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[1.] Utpal Jyoti DAS

VISCO-ELASTIC EFFECT ON HYDROMAGNETIC CONVECTION OF A VISCO-ELASTIC FLUID OVER A CONTINUOSLY MOVING VERTICAL SURFACE WITH CHEMICAL REACTION ADJACENT TO A POROUS REGIME

[1.] Department of Mathematics, Rajiv Gandhi University, Rono Hills, Doimukh, Arunachal Pradesh, INDIA

Abstract: In this paper, hydromagnetic convection of a visco-elastic fluid over a continuously moving vertical surface with uniform suction and heat flux in presence of first-order chemical reaction through porous medium has been investigated. A uniform magnetic field is assumed to be applied perpendicular to the wall. The induced magnetic field assumed to be negligible. The solutions are obtained for velocity, temperature and concentration profiles. The profiles of velocity and skin friction are presented graphically for different combinations of the parameters involved in the flow problem.
Keywords: visco-elastic, hydromagnetic convection, first-order chemical reaction, Hartmann number, porous medium

1. INTRODUCTION

The study of combined heat and momentum transfer from a heated moving surface to a quiescent ambient medium is important in many engineering applications such hot rolling, wire drawing and crystal growing. The heat treatment of materials travelling between a feed roll and a wind-up roll or conveyor belts, the lamination and melt spinning processes in the extrusion of polymers have the characteristics of moving continuous surfaces. In ion propulsion, electromagnetic pumps, power generators, controlled fusion research, plasma jets, chemical synthesis etc; the applications of magnetohydrodynamics is prominent.

Sakiadis [1] studied the two dimensional boundary layer flows over a continuous solid surface moving with constant velocity in an ambient fluid. The flow is quite different from the boundary layer flow over a semi-infinite flat plate due to the entertainment of the ambient fluid. Vajravelu [2] studied the exact solution for hydrodynamic boundary layer flow and heat transfer over a continuous, moving, horizontal flat surface with uniform suction and internal heat generation/absorption. Again, Vajravelu [3] extended the problem (Vajravelu [2]) to vertical surface. Crane [4] studied the boundary layer flow caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point on the surface. Kumar *et al.* [5] discussed the hydromagnetic flow and heat transfer on a continuously moving vertical surface.

The study of heat and mass transfer through and across porous media is receiving considerable attention of many researchers because of its varied applications in the field of cosmical and geophysical sciences. Applications related to porous media can be found in monographs by Nield and Bejan [6] and Pop and Ingham [7]. Kim and Vafai [8] and Harris *et al.* [9] studied the problem of natural convection flow through porous medium past a plate. Magyari *et al.* [10] have investigated analytical solutions for unsteady free convection in porous media. Chaudhary and Jain [11] studied magnetic and mass diffusion effects on the free convection flow, when the plate is made to oscillate with a specified velocity. Mahmoud [12] studied the effects of radiation and variable viscosity on hydromagnetic boundary layer flow along a continuously moving vertical plate with suction and heat flux.

All the above investigators, however, restrict their analysis to the flow of Newtonian fluids. Most fluids such as molten plastics, artificial fibres, drilling of petroleum, blood and polymer solutions are considered non-Newtonian fluids. Choudhury and Das [13] discussed the hydromagnetic flow and heat transfer of a visco-elastic fluid on a continuously moving vertical plate. Das [14] studied the effects of viscoelasticity on unsteady MHD free convection and mass transfer flow of a visco-elastic, incompressible, electrically conducting fluid past an infinite hot vertical porous plate embedded in porous medium.

The main objective of the present work is to investigate the effects of visco-elastic parameter characterised by second-order fluid on steady hydromagnetic flow of a visco-elastic fluid over a continuously moving vertical surface with uniform suction and heat flux in presence of first-order chemical reaction through porous medium. The constitutive equation for the second-order fluid is of the form:

$$S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \tag{1}$$

where S is the stress tensor, p is hydrostatic pressure, I is unit tensor, $A_n\,(n=1,2)$ are the kinematics Rivlin-Ericksen tensors, μ_1, μ_2, μ_3 are the material co-efficients describing the viscosity, elasticity and cross- viscosity respectively. The material co-efficients μ_1, μ_2, μ_3 are taken constants with μ_1 and μ_3 as positive and μ_2 as negative (Coleman and Markovitz [15]). The equation (1) was derived by (Coleman and Noll [16]) from that of simple fluids by assuming that stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

2. MATHEMATICAL FORMULATION

Consider a steady boundary layer convective flow through porous medium of an electrically conducting visco-elastic fluid on a continuous surface, issuing from a slot and moving vertically with a uniform velocity u_w in a fluid and heat is supplied from the plate to the fluid at a uniform rate, in the presence of a uniform magnetic field of strength B_0. Let the x-axis be taken along the direction of motion of the sheet and the y-axis be normal to the surface. The induced magnetic field is assumed to be negligible. It is assumed that there exists a first order chemical reaction between the fluid and the fluid species concentration. The physical model of the problem is shown in Figure 1.

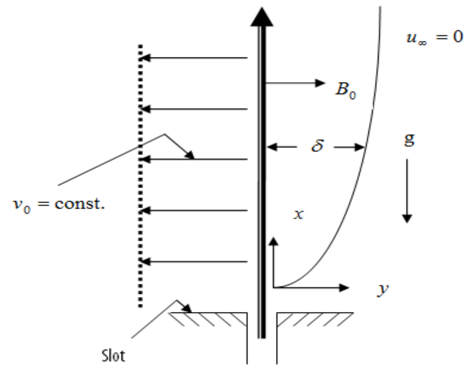

Figure 1: Physical model of the problem

Under the above assumptions, the flow is governed by the following equations:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

Equation of motion:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu_1 \frac{\partial^2 u}{\partial y^2} + \mu_2\left(u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right) - \sigma_1 B_0^2 u + \rho g \beta_T (T' - T'_\infty) + \rho g \beta_C (C' - C'_\infty) - \frac{\mu_1 u}{k_1} \tag{3}$$

Equation of energy:

$$\rho C_p\left(u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial y}\right) = \kappa \frac{\partial^2 T'}{\partial y^2} + \mu_2 u \frac{\partial^2 u}{\partial x \partial y}\frac{\partial u}{\partial y} \tag{4}$$

Equation of mass diffusion:

$$u\frac{\partial C'}{\partial x} + v\frac{\partial C'}{\partial y} = D\left(\frac{\partial^2 C'}{\partial x^2} + \frac{\partial^2 C'}{\partial y^2}\right) \tag{5}$$

where σ_1 is the electrical conductivity, β_T is the thermal expansion coefficient, T' is the temperature, T'_∞ is the temperature of the fluid far away from the plate, β_C is the concentration coefficient, C' is the concentration, C'_∞ is the concentration in the fluid far away from the plate, k_1 is the permeability of the porous medium, κ is the thermal conductivity.

The corresponding boundary conditions are:

$$u = u_w,\ v = v_0 = \text{constant} < 0,\ \frac{\partial T'}{\partial y} = -\frac{q}{\kappa},\ C' = C_\infty + \frac{q\upsilon_1}{\kappa v_0} \quad \text{at } y = 0$$

$$u \to 0,\ T' \to T'_\infty,\ C' = C'_\infty \ \text{as}\ \ y \to \infty \tag{6}$$

The following non-dimensional quantities are introduced:

$$Y = \frac{y v_0}{\upsilon_1},\ U = \frac{u}{u_w},\ T = \frac{T' - T'_\infty}{\left(\frac{q\upsilon_1}{\kappa v_0}\right)},\ Gr = \frac{\upsilon_1 g \beta_T \left(\frac{q\upsilon_1}{\kappa v_0}\right)}{u_w v_0^2},\ Pr = \frac{\mu_1 C_p}{\kappa},\ M = \frac{\sigma_1 B_0^2 \upsilon_1}{\rho v_0^2},$$

$$K = \frac{k_1 v_0^2}{\upsilon_1^2},\ Gm = \frac{\upsilon g \beta_C \left(\frac{q\upsilon_1}{\kappa v_0}\right)}{u_w v_0^2},\ Sc = \frac{\upsilon_1}{D},\ \phi = \frac{C' - C'_\infty}{\left(\frac{q\upsilon_1}{\kappa v_0}\right)} \tag{7}$$

where Gr is the thermal Grashof number, Gm is the solutal Grashof number, Pr is the Prandtl number and M is the Hartmann number, Sc is the Schmidt number, Re is the Reynolds number, K is the parameter of the porous medium, q is the heat flux, D is the mass diffusivity, v_0 is the suction velocity at the plate, and $\upsilon_1 = \frac{\mu_1}{\rho}$. Other physical variables have their usual meanings.

We make use of the assumptions that the velocity and temperature fields are independent of the distance parallel to the surface (as given in [17]) and the Boussinesq's approximation. Equations (3), (4) and (5) are reduced to the following non-dimensional form:

$$\alpha\frac{d^3U}{dY^3}-\frac{d^2U}{dY^2}-\frac{dU}{dY}-GrT-Gm\phi+\left(M+\frac{1}{K}\right)U=0 \tag{8}$$

$$\frac{d^2T}{dY^2}+Pr\frac{dT}{dY}=0 \tag{9}$$

$$\frac{d^2\phi}{dY^2}+Sc\frac{d\phi}{dY}=0 \tag{10}$$

where $\alpha=\frac{\mu_2\rho v_0^2}{\mu_1^2}$ is the visco-elastic parameter.

The corresponding boundary conditions are

$$U=1,\ \frac{\partial T}{\partial Y}=-1,\ \phi=1 \quad \text{at } Y=0,$$

$$U\to 0,\ T\to 0,\ \phi=0 \quad \text{as } Y\to\infty \tag{11}$$

Equations (9) and (10) are solved by using the boundary conditions (11) and their solutions are given by,

$$T(Y)=\frac{e^{-P_rY}}{P_r} \tag{12}$$

$$\phi(Y)=e^{-ScY} \tag{13}$$

The presence of elasticity in the governing fluid flow constitutes a third-order differential equation (8), but for Newtonian fluid $\alpha=0$, then the differential equation reduces to of order two. Also, it is seen that there are insufficient number of boundary conditions for the unique solution of (8). Since α is a measure of elasticity and for small shear rate $|\alpha|<1$, and hence we can assume that

$$U(Y)=U_0(Y)+\alpha U_1(Y)+\alpha^2U_2(Y)+\ldots \tag{14}$$

Substituting (14) in (8) and equating the coefficients of α^0, α^1 and neglecting those of α^2, the following equations are obtained

$$U_0''+U_0'+GrT+Gm\phi-\left(M+\frac{1}{K}\right)U_0=0 \tag{15}$$

$$U_0'''-U_1''-U_1'+\left(M+\frac{1}{K}\right)U_1=0 \tag{16}$$

The corresponding boundary conditions are

$$U_0=1,\ U_1=0,\ \frac{\partial T}{\partial Y}=-1 \quad \text{at } Y=0,$$

$$U_0\to 0,\ U_1\to 0,\ T\to 0 \quad \text{as } Y\to\infty \tag{17}$$

Solving the equations (15) and (16) under conditions (17), and then substituting in (14), we get

$$U=A_2e^{-PrY}+A_3e^{-ScY}+A_4e^{-A_1Y}+\alpha\left(A_9e^{-A_1Y}+A_6e^{-PrY}+A_7e^{-ScY}\right) \tag{18}$$

where: $A_1=\frac{1}{2}\left\{1+\sqrt{1+4(M+1/K)}\right\}$, $A_2=-\frac{Gr}{Pr\left\{Pr^2-Pr-(M+1/K)\right\}}$, $A_3=-\frac{Gm}{Sc^2-Sc-(M+1/K)}$, $A_4=1-A_2-A$,

$A_5=-\frac{A_1^3A_4}{A_1^2-A_1-(M+1/K)}$, $A_6=-\frac{A_2Pr^3}{Pr^2-Pr-(M+1/K)}$, $A_7=-\frac{A_3Sc^3}{Sc^2-Sc-(M+1/K)}$, $A_8=-(A_5+A_6+A_7)$, $A_9=A_5+A_8$.

The local skin friction (wall shear stress) at the surface is given by

$$\tau=\frac{\tau'}{\rho u_wv_0}=-\left(\frac{dU}{dY}\right)_{Y=0}+\alpha\left(\frac{d^2U}{dY^2}\right)_{Y=0}$$

$$=A_4A_1+A_2Pr+A_3Sc+\alpha\left(A_1^2A_4+A_2Pr^2+A_3Sc^2+A_4A_1+A_2Pr+A_3Sc\right) \tag{19}$$

3. RESULTS AND DISCUSSION

The computed solutions for the velocity, temperature, concentration and skin friction are valid at some distance from the slot, even though suction is applied from the slot onward. Selected computations have been depicted graphically in all the figures. In this study, main emphasis is given on the effects of the visco-elastic parameter α on the governing flow with the combination of the other flow parameters.

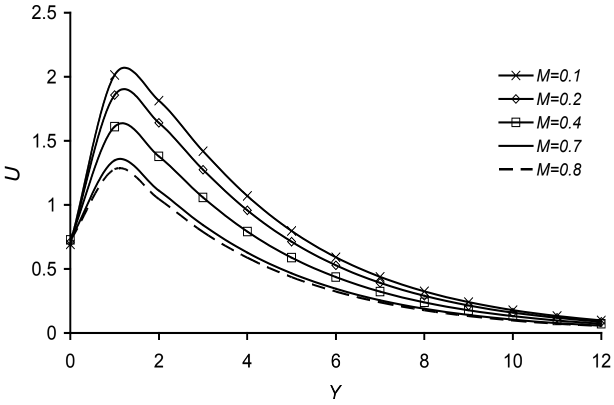

Figure 2: Velocity profiles for different M at $\alpha=-0.3, K=2, Gr=2, Gm=3, Sc=0.3$.

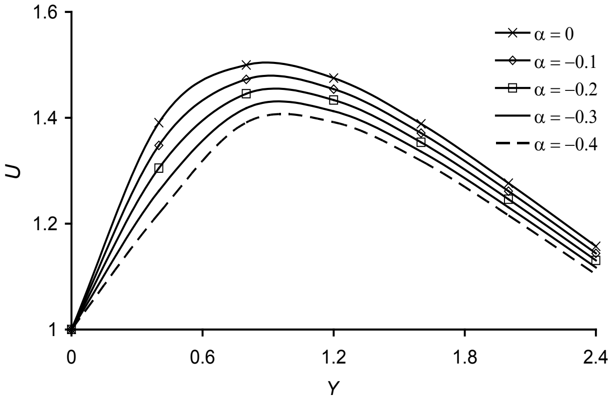

Figure 3: Velocity profiles for different α at $M=0.1, K=2, Gr=2, Gm=2, Sc=0.3$.

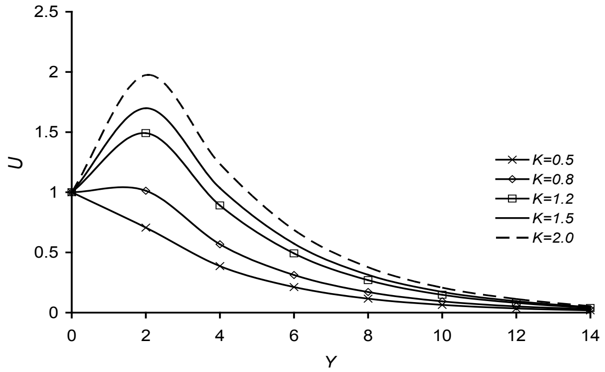

Figure 4: Velocity profiles for different K at $\alpha=-0.3, M=0.1, Gr=2, Gm=3, Sc=0.3$.

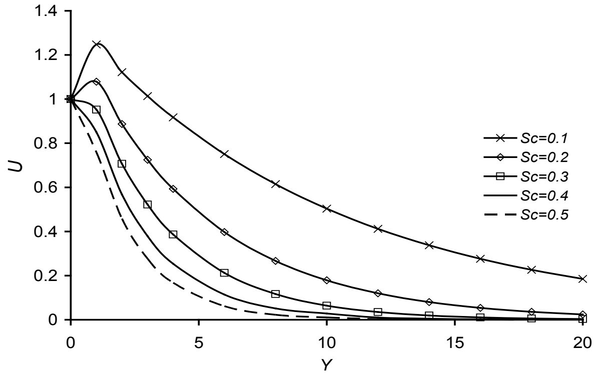

Figure 5: Velocity profiles for different Sc at $\alpha=-0.3, M=0.1, Gr=2, Gm=3, K=0.5$.

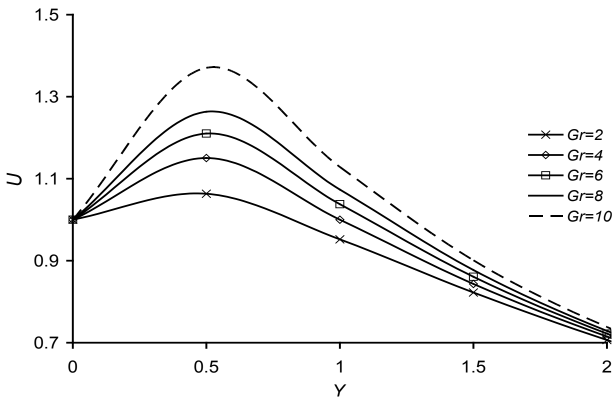

Figure 6: Velocity profiles for different Gr at $\alpha=-0.3, M=0.1, Sc=0.3, Gm=3, K=0.5$.

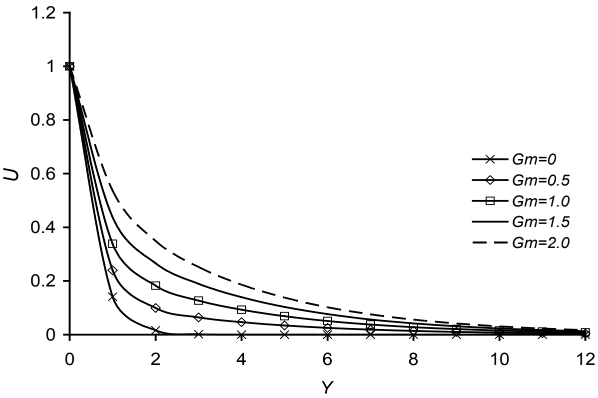

Figure 7: Velocity profiles for different Gm at $\alpha=-0.3, M=0.1, Sc=0.3, Gr=2, K=0.5$.

Figures 2-7 represent the velocity profiles U against Y under the effects of Hartmann number M, visco-elastic parameter α, porous parameter K, Schmidt number Sc, thermal Grashof number Gr and solutal Grashof number Gm respectively, with fixed values of Prantdl number $(Pr=3)$. From these figures, it is evident that the velocity field increases as K, Gr, Gm increase and decreases as $|\alpha|$, M increases. It indicates the fact that the fluid motion is retarded due to application of transverse magnetic field and the growth of magnitude of viscoelasticity of the fluid, whereas it is accelerated under the effects of porosity, thermal diffusion and mass diffusion.

Figures 8-10 represent the effects of the Hartmann number, porous parameter and solutal Grashof number on skin friction, respectively, for various values of the visco-elastic parameter $|\alpha|\left(\alpha=0,-0.02,-0.04,-0.06,-0.08\right)$.

Increasing the Hartmann number clearly enhances the skin friction, whereas increase in porosity parameter and solutal Grashof number reduces the skin friction for both Newtonian and visco-elastic fluid. There is a decrease in the skin friction values at the surface accompanying a rise in the absolute value of the visco-elastic parameter $|\alpha|$ from Newtonian $(\alpha = 0)$ to visco-elastic fluid $(|\alpha| \neq 0)$ with increasing values of Hartmann number and solutal Grashof number, whereas it increase with the increasing values of porosity parameter.

The temperature field and the concentration field are not significantly affected by the visco-elastic parameter.

4. CONCLUSION

An analytic solution of a visco-elastic fluid for hydromagnetic convection at a continuously moving vertical surface in porous medium with chemical reaction and uniform suction is obtained. The solutions are obtained in exponential functions. It was found that the velocity decreases with the increasing values of the Hartmann number, magnitude of the visco-elastic parameter, Schmidt number, whereas increases as porous parameter, thermal Grashof number, solutal Grashof number increase. Skin friction increases in the presence of a magnetic field as compared to its absence in both Newtonian and non-Newtonian cases. The presence of a porous medium and diffusion of mass decreases the skin friction for both Newtonian and visco-elastic fluid.

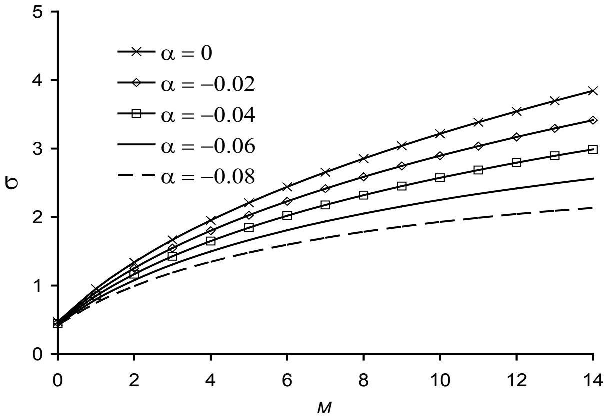

Figure 8: Effects of M on skin friction for different α at $Sc = 0.3, Gr = 2, K = 0.3,\ Pr = 3,\ Gm = 3.$

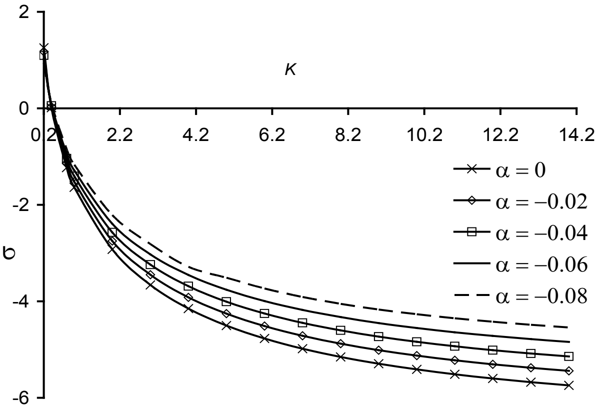

Figure 9: Effects of K on skin friction for different α at $Sc = 0.3, Gr = 2, M = 0.1,\ Pr = 3,\ Gm = 3.$

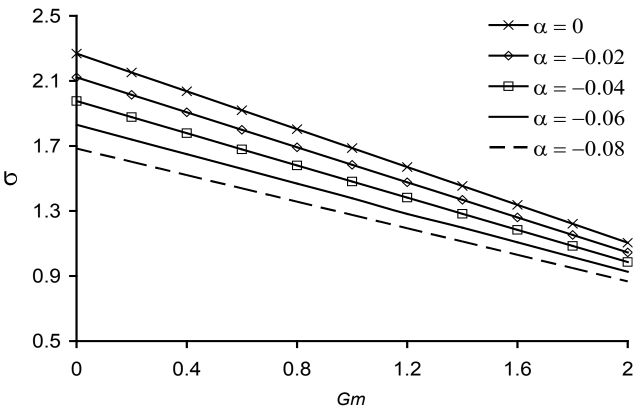

Figure 10: Effects of Gm on skin friction for different α at $Sc = 0.3, Gr = 2, M = 0.1,\ Pr = 3,\ K = 0.3.$

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