ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering

Tome XIII [2015] – Fascicule 2 [May]
ISSN: 1584-2665 [print]; ISSN: 1584-2673 [online]
a free-access multidisciplinary publication of the Faculty of Engineering Hunedoara

[1.] Istvan KUCORA, [2.] Ljiljana RADOVANOVIC, [3.] Ivan PALINKAS [4.] Zivoslav ADAMOVIC

METHODOLOGY FOR APPLYING THE DIFFERENTIAL QUADRATURE (DQ) METHOD TO THE FREE VIBRATION ANALYSIS

[1.] „HIP-Petrohemija" Pancevo, SERBIA
[2-4.] University of NoviSad, Technical Faculty "Mihajlo Pupin", Zrenjanin, SERBIA

Abstract: This work presents a simple way for application of the differential quadrature method on the simply supported and clamped boundray conditions, which avoid weight determination of the angle between the normal to the plate boundaryand the *x*-axis.In this approach, the irregular physical domain is transformed into regular physical domain (square) in curvilinear coordinate system (computing space), and through accompanying equations, the conditions are transformed into relevant forms of curvilinear coordinate system. In this way all the computing is within certain computing domain. By using the present method big operations with matrix multiplications, used by Bert and Malik, are avoided, and as a result, the computing weight has been noticably reduced as well as the size of virtual storage necessary for the important operations. The presentation of the above mentioned is performed by using the application test on the presented approach.
Keywords: differential quadrature method; vibration analysis; arbitrary quadrilateral plates; coordinate transformation

1. INTRODUCTION

A methodology for applying the differential quadrature (DQ) method to the free vibration analysis of arbitrary quadrilateral plates is developed. In our approach, the irregular physical domains transformed into a rectangular domain in the computational space. The governing equation and the boundary conditions are also transformed into relevant forms in the computational space. Then all the computations are based on the computational domain.

In recent years, the differential quadrature method has become one of many popular ways of solving the problem of initial and boundary conditions [2]. The advantage of the differential quadrature method is that it is easy to use and flexible in regards to any spatial coordinate system. In comparison to the conventional computation techniques of lower order, such as finite element methods and finite difference methods, the differential quadrature method gives precise solutions with more mesh points.

2. THE DIFFERENTIAL QUADRATURE METHOD

One of the moot points in differential quadrature method is defining its impact coefficients. For the differential of function *f(x,t)* n-order, in comparison with x and mesh point X_i the approximity of the differential quadrature method can be presented as:

$$f_x^{(n)}(x_i,t)=\sum_{k=1}^{N} c_{ik}^{(n)}\cdot f(x_k,t),\quad n=1,2,\dots,N-1,\qquad i=1,2,\dots,N \tag{1}$$

where N is number of mesh points in the whole domain, and $C_{ik}^{(n)}$ is the impact coefficient determined by the differential quadrature method.

3. PLATES VIBRATION EQUATION IN CURVILINEAR COORDINATE SYSTEM

The plates vibration equation in curvilinear coordinate system which will be presented in this section, can directly apply the traditional rules ofthe differential quadrature method to the problems of vibration of plates with irregular quadrangle domains. The equations of vibration plates conditions can be expresed as follows:

$$w_{xxxx}+2w_{xxyy}+w_{yyyy}=\Omega^2 w \tag{2}$$

where

$$\Omega=\omega a^2\sqrt{\frac{\rho h}{D}}$$

D- plate rigidity, h-plate thickness, ρ-thickness, w-plate buckling, ω-frequency of its own free vibrations

Equations of condition (2), can be transformed in *(ξ,η)* system (computing space) in form of:

$$\bar{D}^{(41)}w_{,\xi\xi\xi\xi}+\bar{D}^{(42)}w_{,\xi\xi\xi\eta}+\bar{D}^{(43)}w_{,\xi\xi\eta\eta}+\bar{D}^{(44)}w_{,\xi\eta\eta\eta}+\bar{D}^{(45)}w_{,\eta\eta\eta\eta}+\bar{D}^{(31)}w_{,\xi\xi\xi}+\bar{D}^{(32)}w_{,\xi\xi\eta}+\bar{D}^{(33)}w_{,\xi\eta\eta}+\bar{D}^{(34)}w_{,\eta\eta\eta}+\bar{D}^{(21)}w_{,\xi\xi}+\bar{D}^{(22)}w_{,\xi\eta}+\bar{D}^{(23)}w_{,\eta\eta}+\bar{D}^{(11)}w_{,\xi}+\bar{D}^{(12)}w_{,\eta}=\Omega^2 w \quad (3)$$

The domain of changeable Eq. (3) is quadrangle. That is shown in Eq. (3) with changeable coefficients $\bar{D}^{(ij)}$ which are much more complex than in the form of Eq. (2). When the computing domain is regular, Eq. (3) can be solved in the same way as the problem of regular domain by using the differential quadrature method.

Simply supported and clamped boundray conditions will also be discused in the present work. They are presented as following:

For clamped:

$$w=0;\ \frac{\partial w}{\partial n}=0 \quad (4a\text{-}b)$$

For supported:

$$w=0;\frac{\partial^2 w}{\partial n^2}+\nu\frac{\partial^2 w}{\partial \tau^2}=0 \quad (5a\text{-}b)$$

n and τ mark the normal and tangent direction, respectivelly. Eq. (4a) and (5a) present null point bending, Eq. (4b) present null point of normal rotation and Eq. (5b) present null normal moment. Null bending condition can be applied easily. In the works of Bert and Malik [1], null point of normal rotation and moment conditions in clamped and simply supported bounderies can be presented as follows:

$$w_{,x}\cos\theta+w_{,y}\sin\theta=0 \quad (6)$$

$$(\cos^2\theta+\nu\sin^2\theta)w_{,xx}+(\sin^2\theta+\cos^2\theta)w_{,yy}+2(1-\nu)\cos\theta\sin\theta\, w_{,xy}=0 \quad (7)$$

where θ- is the angle between the normal to the plate boundary and x – axis. That is shown in Eq. (6) which is equivalent to Eq. (4b), and in the Eq. (7) which is equivalent to Eq. (5b). The following passage presents simplification of Eq. (6) and (7) along ξ-constant and η-constant boundary in the curvilinear coordinate system.

For clamped and simply supported edges, w buckling is always equal to null. The results are as follows:

$$\frac{\partial w}{\partial \eta}=0;\frac{\partial^2 w}{\partial \eta^2}=0 \quad (8a\text{-}b)$$

within ξ-constant boundaries,and

$$\frac{\partial w}{\partial \xi}=0;\frac{\partial^2 w}{\partial \xi^2}=0 \quad (9a\text{-}b)$$

within η -constant boundaries

On the other hand, θ is the angle between normal to the plate boundaryand x-axis. As a result, along ξ constant boundary there is:

$$\cos\theta=\frac{y_\eta}{\sqrt{\alpha}},\sin\theta=\frac{x_\eta}{\sqrt{\alpha}}, \quad (10a\text{-}b)$$

By using Eq. (8) and (10) the null of normal rotation condition (6) along ξ constant boundary can be simplified as follows:

$$\frac{\partial w}{\partial \xi}=0, \quad (11)$$

And the null of normal condition moment *(β)* along ξ-constant boundary can be simplified as follows:

$$\frac{\partial^2 w}{\partial \xi^2}-\frac{2\beta\partial^2 w}{\alpha\partial\xi\partial\eta}+s\frac{\partial w}{\partial \xi}=0 \quad (12)$$

where:

$$s=\frac{1}{J\alpha^2}\left[\left(\alpha^2 y_{\xi\xi}-2\alpha\beta y_{\xi\eta}+\beta^2 y_{\eta\eta}\right)x_\eta-\left(\alpha^2 x_{\xi\xi}-2\alpha\beta x_{\xi\eta}+\beta^2 x_{\eta\eta}\right)y_\eta\right]+\frac{\nu J}{\alpha^2}\left(y_{\eta\eta}x_\eta-x_{\eta\eta}y_\eta\right)$$

Along η -consatant area, cos Θ and sin Θ can be expressed as follows:

$$\cos\theta=\frac{y_\xi}{\sqrt{\gamma}};\sin\theta=-\frac{x_\xi}{\sqrt{\gamma}} \quad (13a\text{-}b)$$

Because the Eq. (6) is reduced to:

$$\frac{\partial w}{\partial \eta}=0 \quad (14)$$

the Eq. (7) is reduced to:

$$\frac{\partial^2 w}{\partial \eta^2} - \frac{2\beta\partial^2 w}{\gamma\partial\xi\partial\eta} + t\frac{\partial w}{\partial \eta} = 0 \tag{15}$$

where:

$$t = \frac{1}{J\gamma^2}\left[\left(\beta^2 x_{\xi\xi} - 2\gamma\beta x_{\xi\eta} + \gamma^2 x_{\eta\eta}\right)y_\xi - \left(\beta^2 y_{\xi\xi} - 2\gamma\beta y_{\xi\eta} + \gamma^2 y_{\eta\eta}\right)x_\xi\right] = \frac{\nu J}{\gamma^2}\left(x_{\eta\eta}y_\xi - y_{\eta\eta}x_\xi\right)$$

4. APPLICATION AND DISCUSION

In this section, the differential quadrature method used for solving the Eq. (3), by defining transversal vibrations of irregular shaped plates, will be presented. Dependant changeable ξ *and* η *in* quadrangle comuting domain take values in a range -1 to 1. The result of the application of the differential quadrature method to Eq. (3), is as follows:

$$\begin{aligned}\bar{D}_{ij}^{(41)}\bar{D}_{ik}^{\xi}w_{kj} + \bar{D}_{ij}^{(42)}C_{ik}^{\xi}A_{jm}^{\eta}w_{km} + \bar{D}_{ij}^{(43)}B_{ik}^{\xi}B_{jm}^{\eta}w_{km} + \bar{D}_{ij}^{(44)}A_{ik}^{\xi}C_{jm}^{\eta}w_{km} + \bar{D}_{ij}^{(45)}\bar{D}_{jm}^{\eta}w_{im}\\ + \bar{D}_{ij}^{(31)}C_{ik}^{\xi}w_{kj} + \bar{D}_{ij}^{(32)}B_{ik}^{\xi}A_{jm}^{\eta}w_{km} + \bar{D}_{ij}^{(33)}A_{ik}^{\xi}B_{jm}^{\eta}w_{km} + \bar{D}_{ij}^{(34)}C_{jm}^{\eta}w_{im}\\ + \bar{D}_{ij}^{(21)}B_{ik}^{\xi}w_{kj} + \bar{D}_{ij}^{(22)}A_{ik}^{\xi}A_{jm}^{\eta}w_{km} + \bar{D}_{ij}^{(23)}B_{jm}^{\eta}w_{im} + \bar{D}_{ij}^{(11)}A_{ik}^{\xi}w_{kj}\\ + \bar{D}_{ij}^{(12)}A_{jm}^{\eta}w_{im} = \Omega^2 w_{i,j}\end{aligned} \tag{16}$$

where i,j=3,...,(N-2), and A_{ij}, B_{ij}, C_{ij} and D_{ij} with exponents ξ *and* η which mark weight coefficients of matrix of first, second, third and fourth order derivation along ξ *and* η direction. N and M are numbers of mesh points along ξ and η direction respectively. Index *k* which is repeated presents the total from 1 to *n* along ξ *and* η direction, and index *m* which is also repeated present the total from 1 to *M* along η direction. Here are presented all the imporatnt coefficients of the differential quadrature method in the Eq. (16), which are obtained the same way as when the differential quadrature method is applied on problems in regular domian. This means that the Eq. (16) only additionally includes N^2M^2 of the scalar product.

There are four types of approaches available for application of multiple boundary conditions. The oldest is called δ-technique, suggested by Bert and his assiciates [3], which is widely used is literature. In this approach, the geometrical boundary conditions are applied on the present boundary points and on the derivative boundary conditions in δ- point, which are at a little distance ($\delta \cong 10^5$ in the units without dimensions [1] from the respective boundary. As already mentioned, the approximately chosen value δ can cause the unexpectd oscilations in the achived results. To overcome the deffects of δ –approach, Wang and Bert [4] developed a new technique, which includes boundary conditions in matrices of impact coefficients of the differential quadrature method in advance, and than the impact coefficients with implemented boundary conditions are directed towards discretisation of the comprehensive equations for the problems in question.

The main idea of this approach is that the boundary conditions, applied during the formulation of the impact coefficients of matrix, are used for inner mesh ponts. The technique increases the accuracy of the differential quadrature method for the problems with simply supported boundary conditions.However, the technique is limited to simple problems, since it can not be applied to problems with discontinuos geometric forms and the measures that are found in derivative boundary conditions.

The validity of the methods was proved by compering the presented results and those of Bert and Malik [1], who have used different approaches for transformation of the coordinates and application of the multiple boundary conditions. We made this program by using the approach in [1], then, we started the program on the personal computer Pentium IV generation and compared the speed of the methods. The Fig. 2 shows the ratio between the processing time and the number of mesh points (N) in direction of *x*-axis for vibration analysis of occasionally excentric plates [5]. In this approach, the same number of mesh points in direction of x and y is adopted. As a result of using our method, much less CPU time was needed for the same number of mesh points than by using the method proposed by Bert and Malik. It is also relevant that computing weight in solving the result scalar, for which there is such non-null vector that the scalar multiplicaton of vectors equals to the vector value under given linear transformation of the equation system is the same in the presented approach as in the Bert and Malik approach.

The comuting efficiency of the presented method lies in the fact that it is not included in the order N^2M^2 of scalar product for obtaining quantization matrix for derivations of of higher order. The Figure 1. shows that for the solution of the result scalar, for which there is such non-null vector that the scalar multiplicaton of vectors equals to the vector value under given linear transformation of the equation system, less CPU time is used in our present work, while on the contrary, the multiplication of matrices in Bert and Malik's approach require more CPU time (Figure 2)[5]. On the other hand, by comparing it with the solutions for plates with changeable boundary conditions, it can be seen that the processing time, when the same number of mesh points is used, does not change much.

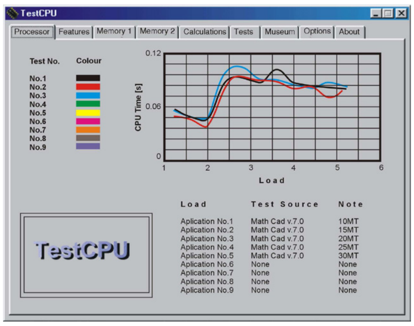

Figure 1. The relation between the processing time necessary for solving problems and the number of sytem points for vibration analysis of occasionally excentric plates (MDRII)

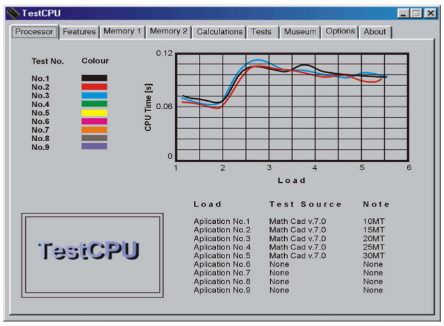

Figure 2. The relation between processing time necessary to solve the problems and numbers of system points for vibration analysis of occasionally excentric plates(Bert&Malik)

The solution for plates with CCCC edges (C-clamped, SS-simply supported), by using PV-2 Ritz method is also included in table 1. Thus we also compared the solutions we achived by using differential quadrature method and Ritz PV-2 method, and came to the conclusion that the difference is very small and insignificant. We came to the same conclusion as regards to symmetric parabolic trapezoidal plates with simply supported boundaries, the present method has faster convergence using uneven number of mesh points than by using even number of mesh points along ξ *and* η direction. This coincides with the approach of Bert and Malik.

Table 1. The convergence of the solutions for the first six frequencies of flexural vibrations of parabolic trapezoidal plate (Figure 2, $a/b=3.0$, $b/c=2.5$, $\Omega=\omega a^2/\pi^2\sqrt{\rho h/D}$)

	FREKVENCY					
N=M	1	2	3	4	5	6
Approach	U-U-U-U					
MDRII	9.3723	13.9641	19.7460	21.8375	27.2672	29.1650
Bert & Malik	9.3645	13.977	19.799	21.843	27.334	29.139
PV-2	9.3428	14.1186	20.0527	21.6208	27.6616	29.2138
	JO-U-JO-U					
MDRII	8.5709	12.880	18.0984	20.7258	24.6152	27.7824
Bert & Malik	8.5694	12.886	18.154	20.746	24.691	27.757
	U-JO-U-JO					
MDRII	5.4742	9.9289	15.4196	16.1466	21.930	24.2740
Bert & Malik	5.4831	9.9535	15.424	16.178	21.942	24.306

At the very end of this discussion, we present the table 2, with the first ten frequencies of the clamped rhombic plates which ratio between the bigger and smaller diagonal of length (b/a) is 1.5:1. In this case, Bert and Malik were not able to get closer to the third frequency [1]. Maybe it was due to the fact that one of the double boundary conditions at certain edges was performed under the influence of δ- point, which is not precisely on the boundary.

Table 2. The convergence of the solutions for the first six frequencies of flexural vibrations rhombic plate (Fig. 5. $\Omega=\omega a^2/\pi^2\sqrt{\rho h/D}$)

Approach	FREKVENCY									
N = M	1	2	3	4	5	6	7	8	9	10
MDRII	12.703	23.369	28.254	34.738	45.948	48.969	49.794	61.130	65.361	74.533
Bert & Malik	12.703	23369	-	34.738	45.948	48.969	49.794	61.130	65.362	74.535
Gorman	12.70	23.37	28.25	34.74	45.95	48.98	49.79	61.13	-	74.54

5. CONCLUSION

This work presents a new approach to the study of vibrations of irregular-shaped plates with simply supported or clamped boundaries. In this approach, the irregular physical domain is transformed into regular physical domain (square) in curvilinear coordinate system (computing space), and through accompanying equations, the conditions are transformed into relevant forms of curvilinear coordinate system. In this way all the computing is within certain computing domain. As long as the computing domain is correct, the application of the differential quadrature method on the irregular-shaped plates in computing space is also correct, as well as the application of the differential quadrature method on the irregular-shaped plates in physical space. The only difference is that there are more members included in the accompnying equation as well as there are more boundary conditions in the curvlinear coordinate system. By using the present method big operations with matrix multiplications, used by Bert and Malik[1], are avoided, and as a result, the computing weight has been noticably reduced as well as the size of virtual storage necessary for the important operations. The presentation of the above mentioned is performed by using the application test on the presented approach, which proved to require less than 1/10 of the processing time in comperison with Bert and Malik's approach when comparing the same number of mesh points. The comparison of speed characteristics of these methods is performed by using the service program Test CPU, and the comparison of the memory space needed for computing the vibration frequenc of four-sided plane plates of irregular shape is performed by using service program *Ram Booster* vl.6. The service program *Ram Booster* vl.6 is also used to establish the involvment of computer processor -Pentium IV, in solving the abovementioned problems.

This work presents a simple way for application of the differential quadrature method on the simply supported and clamped boundray conditions , which avoid weight determination of the angle between the normal to the plate boundaryand the *x*-axis, used in the Bert and Malik approach. From this work, it can be concluded that applying differential quadrature method on the problem of vibration of quadriangle plates of irregular shape is more promising way to improve the efficiency and flexibility of numerical techniques for solving practical engineering problems.

Acknowledgement

This paper is the result of the research within the project TR 34028, financially supported by the Ministry of Science and Technology of Serbia, PD TE – KO Kostolac and Messer Tehnogas.

REFERENCES

[1] C. W. Bert and M. Malik, The differential quadrature method for irregular domains and applications to plate vibration, Int. J. Mech. Sci. 38, 589 (1996).
[2] C. W. Bert and M. Malik, Differential quadrature method in computational mechanics: A review, Appl. Mech. Rev. 49, 1 (1996).
[3] C. W. Bert, S. K. Jang, and A. G. Striz, Two newapproximate methods for analyzing free vibration of structural components, AIAA J. 26, 612 (1988).
[4] X. Wang and C. W. Bert, A new approach in applying differential quadrature to static and free vibrational analyses of beams and plates, J. Sound Vibration 62, 566 (1993).
[5] I.,Kucora, Development of the model of technical system vibration – Magisterial thesis, Techical Faculty "Mihailo Pupin",Zrenjanin, 2005.
[6] K. M. Liew and J. Han, A four-node differential quadrature method for straight-sided quadrilateral Reissner/ Mindlin plates, Commun. Numer. Methods Eng. 13, 73 (1997).
[7] S. S. E. Lam, Application of the differential quadrature method to two dimensional problems with arbitrary geometry, Comput. Struct. 47, 459 (1993).
[8] S. D. Campion and J. L. Jarvis, An investigation of the implementation of the p-version finite element method, Finite Elements Anal. Design 23, 1 (1996).
[9] D. J. Gorman, Accurate free vibration analysis of Rhombic plates with simply supported and fully-clamped edge conditions, J. Sound Vibration 125, 281 (1988).
[10] W. L. Chen, A. G. Striz, and C. W. Bert, A new approach to the differential quadrature method for Fourth–Order equations, Int. J. Num. Methods Eng. 40, 1941 (1997).