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# NON-CONVENTIONAL CONTROL OF LEVEL AND TEMPERATURE IN THE FLOW TANK

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**Abstract**: Control of multivariable processes is greatly hampered due to the presence of mutual coupling (interaction) between their inputs and outputs. Therefore, the analysis of the mutual coupling, i.e. the determination of its measure, is very important for determining further activities towards the achievement of a satisfactory behavior of the controlled object. Present paper contains synthesis of decoupler for flow tank as a multivariable process in order to achieve decentralized control of level and temperature as its outputs. Afterward, PI controllers were tuned, and the quality of this non-conventional control was confirmed through the considerations of system time responses. The research was supported by simulations, and the results represent one of approaches in forming the concept for control of multivariable processes. **Keywords**: multivariable process, non-conventional control, PI controller, flow tank, decoupler

## **1. INTRODUCTION**

One of the characteristics of real processes is their multi-variability. During analysis and synthesis of control for these processes nowadays exists approach, where their coupled inputs and output are observed in separated control loops, i.e. as more SISO (single input single output) systems. This approach gives satisfying results for processes with lower interaction index among inputs and outputs. Industrial development brings increasing the complexity of systems, what means expansion of the multivariable processes. Conventional approach does not give desired control performance here. These problems are overcome by considering of process as multivariable (MIMO - multi input multi output), in other words by respecting its real nature.

Present paper contains analysis of interaction using mathematical postulates given in [1]. Thereafter, the decoupler has been designed, in order to compensate influence of mutual coupling, based on one of ways shown in [2]. This way is attempt to introduce decentralized (non-conventional) control of level and temperature in 2x2 flow tank. Unlike the [2], where transformation of transfer function matrix to its diagonal shape are proposed, in this research PI controllers are tuned based on its main diagonal

elements, when it is not diagonal, what simplifies tuning procedure. Mentioned procedure is simpler in the case when decentralized control provide desired behavior of object, because then multiple relay feedback test can be avoided [3,4] which is one of the ways for designing of multivariable PI controller, i.e. centralized control.

## 2. MODEL OF OBJECT

Flow tank with two inputs and two outputs, shown in Figure 1, was taken into consideration. Water was taken as a fluid, which is supplied through the two valves, 1 and 2. Temperatures of water on these two inputs are different, as Figure 1 shows.

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Figure 1. Shema of the 2x2 flow tank [5]

Controled values (outputs) are level *h* and temperature *t*. Multi-variability is reflected in the fact that change in flow through the any valve ( $Q_1$  or  $Q_2$ ) causes simultaneously changes of the level and temperature in the tank. Flow tanks are very common in industry where two fluids are mixed in order to obtain their blend through the valve 3, with desired density, temperature, concentracion, etc, and constant flow rate  $Q_3$ . Valve 3 is the on/off type, and its position is not controlled in real time. Taking into account speed of mixer, the level should be controlled and maintained at the set value (in this example, 1 m) in order to ensure satisfactory mixing of the two fluids. The set (desired or reference) value of temperature is 30°C. Mathematical model for this type of flow tank is derived in [5] based on expressions in [6] and it follows:

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} = \begin{bmatrix} \frac{K}{T_{s+1}} & \frac{K}{T_{s+1}} \\ \frac{K_{1}}{T_{1}s+1}e^{-L_{1}s} & \frac{K_{2}}{T_{2}s+1}e^{-L_{2}s} \end{bmatrix}$$
(1)

Where are:  $g_{ij}(s)$  – elements of transfer function matrix, K,  $K_1$  and  $K_2$  – gains, T,  $T_1$  and  $T_2$  – time constants,  $L_1$  and  $L_2$  – delay times. **3. ANALYSIS OF OBJECT** 

Analysis of coupling in the considered process and identification of other possible problems in its control were carried out using in literature well known mathematical rules [1]. For that analysis, the gain K (0,01; 0,1; 1) in elements  $g_{11}(s)$  and  $g_{12}(s)$  was varied, i.e. three variants of flow tank were researched. Therefore, transfer function matrix is [5]:

$$G(s) = \begin{bmatrix} \frac{K}{63s+1} & \frac{K}{63s+1} \\ \frac{-0.15}{10s+1}e^{-3s} & \frac{0.4}{10s+1}e^{-2s} \end{bmatrix}$$
(2)

Process does not have zeros, therefore no zeros in the right half plane of the complex plane, so from this point of view, there are no limitations on control. Negative values of poles  $p_1 = -0,0159$  and  $p_2 = -0,1$  are one of the confirmation of process stability. Stability is obvious from Bode plots shown in Figure 2 for each pair of input-output. These diagrams are plotted for value of gain K=1, because in this case the amplitude frequency characteristics has the highest value in comparison with variants of the process where K=0,1 and K=0,01. So it can be concluded that in the other two cases, the process is stable, because smaller values gain K give a lower value of amplitude frequency characteristics and thus higher amplitude margin and hence higher stability margin.

Relative Gain Array (RGA) has been discussed as a measure of the mutual coupling in this process, and it gave relative gain matrix for steady state, which is equal for all three values of gain K(0,01;0,1;1) and it follows:

$$RGA[G(0)] = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} 0,7273 & 0,2727 \\ 0,2727 & 0,7273 \end{bmatrix}$$
(3)

Analysis of the mutual coupling using RGA for crossover frequency  $\omega_c$  was not carried out because researched process does not have that frequency, as it can be seen from diagram in Figure 2. From equation (3) is noticable that there are no negative terms of RGA. That means that there are no closed loops whose static gain changes its sign when the others loops are closed, i.e. during system functioning. This fact makes control easier, because from this point of view, there are no limitations on the pairing of particular inputs and outputs of the process.

However, values of off-diagonal elements of RGA are not equal to zero, so it is evident that the processes of this type have a mutual coupling, in whose presence is very hard or impossible to achieve high control



Figure 2. Bode plots for 2x2 flow tank for K=1

performances using conventional method. That was proved by time responses of the system which will be given in the next chapter. Thus, it is obvious that RGA matrix in equation (3) deviates from the identity matrix, as its most suitable variant for control, so for this system, the decoupler has been designed in order to compensate mutual coupling that was identified during analysis.

#### 4. NON-CONVENTIONAL APPROACH TO CONTROL

Due to [5], using conventional methods, this kind of flow tank can be successfully controlled only when the gains in off-diagonal elements  $g_{12}(s)$  and  $g_{21}(s)$  of transfer function matrix are small enough compared to gains in its diagonal elements  $g_{11}(s)$  and  $g_{22}(s)$ . To prevent this fact to limit possibilities for control of the considered flow tank, regarding range of its characteristics (dimensions of components, temperatures of fluids), decentralized control as a type of non-conventional control is designed below.

#### 4.1. Design of decoupler

To solve this problem the direct decoupler was used [2]. Its design is based on the request that the product of the transfer functions matrix and matrix of decoupler should be diagonal matrix. That enables decoupling as follows [2]:

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \cdot \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = \begin{bmatrix} g_{11}d_{11} + g_{12}d_{21} & g_{11}d_{12} + g_{12}d_{22} \\ g_{21}d_{11} + g_{22}d_{21} & g_{21}d_{12} + g_{22}d_{22} \end{bmatrix} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$$
(4)

where  $d_{ij}(s)$  - elements of the matrix of decoupler, whose block diagram is in Figure 3.



Figure 3. Direct decoupler for 2x2 process [2]

From equation (4) follows matrix of decoupler D(s) and final diagonal matrix Q(s). Equation (5) represents the form of the decoupler for researched flow tank. But in this research, diagonal matrix (6) will not be calculated, in the attempt to make procedure of tuning PI controller be shorter.

$$D(s) = \begin{bmatrix} 1 & \frac{-g_{12}}{g_{11}} \\ \frac{-g_{21}}{g_{22}} & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0,375 \cdot e^{-s} & 1 \end{bmatrix}$$
(5) 
$$Q(s) = \begin{bmatrix} g_{11} - \frac{g_{12} g_{21}}{g_{22}} & 0 \\ 0 & g_{22} - \frac{g_{12} g_{21}}{g_{11}} \end{bmatrix}$$
(6)

#### 4.2. Pl controller tuning

PI controllers were tuned based on elements  $g_{11}(s)$  and  $g_{22}(s)$  on main diagonal of transfer function matrix of the process (flow tank), therefore, as previously stated, without calculation of diagonal matrix in equation (6). Of course, transfer functions of all components between the controller and the process have been taken into account, which will be shown in the next subchapter on the block diagram of the entire control system. Tuning was performed using  $\lambda$ -method derived by Dahlin. According to this method, controller parameters (proportional gain  $K_c$  and integral time constant  $T_i$ ) are:

$$K_c = \frac{1}{K} \frac{T}{L+\lambda}$$
 and  $T_i = T$  (7)

where  $\lambda$  is desired time constant of the process, and it is  $\lambda = T$  (because of emphasising of response speed). Table 1 contains parameters values of PI controller 1 (that form control based on feedback from level as output) for changing gain *K*, and parameters values of PI controller 2 (that form control based on feedback from temperature as output).

Table 1. Parameters values of PI controllers when changing gain K

	PI controller 1		PI controller 2	
	Kc	Ki	Kc	Ki
<i>K</i> =0,01	3,33	0,0529		
<i>K</i> =0,1	0,33	0,005	0,0694	0,0069
<i>K</i> =1	0,03	0,0005		

Parameters values of PI controller 2 are not changed due to gain K variation is performed only in element  $q_{11}(s)$ .

#### 4.3. Evaluation of the results

A good indicator of the effects of introduced changes are time responses of the system. According that, Figure 4 shows block diagram of the control system of level and temperature in flow tank with decoupler, which is given by equation (5).Transfer functions of the components (except decoupler) for system in Figure 4, are taken from [5], where I/P transducer is current-pneumatic transducer. *U* and  $X_i$  are manipulated and controlled variable, respectively.



Figure 4. Block diagram of the control system of level and temperature in flow tank with decoupler

Simulations were carried out for two types of control: first conventional (without decoupler, i.e. elements  $d_{12}$  and  $d_{21}$ ) and second non-conventional (with decoupler). Comparative review of the system responses (level changes in time) in both types of control for three considered values of gains *K* (0,01; 0,1; 1) is given in Figures 5, 6 and 7. Figure 8 shows the same procedure performed for the other output, i.e. temperature, but only for *K*=0,01 because it has very small variation with the change of gain *K* in elements  $g_{11}(s)$  and  $g_{12}(s)$ . Both of conventional and non-conventional type of control were researched using the same parameters of the PI controller, which are shown in Table 1. In the following figures R is desired (reference) value.





Figure 6. Level in flow tank: control with (D) and without (WD) decoupler, when K=0,1



Figure 7. Level in flow tank: control with (D) and without (WD) decoupler, when K=1

Figure 8. Temperature in flow tank: control with (D) and without (WD) decoupler, when K=0,01

In Figures 5, 6 and 7 can be seen that, under control without decoupler, level has acceptable change in time only for lowest value of gain, i.e. K=0,01 (Figure 5), although even for this value of gain overshoot occurs, and it should be eliminated. Processes whose responses are shown in Figures 6 and 7 can not be controlled in a conventional way without decoupler, because the overshoot of level exceeds the dimensions of the tank, and because of its enormous value generally. Although the system is not slow, its settling time is too big.

Mentioned lacks are successfully compensated by decoupler. In fact, it enlarges the set of dimensionally different flow tanks, that can be well controlled. This refers primarily to the elimination of overshoot and shortening of settling time. In comparing the response of the system for cases with and without decoupler for certain values of gain *K*, quantification of indicators of the quality of transition process was not performed, because of obvious improvements with applied decoupler. The specific requirements regarding the response characteristics, can be fulfilled by tuning the controller with an emphasis on the target performance.

Because of the relatively small gain in the element  $g_{21}(s)$  of flow tank model (2) variations that were introduced do not significantly affect the temperature change in time. However, even in this case, improvement of system response with introduced decoupler is evident, because the overshoot disappears, and the system is faster.

#### **5. CONCLUSIONS**

On the basis of this research, it can be concluded that for flow tanks of this type, decentralized control can be used as a nonconventional control with decoupler which provides satisfactory characteristics of both process outputs (level and temperature).

The improvements obtained in this way are reflected in simplifying and shortening the procedure for designing the controller, since it is not necessary to calculate the diagonal transfer functions matrix (6). It is also not necessary to carry out neither decentralized nor individual relay feedback test in order to design multivariable PI controller, thus its multiple repetition can be avoided. Further research directions will refer to examine the possibilities of decentralized PI controller from the viewpoint of disturbance compensation in the 2x2 processes.

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