ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering
Tome XIII [2015] – Fascicule 3 [August]
ISSN: 1584-2673 [CD-Rom; online]
a free-access multidisciplinary publication of the Faculty of Engineering Hunedoara

[1.] Jamshad AHMAD, [2.] Syed Tauseef MOHYUD-DIN

NEW APPLICATIONS OF FRACTIONAL COMPLEX TRANSFORMS TO MATHEMATICAL PHYSICS

[1-2.] Department of Mathematics, Faculty of Sciences, HITEC University TaxilaCantt, PAKISTAN

Abstract: This paper witnesses the coupling of an analytical series expansion method Reduced Differential Transform with fractional complex transforms. The proposed technique is applied on three mathematical models subject to the appropriate initial conditions which arise in mathematical physics.The derivatives are defined in the Jumarie's sense. The accuracy, efficiency, and convergence of the proposed technique are demonstrated through the numerical examples.
Keywords: Fractional differential equation, Jumarie's fractional derivative,Fractional complex transform method, Reduced Differential Transform method

1. INTRODUCTION

Nonlinear partial differential equations (NLPDEs) are mathematical models that are used to describe complex phenomena arising in the world around us. The nonlinear equations appear in many applications of science and engineering such as fluid dynamics, plasma physics, hydrodynamics, solid state physics, optical fibers, acoustics and other disciplines. In the recent years, many authors mainly had paid attention to study solutions of NLPDEs by using various methods including; Adomian Decomposition (ADM) [1], Variational Iteration (VIM) [2], Homotopy Perturbation (HPM) [3], Homotopy Analysis (HAM) [4], F-Expansion [5], Exp-function [6], sine–cosine method[7], Differential Transform (DTM) [8-12],.It has received much attention since it has applied to solve a wide variety of problems by many authors [13–20].

2. JUMARIE'S FRACTIONAL DERIVATIVE

Some useful formulas and results of Jumarie's fractional derivative were summarized [25].

$$D_x^{\alpha} c = 0, \alpha \geq 0, \ c = \text{constant}. \tag{1}$$

$$D_x^{\alpha}[c f(x)] = c D_x^{\alpha} f(x), \alpha \geq 0, \ c = \text{constant}. \tag{2}$$

$$D_x^{\alpha} x^{\beta} = \frac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)} x^{\beta-\alpha}, \beta \geq \alpha \geq 0. \tag{3}$$

$$D_x^{\alpha}[f(x)g(x)] = [D_x^{\alpha} f(x)]g(x) + f(x)[D_x^{\alpha} g(x)]. \tag{4}$$

$$D_x^{\alpha} f(x(t)) = f_x'(x) x^{\alpha}(t). \tag{5}$$

3. FRACTIONAL COMPLEX TRANSFORM METHOD (FCTM)

The fractional complex transform was first proposed [26] and is defined as

$$\begin{cases} T = \dfrac{p t^{\alpha}}{\Gamma(\alpha+1)} \\ X = \dfrac{q x^{\beta}}{\Gamma(\beta+1)} \\ Y = \dfrac{k y^{\gamma}}{\Gamma(1+\gamma)} \\ Z = \dfrac{l z^{\lambda}}{\Gamma(1+\lambda)} \end{cases} \tag{6}$$

where *p*, *q*, *k*, and *l* are unknown constants, $0 < \alpha \leq 1, \ 0 < \beta \leq 1, \ 0 < \gamma \leq 1, \ 0 < \lambda \leq 1.$

4. REDUCED DIFFERENTIAL TRANSFORM METHOD (RDTM)

To illustrate the basic idea of the DTM, The differential transform of k^{th} derivative of a function $u(x,t)$, which is analytic and differentiated continuously in the domain of interest, is defined as

$$U_k(x)=\frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=t_0}, \tag{7}$$

The differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t)=\sum_{k=0}^{\infty} U_k(x)(t-t_0)^k, \tag{8}$$

Equation (8) is known as the Taylor series expansion of $u(x,t)$, around $t=t_0$. Combining equations (7) and (8)

$$u(x,t)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=t_0}(t-t_0)^k, \tag{9}$$

when $t_0=0$, above equation reduces to

$$u(x,t)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=t_0} t^k, \tag{10}$$

and equation (2) reduces to

$$u(x,t)=\sum_{k=0}^{\infty} U_k(x)t^k. \tag{11}$$

Theorem 1: If the original function is $u(x,t)=w(x,t)+v(x,t)$, then the transformed function is $U_k(x)=W_k(x)+V_k(x)$

Theorem 2: If $u(x,t)=\alpha w(x,t)$, then $U_k(x)=\alpha W_k(x)$.

Theorem 3: If $u(x,t)=\frac{\partial^m w(x,t)}{\partial t^m}$, then $U_k(x)=\frac{(k+m)!}{k!}W_k(x)$.

Theorem 4: If $u(x,t)=\frac{\partial w(x,t)}{\partial x}$, then $U_k(x)=\frac{\partial}{\partial x}W_k(x)$.

Theorem 5: If $u(x,y,t)=\frac{\partial w(x,y,t)}{\partial x}$, then $U_k(x,y)=\frac{\partial}{\partial x}W_k(x,y)$.

Theorem 6: If $u(x,y,z,t)=\frac{\partial w(x,y,z,t)}{\partial x}$, then $U_k(x,y,z)=\frac{\partial}{\partial x}W_k(x,y,z)$.

Theorem 7: If $u(x,t)=x^m t^n w(x,t)$, then $U_k(x)=x^m W_{k-n}(x)$.

Theorem 8: If $u(x,t)=w^2(x,t)$, then $U_k(x)=\sum_{r=0}^{k} W_r(x)W_{k-r}(x)$.

5. NUMERICAL APPLICATIONS

To show the efficiency of the fractional complex transform method coupled with reduced differential transform method described in the previous part, we present some examples.

Ξ **The Fractional Kaup–Kupershmidt(FKK) Equation**

Consider the nonlinear KK equation [23,24]:

$$\frac{\partial^\alpha u}{\partial t^\alpha}-\frac{\partial^5 u}{\partial x^5}-5u\frac{\partial^3 u}{\partial x^3}-\frac{25}{3}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}-5u^2\frac{\partial u}{\partial x}=0, \tag{12}$$

with the initial condition

$$u(x,0)=-2k^2+\frac{24k^2}{1+e^{kx}}-\frac{24k^2}{\left(1+e^{kx}\right)^2}, \tag{13}$$

where *k* is an arbitrary constant.

Applying the transformation [26], we get the following partial differential equation

$$\frac{\partial u}{\partial T}-\frac{\partial^5 u}{\partial x^5}-5u\frac{\partial^3 u}{\partial x^3}-\frac{25}{3}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}-5u^2\frac{\partial u}{\partial x}=0, \tag{14}$$

Applying the Differential Transform to Eq. (14) and Eq. (13), we obtain the following recursive formula

$$(k+1)U_{k+1}(x)=\frac{\partial^5 U_k(x)}{\partial x^5}+5\sum_{r=0}^{k}U_{k-r}(x)\frac{\partial^3 U_r(x)}{\partial x^3}+\frac{25}{3}\sum_{r=0}^{k}U_{k-r}(x)\frac{\partial^2 U_r(x)}{\partial x^2}+5\sum_{r=0}^{k}\sum_{s=k}^{r}U_{k-r}(x)U_{r-s}(x)\frac{\partial U_s(x)}{\partial x}. \tag{15}$$

Using the initial condition, we have

$$U_0(x)=-2k^2+\frac{24k^2}{1+e^{kx}}-\frac{24k^2}{\left(1+e^{kx}\right)^2}. \tag{16}$$

Now, substituting Eq. (16) into (15), and by straightforward iterative steps, yields

$$U_1(x)=-\frac{264k^7e^{kx}\left(-1+e^{kx}\right)}{\left(1+e^{kx}\right)^3},\ U_2(x)=-\frac{1452k^{12}e^{kx}\left(4e^{kx}-e^{2kx}-1\right)}{\left(1+e^{kx}\right)^4},\ U_3(x)=\frac{3524k^{17}e^{kx}\left(-11e^{kx}+11e^{2kx}-e^{3kx}+1\right)}{\left(1+e^{kx}\right)^5},\vdots$$

and so on.

The series solution is given by

$$u(x,T)=-2k^2+\frac{24k^2}{1+e^{kx}}-\frac{24k^2}{\left(1+e^{kx}\right)^2}-\frac{264k^7e^{kx}\left(-1+e^{kx}\right)}{\left(1+e^{kx}\right)^3}T-\frac{1452k^{12}e^{kx}\left(4e^{kx}-e^{2kx}-1\right)}{\left(1+e^{kx}\right)^4}T^2+\frac{3524k^{17}e^{kx}\left(-11e^{kx}+11e^{2kx}-e^{3kx}+1\right)}{\left(1+e^{kx}\right)^5}T^3+\dots$$

The inverse transformation will yield

$$u(x,t)=-2k^2+\frac{24k^2}{1+e^{kx}}-\frac{24k^2}{\left(1+e^{kx}\right)^2}-\frac{264k^7e^{kx}\left(-1+e^{kx}\right)}{\left(1+e^{kx}\right)^3}\frac{t^\alpha}{\Gamma(\alpha+1)}$$
$$-\frac{1452k^{12}e^{kx}\left(4e^{kx}-e^{2kx}-1\right)}{\left(1+e^{kx}\right)^4}\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)}+\frac{3524k^{17}e^{kx}\left(-11e^{kx}+11e^{2kx}-e^{3kx}+1\right)}{\left(1+e^{kx}\right)^5}\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)}+\dots$$

This solution is convergent to the exact solution

$$u(x,t)=-2k^2+\frac{24k^2}{1+e^{kx+11k^5t}}-\frac{24k^2}{\left(1+e^{kx+11k^5t}\right)^2}. \tag{17}$$

Ξ **The Generalized Fractional Drinfeld–Sokolov (GFDS) Equations**

We consider the system of generalized Fractional Drinfeld–Sokolov (GFDS) equations [21,22]:

$$\begin{cases}\dfrac{\partial^\beta u}{\partial t^\beta}+\dfrac{\partial^3 u}{\partial x^3}-6u\dfrac{\partial u}{\partial x}-6\dfrac{\partial v^\alpha}{\partial x}=0,\\ \dfrac{\partial^\beta v}{\partial t^\beta}-2\dfrac{\partial^3 v}{\partial x^3}+6u\dfrac{\partial v^\alpha}{\partial x}=0,\end{cases}\quad 0<x,t<\pi,\ 0<\beta\le 1, \tag{18}$$

with the initial conditions

$$u(x,0)=\frac{-b^2-4k^4}{4k^2}+2k^2\tanh^2(kx),\ v(x,0)=b\tanh(kx). \tag{19}$$

where α is a constant.

Applying the transformation [26], we get the following partial differential equations

$$\begin{aligned}&\frac{\partial u}{\partial T}+\frac{\partial^3 u}{\partial x^3}-6u\frac{\partial u}{\partial x}-6\frac{\partial v^\alpha}{\partial x}=0,\\ &\frac{\partial v}{\partial T}-2\frac{\partial^3 v}{\partial x^3}+6u\frac{\partial v^\alpha}{\partial x}=0,\end{aligned} \tag{20}$$

Applying the Differential Transform to Eq. (20) and Eq. (19), we obtain the following recursive formula

$$(k+1)U_{k+1}(x)=-\frac{\partial^3 U_k(x)}{\partial x^3}+6\sum_{r=0}^{k}U_{k-r}(x)\frac{\partial U_r(x)}{\partial x}+6\frac{\partial V_k^{\alpha}(x)}{\partial x}$$
$$(k+1)V_{k+1}(x)=2\frac{\partial^3 V_k(x)}{\partial x^3}-6\sum_{r=0}^{k}U_{k-r}(x)\frac{\partial V_r(x)}{\partial x}. \tag{21}$$

Using the initial condition, we have

$$U_0(x)=\frac{-b^2-4k^4}{4k^2}+2k^2\tanh^2(kx)$$
$$V_0(x)=b\tanh(kx). \tag{22}$$

Now, substituting Eq. (22) into Eq. (21) when ($\alpha=2$), and by straightforward iterative steps, yields

$$U_1(x)=\frac{2k(4k^2+3b^2)\sinh(kx)}{\cosh(kx)^3},V_1(x)=\frac{1}{2}\frac{b(4k^2+3b^2)}{\cosh(kx)^2 k},$$

$$U_2(x)=-\frac{1}{2}\frac{(2\cosh(kx)^2-3)(4k^2+3b^2)^2}{\cosh(kx)^4},V_2(x)=-\frac{1}{4}\frac{b(4k^2+3b^2)^2\sinh(kx)}{\cosh(kx)^3k^2},$$

$$U_3(x)=\frac{1}{3}\frac{\sinh(kx)(2\cosh(kx)^2-3)(4k^2+3b^2)^3}{k\cosh(kx)^5},V_3(x)=\frac{1}{24}\frac{b(2\cosh(kx)^2-3)(4k^2+3b^2)^3}{k^3\cosh(kx)^4},\vdots$$

and so on.

The series solution is given by

$$u(x,T)=-2k^2+\frac{24k^2}{1+e^{kx}}-\frac{24k^2}{(1+e^{kx})^2}+\frac{2k(4k^2+3b^2)\sinh(kx)}{\cosh(kx)^3}T-\frac{1}{2}\frac{(2\cosh(kx)^2-3)(4k^2+3b^2)^2}{\cosh(kx)^4}T^2$$
$$+\frac{1}{3}\frac{\sinh(kx)(2\cosh(kx)^2-3)(4k^2+3b^2)^3}{k\cosh(kx)^5}T^3+\ldots$$

$$v(x,T)=b\tanh(kx)+\frac{1}{2}\frac{b(4k^2+3b^2)}{\cosh(kx)^2k}T-\frac{1}{4}\frac{b(4k^2+3b^2)^2\sinh(kx)}{\cosh(kx)^3k^2}T^2+\frac{1}{24}\frac{b(2\cosh(kx)^2-3)(4k^2+3b^2)^3}{k^3\cosh(kx)^4}T^3+\ldots$$

The inverse transformation will yield

$$u=-2k^2+\frac{24k^2}{1+e^{kx}}-\frac{24k^2}{(1+e^{kx})^2}+\frac{2k(4k^2+3b^2)\sinh(kx)}{\cosh(kx)^3}\frac{t^{\alpha}}{\Gamma(\alpha+1)}-\frac{1}{2}\frac{(2\cosh(kx)^2-3)(4k^2+3b^2)^2}{\cosh(kx)^4}.$$
$$\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)}+\frac{1}{3}\frac{\sinh(kx)(2\cosh(kx)^2-3)(4k^2+3b^2)^3}{k\cosh(kx)^5}\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)}+\ldots$$

$$v=b\tanh(kx)+\frac{1}{2}\frac{b(4k^2+3b^2)}{\cosh(kx)^2k}\frac{t^{\alpha}}{\Gamma(\alpha+1)}-\frac{1}{4}\frac{b(4k^2+3b^2)^2\sinh(kx)}{\cosh(kx)^3k^2}\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)}+\frac{1}{24}\frac{b(2\cosh(kx)^2-3)(4k^2+3b^2)^3}{k^3\cosh(kx)^4}\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)}+\ldots$$

This solution is convergent to the exact solution

$$u(x,t)=\frac{-b^2-4k^4}{4k^2}+2k^2\tanh^2\left(kx+\frac{3b^2+4k^4}{2k}t\right).,v(x,t)=b\tanh\left(kx+\frac{3b^2+4k^4}{2k}t\right). \tag{23}$$

Ξ System of Coupled Fractional Sine-Gordon Equations

We now consider a system of coupled sine-Gordon equations [27,28]:

$$\begin{cases}\dfrac{\partial^{2\alpha}u}{\partial t^{2\alpha}}-\dfrac{\partial^2 u}{\partial x^2}=-a^2\sin(u-v),\\ \dfrac{\partial^{2\alpha}v}{\partial t^{2\alpha}}-c^2\dfrac{\partial^2 v}{\partial x^2}=\sin(u-v),\end{cases}\quad 0<x,t<\pi,0<\alpha\le 1, \tag{24}$$

with the initial conditions

$$u(x,0)=A\cos(kx),u_t(x,0)=0.$$
$$v(x,0)=0,v_t(x,0)=0. \tag{25}$$

Applying the transformation [26] to Eq. (24), we get the following partial differential equations

$$\frac{\partial^2 u}{\partial T^2}-\frac{\partial^2 u}{\partial x^2}=-a^2\sin(u-v),\qquad \frac{\partial^2 v}{\partial T^2}-c^2\frac{\partial^2 v}{\partial x^2}=\sin(u-v), \tag{26}$$

Applying the Differential Transform to Eq. (26) and Eq. (25), we obtain the following recursive formula

$$\frac{(k+2)!}{k!}U_{k+2}(x)=\frac{\partial^2 U_k(x)}{\partial x^2}-a^2N_k(x),\ \frac{(k+2)!}{k!}V_{k+2}(x)=c^2\frac{\partial^2 U_k(x)}{\partial x^2}+N_k(x) \tag{27}$$

Using the initial condition, we have

$$U_0(x)=A\cosh(kx),U_1(x)=0,V_0(x)=0,\ V_1(x)=0. \tag{28}$$

Now, substituting Eq. (28) into Eq. (27), and by straightforward iterative steps, yields

$$U_2(x)=-\frac{Ak^2\cosh(kx)}{2}-\frac{a^2\sin(A\cosh(kx))}{2},V_2(x)=\frac{\sin(A\cosh(kx))}{2},\ U_3(x)=0,V_3(x)=0,$$

$$U_4(x)=\frac{Ak^4\cosh(kx)}{24}+\frac{a^2A^2k^2\sin(A\cosh(kx))}{24}-\frac{a^2A^2k^2\sin(A\cosh(kx))\cos^2(kx)}{24}$$
$$+\frac{a^2Ak^2\cos(A\cosh(kx))\cos(kx)}{12}+\frac{a^4\cos(A\cosh(kx))\sin(A\cosh(kx))}{24}+\frac{a^2\cos(A\cosh(kx))\sin(A\cosh(kx))}{24},$$

$$V_4(x)=\frac{c^2A^2k^2\sin(A\cosh(kx))}{24}+\frac{c^2A^2k^2\sin(A\cosh(kx))\cosh^2(kx)}{24}-\frac{c^2Ak^2\cos(A\cosh(kx))\cosh(kx)}{24}$$
$$-\frac{Ak^2\cos(A\cosh(kx))\cosh(kx)}{24}-\frac{a^2\cos(A\cosh(kx))\sin\cosh(kx)}{24}-\frac{\cos(A\cosh(kx))\sin\cosh(kx)}{24},$$
$$\vdots$$

and so on.

The series solution is given by

$$u(x,T)=A\cosh(kx)-\left(\frac{Ak^2\cosh(kx)}{2}+\frac{a^2\sin(A\cosh(kx))}{2}\right)T^2+\left(\begin{array}{l}\frac{Ak^4\cosh(kx)}{24}+\frac{a^2A^2k^2\sin(A\cosh(kx))}{24}-\frac{a^2A^2k^2\sin(A\cosh(kx))\cos^2(kx)}{24}\\+\frac{a^2Ak^2\cos(A\cosh(kx))\cos(kx)}{12}+\frac{a^4\cos(A\cosh(kx))\sin(A\cosh(kx))}{24}\\+\frac{a^2\cos(A\cosh(kx))\sin(A\cosh(kx))}{24}\end{array}\right)T^4+\dots$$

$$v(x,T)=\frac{\sin(A\cosh(kx))}{2}T^2+\left(\begin{array}{l}\frac{c^2A^2k^2\sin(A\cosh(kx))}{24}+\frac{c^2A^2k^2\sin(A\cosh(kx))\cosh^2(kx)}{24}-\frac{c^2Ak^2\cos(A\cosh(kx))\cosh(kx)}{24}\\-\frac{Ak^2\cos(A\cosh(kx))\cosh(kx)}{24}-\frac{a^2\cos(A\cosh(kx))\sin\cosh(kx)}{24}-\frac{\cos(A\cosh(kx))\sin\cosh(kx)}{24}\end{array}\right)T^3+\dots$$

The inverse transformation will yield

$$u(x,T)=A\cosh(kx)-\left(\frac{Ak^2\cosh(kx)}{2}+\frac{a^2\sin(A\cosh(kx))}{2}\right)\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)}+\left(\begin{array}{l}\frac{Ak^4\cosh(kx)}{24}+\frac{a^2A^2k^2\sin(A\cosh(kx))}{24}-\frac{a^2A^2k^2\sin(A\cosh(kx))\cos^2(kx)}{24}\\+\frac{a^2Ak^2\cos(A\cosh(kx))\cos(kx)}{12}+\frac{a^4\cos(A\cosh(kx))\sin(A\cosh(kx))}{24}\\+\frac{a^2\cos(A\cosh(kx))\sin(A\cosh(kx))}{24}\end{array}\right)\frac{t^{4\alpha}}{\Gamma^4(\alpha+1)}+\dots$$

$$v(x,T)=\frac{\sin(A\cosh(kx))}{2}\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)}+\left(\begin{array}{l}\frac{c^2A^2k^2\sin(A\cosh(kx))}{24}+\frac{c^2A^2k^2\sin(A\cosh(kx))\cosh^2(kx)}{24}\\-\left(1+c^2\right)\frac{Ak^2\cos(A\cosh(kx))\cosh(kx)}{24}-\left(1+a^2\right)\frac{\cos(A\cosh(kx))\sin\cosh(kx)}{24}\end{array}\right)\frac{t^{4\alpha}}{\Gamma^4(\alpha+1)}+\dots$$

This solution is convergent to the Adomian's decomposition method solution [27,28].

6. CONCLUSION

In this research, we present new applications of the fractional complex transform method with coupling reduced differential transform method (RDTM) by handling three nonlinear physical fractional dynamical models. This coupling is an alternative approach to overcome the demerit of complex calculation of fractional differential equations. The proposed technique, which does not require linearization, discretization or perturbation, gives the solution in the form of convergent power series with elegantly computed components. All the examples show that the proposed combination is a powerful mathematical tool to solving other nonlinear equations.

References

[1] G. Adomian, A new approach to nonlinear partial differential equations, J. Math. Anal. Applic. 102 (1984) 420–434.
[2] S. Abbasbandy, Numerical solutions of nonlinear Klein-Gordon equation by variational iteration method. Internat. J. Numer. Math. Engrg. 70 (7) 2007 876–881.
[3] M.A. Noor, S.T. Mohyud-Din, Homotopy Perturbation Method for Solving Thomas-Fermi Equation Using Pade Approximants, International Journal of Nonlinear Science, Vol.8(1) (2009) 27-31.
[4] S.J. Liao, Beyond Perturbation: Introduction to the HomotopyAnalysis Method, Chapman & Hall/CRC Press, Boca Raton, 2003.
[5] J.L. Zhang, M.L. Wang, Y.M. Wang, Z.D. Fang, The improvedF-expansion method and its applications, Phys. Lett. A 350 (2006) 103–109.
[6] S. T. Mohyud-Din, M. A. Noor, K. I. Noor and M. M. Hosseini, Variational iteration method for re-formulated partial differential equations, Comm. Nonlin. Sci. Numer. Simul. 11 (2) (2010), 87-92.
[7] A.M. Wazwaz, A sine–cosine method for handling nonlinearwave equations, Math. Comput. Model. 40 (2004) 499–508.
[8] J.K. Zhou, Differential Transformation and its Application for Electrical Circuits, Huazhong University Press, Wuhan, China, 1986.
[9] J. Ahmad and S. T. Mohyud-Din, An Efficient Algorithm to Some Highly Nonlinear Fractional PDEs, European Journal of Scientific Research, Vol. 117(2) (2014) 242-251.
[10] Arikoglu and Ozkol I., Solution of fractional differential equations by using differential transform method, Chaos, Solitons& Fractals, 34 (5) (2007) 1473-1481.
[11] Kurnaz and G., (2005). The differential transforms approximation for the system of ordinary differential equations, International Journal of Computer Mathematics, 82 (6): 709-719.
[12] J. Ahmad and S. T.Mohyud-Din, An Efficient Algorithm for Some Highly Nonlinear Fractional PDEs in Mathematical Physics, PLoS ONE 9(12): (2014) e109127. doi:10.1371/journal.pone.0109127.
[13] A. Saravanan, N. Magesh, A comparison between the reduced differential transform method and the Adomian decomposition method for the Newell–Whitehead–Segel equation, J. Egypt. Math. Soc. 21 (3) (2013) 259–265.
[14] R. Abazari, M. Abazari, Numerical study of Burgers–Huxleyequations via reduced differential transforms method, Comput.Appl. Math. 32 (1) (2013) 1–17.
[15] B. bis, M. Bayram, Approximate solutions for some nonlinearevolutions equations by using the reduced differential transformmethod, Int. J. Appl. Math. Res. 1 (3) (2012) 288–302.
[16] R. Abazari, B. Soltanalizadeh, Reduced differential transformmethod and its application on Kawahara equations, Thai J. Math. 11 (1) (2013) 199–216.
[17] M.A. Abdou, A.A. Soliman, Numerical simulations ofnonlinear evolution equations in mathematical physics, Int. J. Nonlinear Sci. 12 (2) (2011) 131–139.
[18] M.A. Abdou, Approximate solutions of system of PDEEs arising in physics, Int. J. Nonlinear Sci. 12 (3) (2011) 305–312.
[19] P.K. Gupta, Approximate analytical solutions of fractionalBenney–Lin equation by reduced differential transform method and the homotopy perturbation method, Comput. Math. Applic. 61 (2011) 2829–2842.
[20] R. Abazari, M. Abazari, Numerical simulation of generalizedHirota–Satsuma coupled KdV equation by RDTM and comparison with DTM, Commun. Nonlinear Sci. Numer. Simul. 17 (2012) 619–629.
[21] Y. Ugurlu, D. Kaya, Exact and numerical solutions ofgeneralized Drinfeld–Sokolov equations, Phys. Lett. A 372 (2008) 2867–2873.
[22] G.A. Afrouzi, J. Vahidi, M. Saeidy, Numerical solutions ofgeneralized Drinfeld–Sokolov equations using the homotopyanalysis method, Int. J. Nonlinear Sci. 9 (2) (2010) 165–170.
[23] A. Mohebbi, Numerical solution of nonlinear Kaup–Kupershmit equation, KdV-KdV and Hirota–Satsuma systems, Int. J. Nonlinear Sci. Numer. Simul. 13 (7–8) (2012)479–486.
[24] K.A. Gepreel, S. Omran, S.K. Elagan, The traveling wavesolutions for some nonlinear PDEs in mathematical physics, Appl. Math. 2 (2011) 343–347.
[25] G.Jumarie, Modified Riemann-Liouville Derivative and Fractional Taylor series of Non-differentiable Functions Further Results, Comput. Math. Appl. 51 (9-10) (2006) 1367-1376.
[26] Z.B.Li, and J. H.He, Fractional Complex Transform for Fractional Differential Equations, Math. Comput. Appl. 15 (5)(2010)970-973.
[27] S. S. Ray, A numerical solution of the coupled sine-Gordon equation using the modified decomposition method, Applied Mathematics and Computation, 175 (2006) 1046-1054.
[28] A. Sadighi, D. D. Ganji, et al., Traveling Wave Solutions of the Sine-Gordon and the Coupled Sine-Gordon Equations Using the Homotopy-Perturbation Method, ScientiaIranica Transaction BMechanical Engineering, 16 (2009) 189-195.

ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering

http://annals.fih.upt.ro