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DESIGN OPTIMIZATION OF 3-PHASE DRY TYPE POWER TRANSFORMER BY POWELL'S METHOD USING CONJUGATE DIRECTIONAL SEARCH

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Abstract: Powell's method gives a computationally efficient algorithm for searching and finding out the optimal solution. It is a direct search method which can be applied to single variable or multiple variable problems. If the no of variables is more than one, Powell's conjugate direction method is made use of. In this paper, the method has been applied to find out the optimal design of a 3-phase 3-limbed dry type transformer. The cost of production has been taken as the objective function. The key design variables have been identified and arrayed in descending order of their vulnerability. The cost function has been computed by a special subroutine- no attempt has been taken to frame an objective function in terms of the design variables to avoid complexity. Single variable searches have been made in the conjugate directions defined by the key variables retaining the previously obtained best values. As the cost function is quadratic by nature, the optimal solution has been obtained in one pass. A case-study on application of Powell's method to transformer design has been made and the results obtained have been given at the end.

Keywords: Three-phase dry-type transformers, optimization, direct search, conjugate direction, objective function, constraints

1. INTRODUCTION

3-phase transformers are very widely used in industry for stepping up or stepping down the voltage. They are used in generating substations as step-up transformersand in receiving substations as step-down transformers. In the generating substations, the voltage is stepped up to economic transmission voltage [11, 9] (as high as 400/220/132 kV). The voltage is stepped down in two stages in the receiving substations- from high value of transmission voltage to primary distribution voltage (11/6.6/3.3 kV) and then to 400/230 V for secondary distribution. As the core type construction is more economic than the shell type in view of the relative costs of conductor and core materials, the core type construction is universally adopted. In view of rising tariff-rates, it is expedient to design the machine for low loss to achieve running economy. Therefore, good grades of cold-rolled steel stampings are to be used. Copper should be used as conductor for lower losses and for compact design. However for outdoor applications, where there is no space constraint, aluminium is used as conductor to reduce the cost of production.

The transformers may be oil-filled or dry type. Transformers of large ratings for outdoor applications must be oil-filled, and if required with forced cooling arrangements, to make the design technically feasible. But for smaller ratings, particularly to avoid oil-related hazards, dry type transformers are preferred [2, 12].

2. DRY TYPE vs. OIL-FILLED TYPE TRANSFORMER

The dry type transformers use air as cooling medium. Though a little costlier, they are very much preferred for indoor applications. They are free from fire and safety hazards and can be located near to the load- it saves in the cost of connecting cables and reduces the distribution losses. Their maintenance requirement is very low and the operation is trouble-free.

Oil is a much better coolant. Therefore oil-filled transformers can be designed for higher current density, thus giving an economic design. But they are subject to fire and safety hazards- reliable fire safety and extinction arrangements are to be provided for them. Moreover, they are to be specially located and they need civil construction for their installation. They need periodic sampling of oil and exhaustive maintenance [5].

Dry type transformers are designed up to a power rating of 3-5 MVA and a voltage rating of 6.6/11 kV. Efforts are being made for still higher ratings. The cost of production is on the higher side as it must be designed for a relatively lower flux-density and current density but the higher price is partially offset by the reduction in distribution losses and lower maintenance charges.

3. OPTIMAL DESIGN

Optimization itself is an old concept. It was known in principle from many centuries back in history but in the earlier regimes it could not be effectively applied due to absence of computational facilities. The advent of fast-acting digital computer with enormous memory has paved the path to optimal design of industrial equipment following one of the several paths suggested by the mathematicians [1].

Optimal design is choosing the best possible design from a set of feasible designs, generally in presence of equality and inequality constraints [7]. The first step in the process is to identify the design variables, particularly the key variables which significantly affect the cost. The second step is to frame the objective function or the cost function- its minimality or maximality has to be found out as the case may be. The third step is to note down the customer's specifications or the design constraints- the design must conform to the specifications without violating any constraint. The fourth step is to write down the algorithm or flow chart for the chosen optimization technique and then to write down the computer program. In the next step, the program is to be run for the specific cases and results obtained. Recourse has to be made to computer [13] for going through these steps.

There are several optimization techniques viz.

a) Bracketing method - (i) exhaustive search and (ii) bounding phase
b) Region elimination method - (i) interval halving, (ii) Fibonacci search and (iii) golden section search
c) Point estimation method - (i) successive quadratic approximation
d) Gradient search method - (i) Newton-Raphson, (ii) Repeated bisection (iii) Secant and (iv) Cubic search

The choice is to be made according to the specific problem [6, 3].

4. POWELL'S METHOD- SINGLE AND MULTIPLE VARIABLE SEARCH

Powell's method is a point estimation method- it is apattern search technique with quadratic convergence. Most of the functions can be approximated by a quadratic (or a 3-point parabola) without much loss of accuracy. Powell's method uses successive quadratic approximation which closely fits the original. Therefore, it is most efficient out of all direct search methods. The simplest is the single variable search. It starts with the choice of three initial points. A quadratic curve is fitted to these points. The minimum of this curve is used as a candidate point for the next iteration. The process is continued until convergence.

The algorithm for single variable search can be extended to multiple variables. This is Powell's conjugate direction method and has been claimed to be most successful one. It uses the history of the previous searches to find out new directions of search. Even non-quadratic functions can be solved by this method.

The process starts with finding out a no of linearly independent search directions. Then a series of unidirectional searches are made along each direction, starting every time from the previous best point. The optimal solution is obtained in one pass for quadratic functions. One or two more passes may be required for non-quadratic functions [3].

5. APPLICATION TO TRANSFORMER DESIGN

The cost function of a transformer generally depends on many design variables [9, 8]. So it is necessary to search for the optima in different directions corresponding to each variable. Application of Powell's method becomes a simple exercise by avoiding the formulation of an objective function in terms of the design variables. The design of a transformer goes through multiple steps. Non-linear functions and integer variables are involved, also graphs or data-files. Therefore, the expression for objective function becomes very cumbrous and untraceable. It becomes much easier to apply Powell's method to a cost function that is computed by a subroutine dedicated only for the design of the transformer.

At first, the design variables are identified and arrayed in descending order of their vulnerability. The constraints are also identified. Then, Powell's conjugate direction method is applied. A single variable search is started with the most sensitive variable. The best value obtained (without violating any constraint) is retained for the next search with the second sensitive variable. This practice is continued until single variable searching for all the variables is complete. This is one cycle of searching.

The hyper-surface generated by the variables is concave for a quadratic function. So the optimal solution is reached in one cycle [3]. One or two more cycles of searching has to be made for non-quadratic functions with the best possible values obtained so far. This practice has been followed in this work.

6. DESIGN CONSIDERATIONS- VARIABLES AND CONSTRAINTS

A keen competition is going on in the market for cheaper engineering products of good quality. This trend has reflected itself in the design procedure [4]. A 3-limbed core construction has been used as the shell construction gives rise to higher price. CRGOS has been chosen as core material and copper as conductor material to reduce the losses and to get a compact design. The transformer is of dry type and open. The objective function has been taken as the cost of production. Two key variables have been identified which affect the cost viz.

≡ e.m.f. constant K in $E_t = K\sqrt{S}$, where E_t = e.m.f./turn and S = KVA-rating of the transformer.

≡ window height : width ratio $R_w = H_w / W_w$ where H_w & W_w : height & width of the window.

The approximate value of K and R_w are known from designers' accumulated experience.

The flux-density in the core and the current density in the conductor have been fixed at maximum possible values, without violating constraints, to reduce the cost of production [11]. Other variables e.g. thickness of lamination, no of core-steps etc. have minor effects on the cost function- no search has been made in those directions.

7. ALGORITHM

The developed algorithm using Powell's method for searching along conjugate direction [3] is given below:

Search along e.m.f. constant K

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1  Choose initial point (for e.m.f. constant), step size, convergence constants: $x_1$, V, $\varepsilon_c$, $\varepsilon_k$
2  $K \leftarrow x_1$ : Go to transformer design subroutine
3  $x_2 \leftarrow x_1 + V$
4  $f_1 \leftarrow$ cost, obtained from subroutine.
5  $K \leftarrow x_2$ : Go to transformer design subroutine
6  $f_2 \leftarrow$ cost, obtained from subroutine.
7  If $f(x_1) > f(x_2)$ then $x_3 \leftarrow x_1 + 2V$ else $x_3 \leftarrow x_1 - V$
8  $K \leftarrow x_3$ : Go to transformer design subroutine
9  $f_3 \leftarrow$ cost, obtained from subroutine.
   $(x_1, f_1), (x_2, f_2), (x_3, f_3)$ are three points of the cost function
10 $f_{\min} = \min(f_1, f_2, f_3)$; $x_{\min}$ corresponds to $f_{\min}$
11 $a_0 \leftarrow f_1$
12 $a_1 \leftarrow (f_2 - f_1)/(x_2 - x_1)$
13 $a_2 \leftarrow \{(f_3 - f_1)/(x_3 - x_1) - a_1\}/(x_3 - x_2)$
14 $\bar{x} = (x_1 + x_2)/2 - a_1/(2a_2)$
15 $K \leftarrow \bar{x}$ : Go to transformer design subroutine
16 $f_4 \leftarrow$ cost, obtained from subroutine.
17 If $|f_{\min} - f_4| < \varepsilon_c$ and $|x_{\min} - x_4| < \varepsilon_k$ then go to step 20
18 Choose the best three out of these four points.
19 Re-label them according to: $x_1 < x_2 < x_3$ : go to step 4
20 Optimum: $f_{\min} \leftarrow \min(f_1, f_2, f_3, f_4)$ : $x_{\min}$ corresponds to $f_{\min}$
21 $K \leftarrow x_{\min}$
22 Choose initial point (for window height : width ratio), step size, convergence constants: $x_1$, V, $\varepsilon_c$, $\varepsilon_k$
23 $R_w \leftarrow x_1$ : Go to transformer design subroutine
24 $x_2 \leftarrow x_1 + V$
25 $f_1 \leftarrow$ cost, obtained from subroutine.
26 $R_w \leftarrow x_2$ : Go to transformer design subroutine
27 $f_2 \leftarrow$ cost, obtained from subroutine.
28 If $f(x_1) > f(x_2)$ then $x_3 \leftarrow x_1 + 2V$ else $x_3 \leftarrow x_1 - V$
29 $R_w \leftarrow x_3$ : Go to transformer design subroutine
30 $f_3 \leftarrow$ cost, obtained from subroutine.
   $(x_1, f_1), (x_2, f_2), (x_3, f_3)$ are three points of the cost function
31 $f_{\min} = \min(f_1, f_2, f_3)$; $x_{\min}$ corresponds to $f_{\min}$
32 $a_0 \leftarrow f_1$
33 $a_1 \leftarrow (f_2 - f_1)/(x_2 - x_1)$
34 $a_2 \leftarrow \{(f_3 - f_1)/(x_3 - x_1) - a_1\}/(x_3 - x_2)$
35 $\bar{x} = (x_1 + x_2)/2 - a_1/(2a_2)$
36 $R_w \leftarrow \bar{x}$ : Go to transformer design subroutine
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37  $f_4 \leftarrow$ cost, obtained from subroutine.
38  If $\left|f_{\min} - f_4\right| < \varepsilon_c$ and $\left|x_{\min} - x_4\right| < \varepsilon_k$ then go to step 41
39  choose the best three out of these four points.
40  Re-label them according to: $x_1 < x_2 < x_3$ : go to step 25
41  Optimum: $f_{\min} \leftarrow \min(f_1, f_2, f_3, f_4)$ : $x_{\min}$ corresponds to $f_{\min}$
42  $R_w \leftarrow x_{\min}$ : go to transformer print subroutine
43  Stop
44  End
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8. CASE-STUDY

The case-study has been made on design optimization of a 3-phase dry type power transformer of rating 10 kVA, 3300/433 V, 50 Hz. by using Powell's method.At first a single variable search has been made based on e.m.f. constant K . Then a second single variable search has been made based on window height : width ratio R_w . During this search, the value of K has been fixed at the best value obtained in the first search. No further iteration had to be made as the minimality was obtained in one pass.

a) Single variable search on e.m.f. constant K . Initially chosen value: $K = 0.6$; The step length has been chosen as 0.005. The iterations are shown in Table-1.

Table-1: Iterations with change in e.m.f. constant K

Count	x(i) = K	f(i), Rs.	Count	x(i) = K	f(i), Rs.	Count	x(i) = K	f(i), Rs.
1	0.6	23443.85	2	0.5642181	23203.75	3	0.4630825	22921.53
	0.605	23481.19		0.5692181	23229.86		0.4680825	22916.91
	0.595	23411.39		0.5592181	23178.90		0.4730824	22911.85
	0.5642181	23203.75		0.4630825	22921.53		0.4139886	23111.93

The problem converges to solution for K= 0.4730824. It has taken only 3 iterations to get the minimum cost of Rs. 22911.85. The convergence constant has been set at Rs. 10/-

b) Single variable search on window height/width ratio R_w is being conducted. The value of e.m.f. constant has been fixed at: K= 0.4730824, obtained from the earlier single variable search on K. The convergence constant has been set at Re. 1/-. The iterations are shown in Table-2.

Table-2 – Iterations with change in window height : width ratio

Count	x(i) =R_w	f(i), Rs.	Count	x(i) =R_w	f(i), Rs.
1	3.0	22911.85	2	3.40824	22886.77
	3.05	22906.49		3.45824	22886.28
	3.1	22901.83		3.50824	22886.29
	3.40824	22886.77		3.48304	22886.23

The problem converges to solution for window height: width ratio, R_w= 3.48304. The initially chosen value is 3.0. It has taken only 2 iterations. The minimum cost has been found out to be Rs. 22886.23 p. The difference is only Rs. 25.62 p. No improvement is found in the further pass- so the process is terminated. The design data-sheet for the optimized machine is given below:

- Rating: 10KVA, 3300/433 V, 50 Hz., Dy-connected, 3-phase, 3-limbed, dry-type transformer.
- Constraints: efficiency close to 96% ; voltage regulation below 3%; no load current below 2%
- EMF/turn= 1.496962 volts
- Primary/secondary current in A: 1.01 / 13.33
- No of primary/secondary turns: 2204 / 167
- No tapping have been used to avoid problems arising out of it.
- Maximum core flux-density chosen= 1.5 wb/m^2
- Core flux= 6.7431E-03 wb
- Net area of iron= 4.4954E-03 m^2
- Cold Rolled Steel (CRS) type core has been chosen. The stacking factorhas been taken as 0.92
- Gross area of iron= 4.8863E-03 m^2
- Diameter of the circumscribing circle= 8.5591E-02 m
- 3-stepped core has been used.
- The length of the core-stampings in mm: 77 /61 /34
- The current density has been chosen as 3 A/ m^2

- The window space factor has been taken as 0.3003
- The window area= 9.8867E-03 m^2
- The width, height of the window in mm: 53 /186
- The width, height of the yoke in mm: 77 /63
- The distance between core centers= 139 mm
- The total width, height in mm: 355 /312
- The length of mean turn of primary winding= 0.38611 m
- The length of mean turn of secondary winding= 0.31078 m
- The resistivity of copper at operating temperature has been taken as 0.022 ohm/ m^3
- The resistance of primary winding= 55.603Ω
- The resistance of secondary winding= 0.25690Ω
- The total copper loss = 307.21 Watts.
- Per cent copper loss= 3.0721
- Iron loss/Kg for the material at the chosen flux-density= 1.627 Watt
- Total iron loss= 70.898 Watts
- Percent iron loss= 0.70898
- Efficiency at full load and 0.9 lagging p.f. = 0.9597
- Leakage reactance referred to primary= 29.402Ω
- Per cent resistance/ leakage reactance: 3.0721/ 0.89996
- Voltage regulation at full load and 0.9 lagging p.f= 2.7649
- Magnetizing current w.r.t. primary = 1.5986E-02 A
- Per cent magnetizing current= 1.5826
- Percent iron loss current= 0.70898; Percent no-load current= 1.7341
- The volumein /weight of copper: 1.5516E-03 m^3/ 13.809 Kg
- The specific cost of copper, and total cost of copper: Rs. 600/-per Kg / Rs. 8285/-
- The volume/weight of iron: 6.1914E-03 m^3/43.575Kg
- The specific cost of iron, and total cost of iron: Rs. 160/- per Kg /Rs. 6972/-
- Total cost of copper and iron =Rs. 15257/-
- Considering other direct and indirect cost as 50%, the selling cost =Rs. 22886/-

9. CONCLUSION

Powell's method is a powerful direct search method for finding out the optimal solution. It can be applied to both univariate and multivariate problems. The method of conjugate direction is applied to multivariate problems. The method minimizes a quadratic function in finite no of steps. Any non-linear problem can be approximated as a quadratic near its minimum. Therefore, the conjugate direction method gives quick convergence.

Powell's method is an extension of the basic pattern search method. Though it is best-suited for quadratic functionsit can be extended to non-quadratic functions also. For non-quadratic functions the convergence may not be obtained in a single pass. A single variable search may be sufficient forsome simple design problems, but in most cases we have to conduct a multivariablesearch using the method of conjugate direction.

In this paper the design optimization of a 3-phase 3-limbed dry type transformer has been made using Powell's conjugate direction method. The cost of production has been taken as the objective function. Two key variables $K \,\&\, R_w$ which affect the cost function appreciably have been chosen as key variables. The variables have been initialized from designers' accumulated experience. Then conjugate directional search has been made to reach the minimal solution. The minimality has been obtained in a single pass as the function is quadratic. Compared to other methods, the minimal point has been reached in a much less no of iterations. The cost function for specified values of design variables has been evaluated by a specially constructed subroutine. The dimensions, performance variables and the cost elements have been given at the end.

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