In particular, for applications involving lubricants which exhibit shear-thinning behaviour, the use of an appropriate non-Newtonian fluid model is required to predict the EHL behaviour more accurately.

The exponential [1] and Roelands equations [2] are widely used to describe the pressure-viscosity relationship in EHL simulations. However, the real responses of most lubricants are often non-linear [3]. It is usually difficult to predict the non-linearity accurately by the exponential equation, and the value is likely to be underestimated by the Roelands equation [4].

In the light of the above evidences, with the purpose of more accurate modelling of the lubricant rheology Doolittle [5] developed the first free-volume model based on a physical meaning: the resistance to flow in a liquid depends upon the relative volume of molecules present per unit of free volume. Using an exponential function, Doolittle related viscosity to the fractional free volume. The Williams–Landel–Ferry (WLF) equation [6] can be derived from the Doolittle free-volume equation; this one, subsequently improved by Yasutomi [7,8] and Cook [9], may accurately describe the temperature variation of viscosity and, above all, lead to accurate description of viscosity variation with pressure.

In this paper the effect of different pressure–viscosity relationships on film thickness, including the Barus model, the Roelands model, and the free-volume model, is investigated through numerical simulations by taking into account the physical properties of the lubricant.

2. THEORETICAL MODELS AND NUMERICAL RESULTS

The classical EHL problem mainly consists of three equations which have to be solved simultaneously. These are the Reynolds equation, the film thickness equation including the elastic deformation and the force-balance equation. The desired outputs related to the resolution of this system are, usually, the pressure distribution and the film thickness variation. In addition to these three equations, the density and viscosity-pressure relations have to be calculated inside the lubricated meatus.

Reynolds Equation. According to isothermal EHL theory, the film profile and pressure distribution in line EHL contacts are expressed by the equivalent elastic modulus E', equivalent radii in the x direction R, properties of lubricant, average velocity u_m and the applied load w. The Reynolds equation for the case of smooth surfaces is:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial x}\right)-6\frac{\partial}{\partial x}(u_m\rho h)=0$$

where $u_m=\frac{U_1+U_2}{2}$, U_1 and U_2 are the surface velocities and in this analysis no slip condition has been supposed so that the surface velocities are identical to the lubricant velocities.

The boundary conditions are *p=0* at the boundaries of the calculation domain and the cavitation condition is *p=0* at the cavitation boundary.

Film Thickness Equation. The film thickness equation, describing the distance between the two contacting surfaces, consists of two components, the gap between the undeformed surfaces and the elastic deformation of the surfaces. Consequently, the gap between the elastically deformed surfaces, generally described by the film thickness equation, is calculated in each point of the x direction as:

$$h(x)=h_0+\frac{x^2}{2R}-\frac{4}{\pi E'}\int_{-\infty}^{+\infty}p(x')\ln\left(\frac{|x-x'|}{b}\right)dx'$$

Force Balance Equation. An equation is also needed to make sure that the load and the pressure in the contact are in equilibrium. This equation is usually called the force balance equation. In full film conditions, the load per width unit is carried by the lubricant film and the calculation is an integration of the lubricant film pressure.

$$w=\int_{-\infty}^{+\infty}p(x)dx$$

In conjunction with these three equations, a density model has been used for these EHL simulations. Due to the isothermal conditions assumed in this work, the density model used is only pressure dependent and it has been developed by Dowson and Higginson [10], in which a model for the relation between the density and the pressure for a lubricant is presented.

$$\rho(p)=\rho_0\frac{5.9\cdot 10^8+1.34p}{5.9\cdot 10^8+p}$$

3. LUBRICANT PIEZO-VISCOUS RESPONSE

An important factor which plays a decisive role in the prediction of film thickness and pressure distribution is the piezo-viscous response of lubricants.

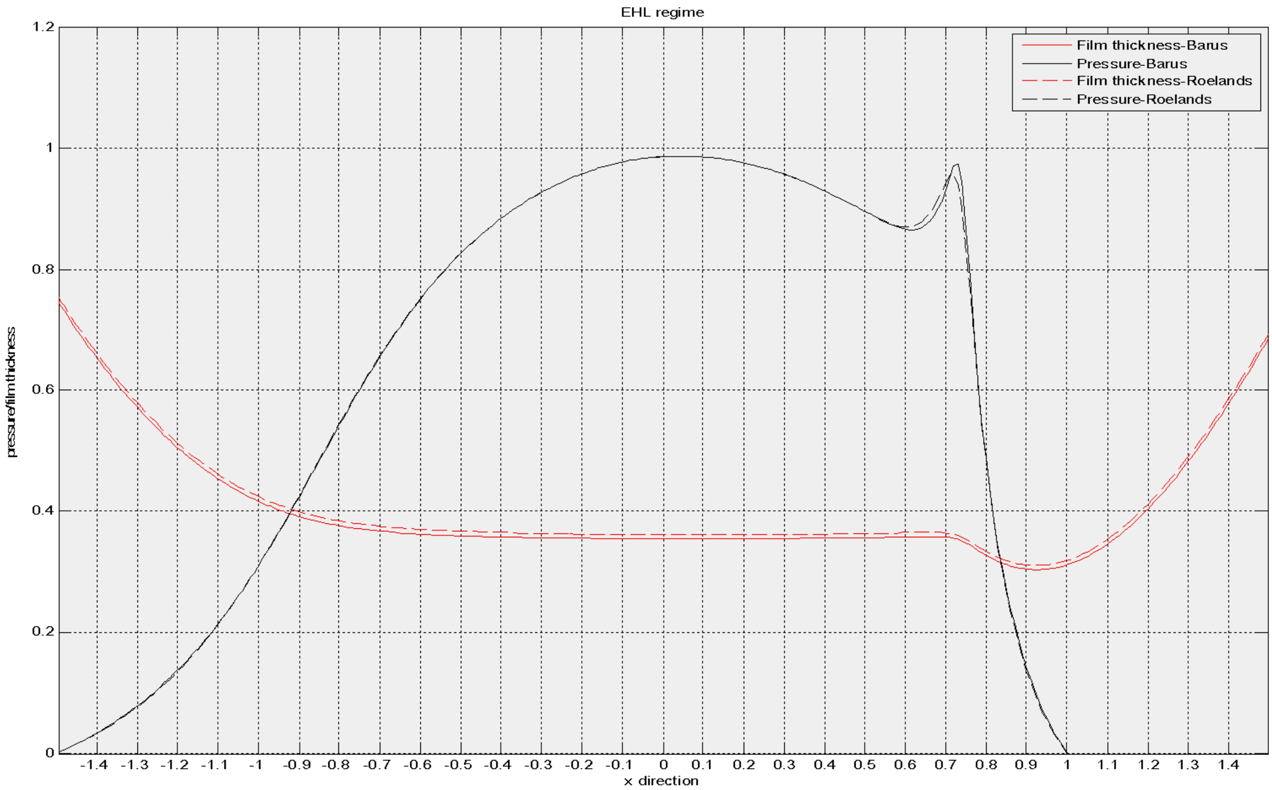

Figure 1: Film thickness profile and pressure distribution using Roleands' law (solid line) and Barus law (dashed line) with $W=3.0\times10^{-4}$; $U=2.0\times10^{-11}$.

3.1. Barus and Roelands models

The majority of available studies on EHL contacts often use one of the two well-known pressure-viscosity equations given below:

- Barus' model: $\mu = \mu_0 \exp(\alpha \cdot p)$
- Roelands' model:

$$\mu = \mu_0 \exp\left[\left(\ln \mu_0 + 9.67\right)\left(-1 + \left(1 + 5.1 \cdot 10^{-9} p\right)^z\right)\right]$$

particularized at z=0.6 (Fig. 1).

Dimensionless pressure profiles and film shapes variations with dimensionless load and sliding speed parameters in a domain *-1.5<X<1.5* are presented in the Figures 2 and 3. The model predicts that a pressure spike occurs on the outlet side of the contact, accompanied by a constriction in oil-film thickness. Increasing speed or increasing applied load causes the increasing of the pressure spike in height and its moving from the outlet side of the contact toward the inlet.

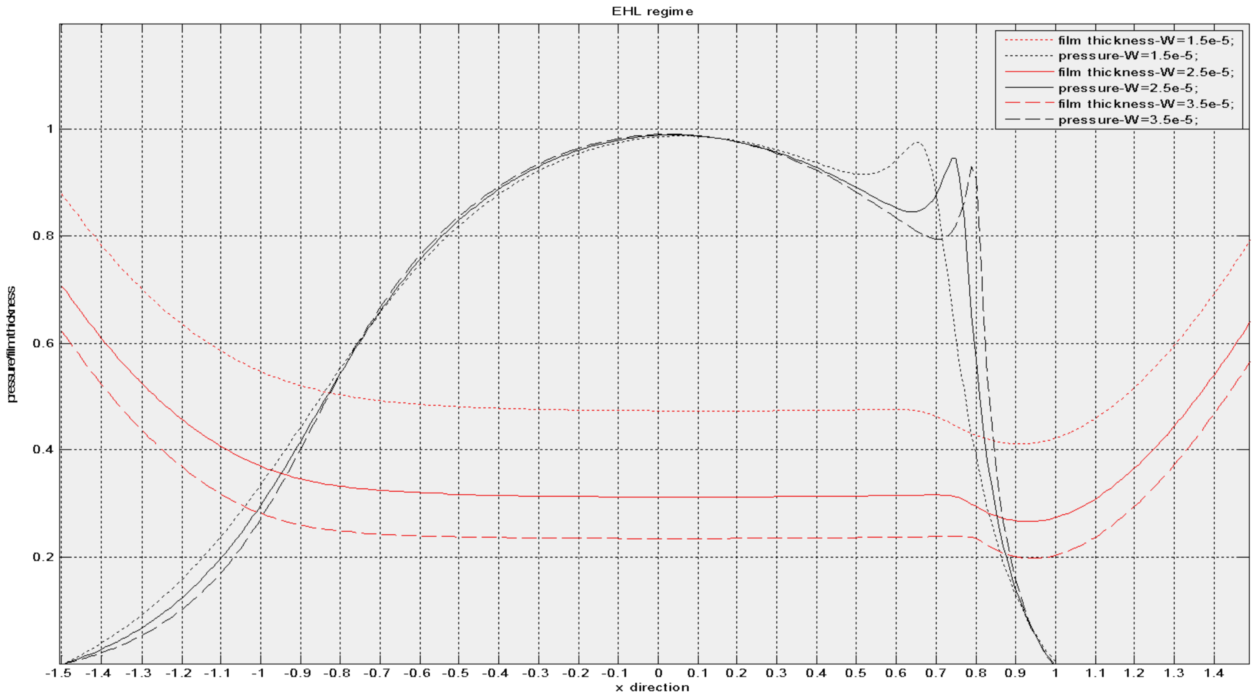

Figure 2: Film thickness profile and pressure distribution comparison using Roelands' law, with W=1.5×10^{-4}(dot line); W=2.5×10^{-4} (solid line), W=3.5×10^{-4} (dashed line)

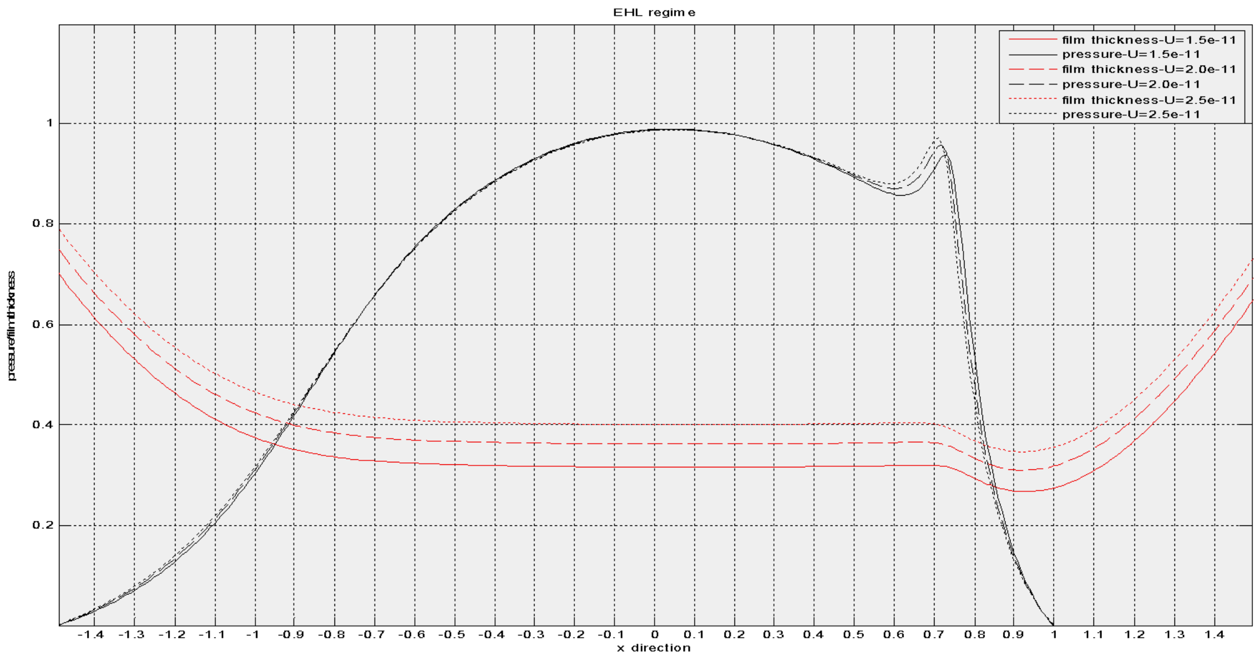

Figure 3: Film thickness profile and pressure distribution comparison using Roelands' law, with U=1.5×10^{-11}(solid line); U=2.0×10^{-11} (dashed line), U=2.5×10^{-11} (dot line)

On the other side, for the film thickness behaviour, increasing the dimensionless sliding speed, the height of the meatus increases, while for an increase in applied load a reduction of the film thickness is observed.

These results are due to progressive variation of the lubrication regime as function of current speed and applied load. In fact, an increase of the load causes a progressive predominance of the boundary lubrication regime condition, while increasing the sliding speed hydrodynamic lubrication regime is expected with a higher fluid meatus. This result is well described by the Stribeck curve [11] in which the friction coefficient is usually represented as a function of a dimensionless lubrication parameter ηv/P, where η is the dynamic viscosity, v the sliding speed and P the load projected on to the geometrical surface dimensionless per unit length.

3.2. Doolitle's free-volume model

Doolittle [5] developed the first free-volume model based on a physical meaning, that the resistance to flow in a liquid depends upon the relative volume of molecules present per unit of free volume. The free volume of a liquid was originally considered to be the volume resulting from the thermal expansion without phase change. By using an exponential function, Doolittle related viscosity to the fractional free volume. In this paper, the Doolittle's viscosity–pressure relationship is represented by:

$$\mu = \mu_0 \exp\left(B\frac{V_\infty}{V_0}\left[\frac{1}{\frac{V}{V_0}-\frac{V_\infty}{V_0}} - \frac{1}{1-\frac{V_\infty}{V_0}} \right]\right)$$

where B and V_∞/V_0 are constants and V/V_0 can be calculated using the Tait's equation [12,13]:

$$\frac{V}{V_0} = 1 - \frac{1}{1+K'_0}\ln[1+\frac{p}{K_0}(1+K'_0)]$$

The lubricant in the proposed model is characterized by

$$\mu_0 = 1.3\ \text{Pa·s};\ B=4.422;\ V_\infty/V_0 = 0.6694;\ K'_0 = 12.83,\ K_0 = 1.4252\ \text{GPa [14]}.$$

Fig. 4 shows the pressure distribution and film thickness profile obtained from the analysis of static load EHL contact with the free-volume model. The dimensionless load is $W=3.0\times10^{-4}$, the dimensionless sliding speed is $U=2.0\times10^{-11}$ and the dimensionless material parameter is G=4000. The corresponding maximum Hertzian contact pressure p_h for these operating conditions is equal to 1.25 GPa.

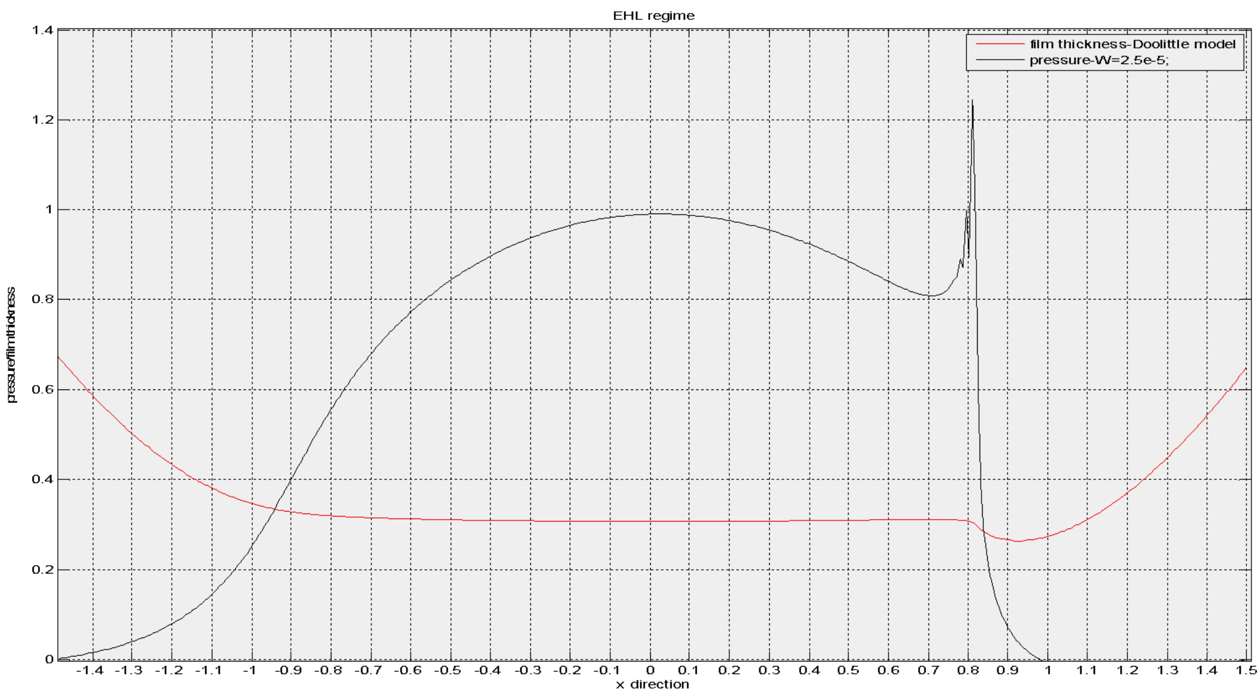

Figure 4: Film thickness profile and pressure distribution using Doolittle model with $W=3.0\times10^{-4}$; $U=2.0\times10^{-11}$.

Analogous behaviour to those reported previously using the Roelands' law, for the pressure distribution and film shape, in function of the dimensionless applied load and sliding speed, has been observed with the use of the free-volume model. In fact an increase in sliding speed or applied load causes the increasing of the pressure spike in height and its moving from the outlet side of the contact toward the inlet, Fig. 5.

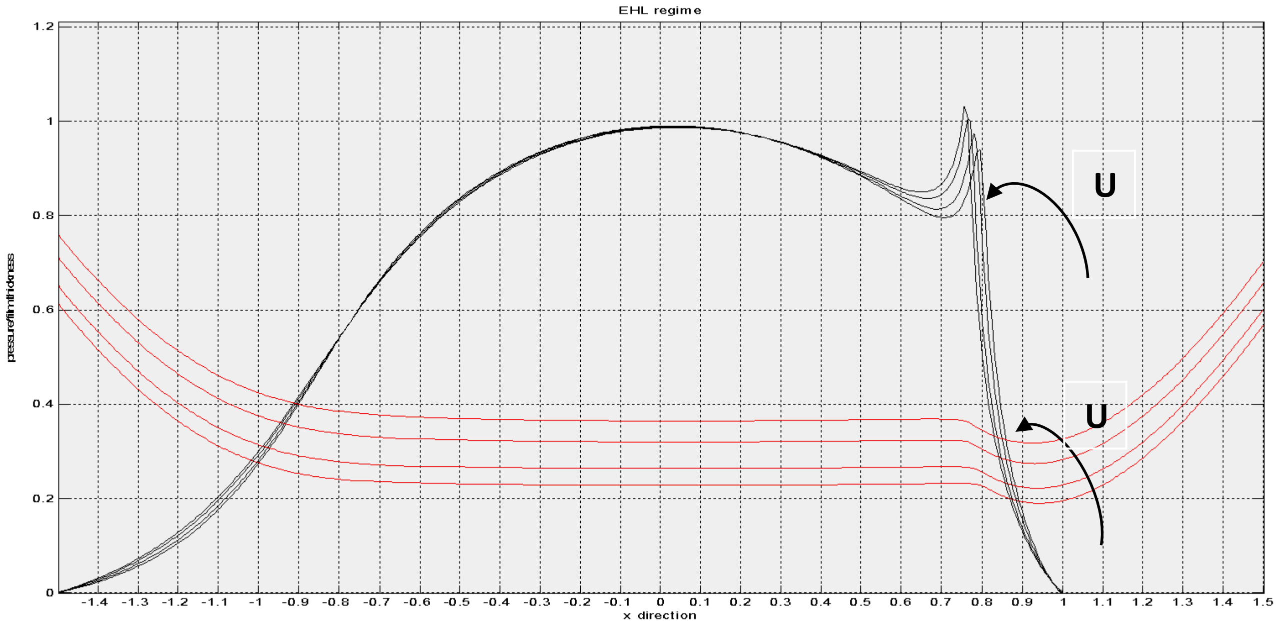

Figure 5: Film thickness profile (red line) and pressure distribution (black line) comparison using Doolittle model for different level of dimensionless sliding speed (from $U=1.0\times10^{-11}$to $U=6.0\times10^{-11}$)

Comparing the results obtained in terms of pressure distribution and film thickness, it is possible to observe a pressure spike amplitude of about 25% higher using the model based on the free-volume theory (Fig. 6). It has also seen that the difference in central pressure is approximately 2%.

Similarly, it can be seen that the height of the film thickness is smaller for the proposed model even if in a small percentage of about 3%.

The remarkable difference as regards the pressure magnitude inside the contact area could also influence the sub surface stresses, for this reason the risk of surface fatigue can be estimated more accurately.

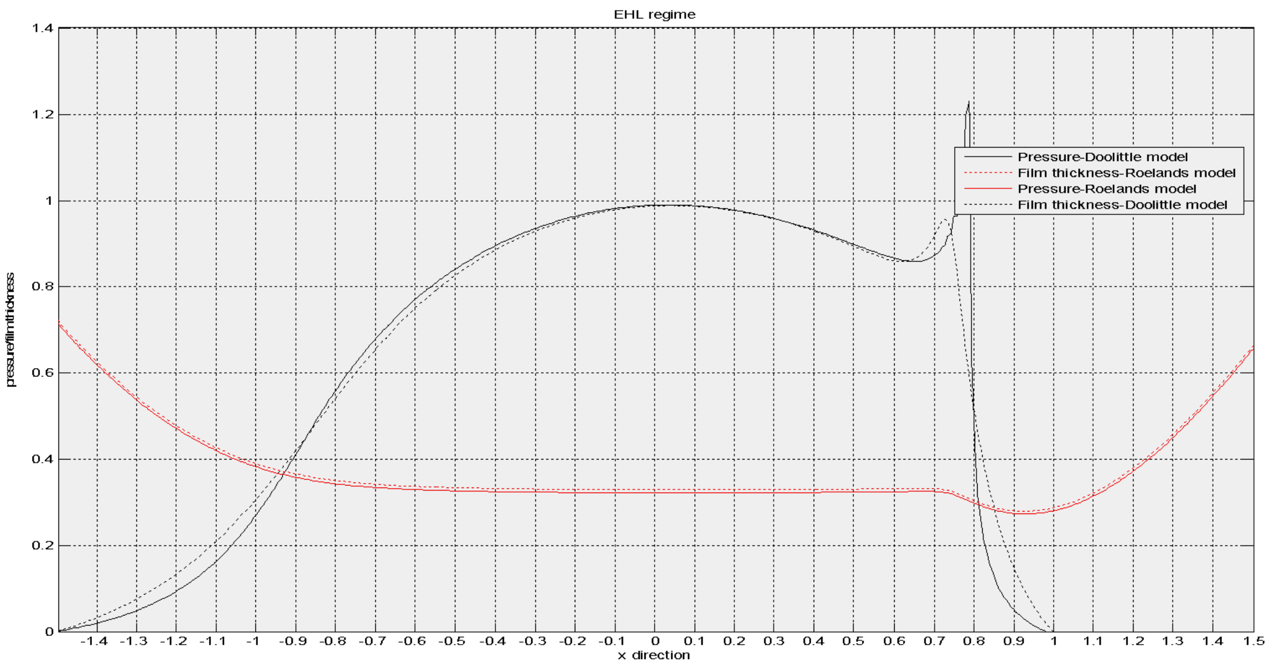

Figure 6: Film thickness profile and pressure distribution using Doolittle' model (solid line) and Reolands' law (dashed line) with $W=3.0\times10^{-4}$; $U=2.0\times10^{-11}$.

4. CONCLUSIONS

This focus of this paper is the analysis of the results ensuing the modelling of EHL line conjunctions through different relationships aiming at describing the piezo-viscous properties of the lubricant fluids. The Roelands' law, usually used for modelling the pressure-viscosity relationship of the lubricants has been observed to weakly predict the film thinning behaviour. For this reason, the free-volume based theory has been used to investigate about the EHL main characteristics, i.e. pressure distribution and film thickness.

The outcomes underline that the film thickness becomes thinner using the free-volume model and in particular the pressure spike magnitude significantly increases.

Appendix

Dimensional parameters

B= half width of hertzian contact zone $b = 4R\sqrt{\frac{W}{2\pi}}$ (m)

E′= effective elastic modulus of bodies 1 and 2 (Pa)

h= film thickness (m)

p=pressure (Pa)

p_h=maximum hertzian pressure $p_h = \frac{E'b}{4R}$ (Pa)

R= radius of contact (m)

u_m= average sliding speed (m/s)

w=applied load per unit length (N/m)

μ_0= inlet viscosity of the lubricant (Pa s)

ρ_0= inlet mass density of the lubricant (kg/m^3)

μ= lubricant viscosity at the local pressure (Pa s)

ρ= lubricant mass density at the local pressure (kg/m^3)

α=piezo-viscous coefficient (Pa^{-1})

x= coordinate in the direction of surface velocity (m)

Dimensionless parameters

G=dimensionless load parameter G= αE′

H= dimensionless film thickness $H = \frac{hR}{b^2}$

P= dimensionless pressure $P = \frac{p}{p_h}$

U= dimensionless speed parameter $U = \frac{\mu_0 u_m}{E'R}$

W= dimensionless load parameter $W = \frac{w}{E'R}$

$\bar{\mu}$ = dimensionless viscosity $\bar{\mu} = \frac{\mu}{\mu_0}$

$\bar{\rho}$ = dimensionless mass density $\bar{\rho} = \frac{\rho}{\rho_0}$

Z= Roelands parameter

X=dimensionless coordinate in x direction $X = \frac{x}{b}$

B=Doolittle parameter

V_{occ}= occupied volume

V= volume

V_0= volume for p=0

K_0= isotherm bulk modulus at p=0 (Pa)

K_0'= pressure rate of change of isothermal bulk modulus at p=0

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