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MODELING TRAJECTORY ON THE ROBOTIC SYSTEMS OF INTERPOLATION METHODS

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Abstract: This paper deals with the interconnection of virtual reality and robotics systems. Data glove belongs to the special peripheral equipment that provides visual, tactile, auditory and positional interactions. This contribution presents a mathematical model of the motion of the robot. Trajectory of robot are described by means of interpolation polynomials, namely the Lagrange polynomial, Newton polynomial and using spline-functions (splines). Degree of the interpolating polynomial (Lagrange, Newton) depends on the number of control points, where a high degree interpolating polynomial can cause the undesired changes in curvature. Splines are mathematical model curve, which simply represents the "best match" of any finite set of points. Thus this is sort of smooth, easy to express curve from mathematically from mathematical point of view. Splines are very important part of computer graphics. Prediction of trajectory allows for example simulate robot´s movements. Upon completing the operator´s movements, the trajectories can be saved as a program and download it directly to a robotic device. Operator through these means can accurately simulate the movement of the robot.
Keywords: virtual reality, robot, interpolation, polynomial

1. INTRODUCTION

In the present industry but also other sectors are using many technical achievements, whether in design or process simulation. One of these technologies is a virtual reality, which finds application in various industries. Virtual reality gets very quickly out of the field of science fiction movies and is more often seen on computer monitors of scientists, as well as in ordinary users. This collocation represents at the same time something imaginary - virtual and real - real. Virtual reality is typically presented as a computer simulation used for action games. The basis of virtual reality is an effort to more accurately display of spatial models and scenes, handling, creating real world, the particular part with all its regularities and rules of movement in three dimensional space all in real time, while the basic procedures are used in the field of computer graphics. These methods are compounded by the use of special peripherals that provide visual, tactile, auditory and site interaction (goggles , helmets with built-in screens and headphones, position sensors, data gloves or full suits, pointing devices, different simulation cab or motion devices etc.). Using these devices is potentiated by the impression of the real world (all the actions happening in real time as possible with instant response to user action, virtual world and the objects in it are placed the three-dimensional character and the user is allowed to enter the virtual world and move it along different paths, the virtual world is not static, virtual bodies move along the animation curves, integrate with users and with each other).

2. ROBOTICS SYSTEMS

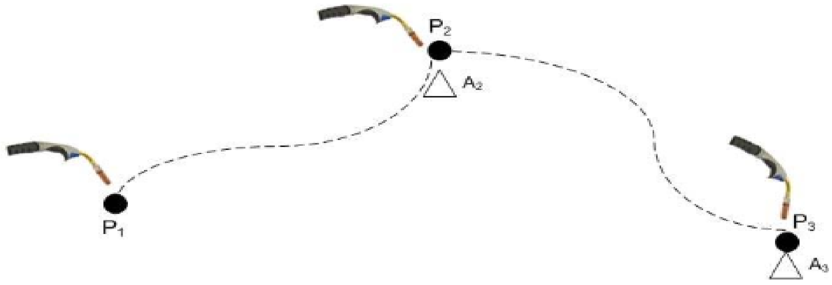
According to the nature of motion control are differentiated management of industrial robots:

- Point control (PTP - Point to Point) this type of management is used when it is necessary in the workspace of the robot to reach certain points, between whom there is no connection.
- Path control (CP - continuous path) this kind of control is used when we need to manage robot across its path.

Point control the sequence of passing discrete points in space. Figure 1 shows a possible sequence of movements P1-P2-P3 at various points with the example of a program formulated in natural language. The path of travel between the points is not defined, the axis move without a functional connection. Point control is used mainly for handling and spot welding.

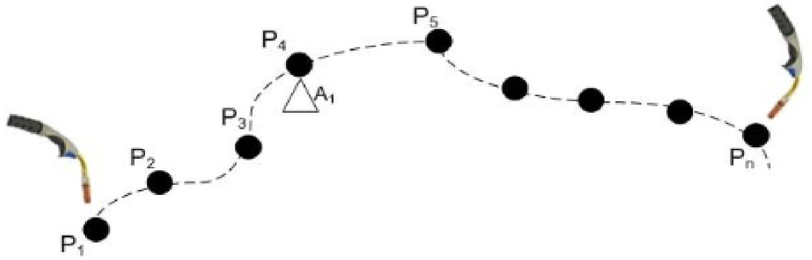
Management with path behavior allows programming and browsing defined motion paths with a functional relationship in movement in different axes. In this case there are two possibilities: multipoint and track management. *Multipoint control* (MP) including the running program as a thick sequence of points in space that you enter in a short time period (after 10 to 100 ms) of the positioner axes. Multipoint control are mainly used in paint spraying robots, spot welding, and surface treatment of polishing (Figure

2). *Path control* (CP) allows you to scroll through mathematically defined motion paths. It is used where it is necessary arc welding. Computer - interpolator determines when evaluating motion commands a number of intermediate values for certain desired path curve and enter them into the positioner (Figure 3).



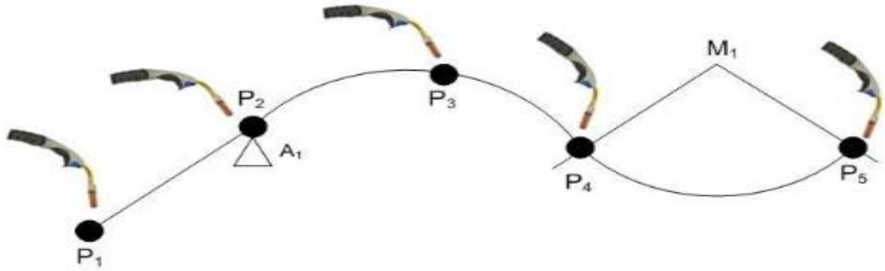
P_i - position the robot head; A_i - operation of the robot in its current position

Figure 1: Point control robot motion



P_i - position the robot head; A_i - operation of the robot in its current position

Figure 2: Multipoint control robot motion



P_i - position the robot head; A_i - operation of the robot in its current position

Figure 3: Path control robot motion

Interpolation is the process of defining the functions which, according to certain values passes given points. The robotic technology is used interpolation points, linear and circular. To create lifelike animation or simulation of human movement is difficult and tedious. One possibility is to describe the movement with creating of mathematical model. On the basis of the trajectory are known coordinates of individual points. These points are node points of the interpolating polynomial, which can be described as the shape of the trajectory. By interpolation construct a curve that goes through each coordinates - nodal points of an approximation to replace the function $f(x)$ - approximation function.

3. INTERPOLATING POLYNOMIAL

Let the function $f(x)$ is defined on the interval $\langle a,b \rangle$ and points $x_0, x_1, x_2, ..., x_n$ are arbitrary and different points of the interval $\langle a,b \rangle$ and $f(x_i)=y_i$ functional values are given, then there exists an interpolating polynomial

$$P_n(x)=a_0+a_1x+a_2x^2+....a_nx^n$$

most of the *n-th* degree, for which apply: $y_i=P_n(x_i)$ for each $i=0,1,2......n$.

Polynomial of the *n-th* degree is uniquely determined by $n+1$ various nodal points. e.g. polynomial second instance is uniquely determined by three different points.

Interpolation task is to find a continuous function $y=f(x)$ such that for every $i=0,1,2......n$ valid $f(x_i)=y_i$. It is the determination of polynomial functions specified by the coordinates of its nodes. Values $x_1, x_2, ..., x_n$ are interpolation nodes. Interpolating polynomial we can arrange a variety of methods, for example Lagrange and Newton polynomial interpolation polynomials are identical, different are only in way how they are constructed.

Lagrange interpolating polynomial

Lagrange interpolating polynomial is given $L_n(x)=\sum_{i=0}^{n} y_i l_i(x)$ where

$$l_i(x)=\frac{(x-x_0)\ldots(x-x_{i-1})(x-x_{i+1})\ldots(x-x_n)}{(x_i-x_0)\ldots(x_i-x_{i-1})(x_i-x_{i+1})\ldots(x_i-x_n)} \qquad l_i(x)=\prod_{\substack{j=0\\ j\neq i}}^{n}\frac{x-x_j}{x_i-x_j}.$$

If we replace the function $y=f(x)$ of the Lagrange interpolation polynomial of committing errors:

$$\left|f(x)-L_n(x)\right|\leq\frac{M_{n+1}}{(n+1)!}\left|(x-x_0)(x-x_1)\ldots(x-x_n)\right|,\ \text{where}\ M_{n+1}\geq\max_{0\leq x\leq x_n}\left|f^{(n+1)}(x)\right|$$

Newton interpolating polynomial

Newton interpolating polynomial is given

$$N_n(x)=f[x_0]+(x-x_0)f[x_0,x_1]++(x-x_0)(x-x_1)f[x_0,x_1,x_2]+\cdots$$
$$\ldots+(x-x_0)(x-x_1)\cdots(x-x_{n-1})f[x_0,x_1,\ldots x_n],$$

where $f[x_i]=y_i$ $i=0,1,\ldots n$ is the relative difference of zero-order node x_i,

$f[x_i,x_{i+1}]=\dfrac{f[x_{i+1}]-f[x_i]}{x_{i+1}-x_i}$ $i=0,1,\ldots n-1$ is the relative difference of first-order nodes x_i, x_{i+1},

$f[x_i,x_{i+1},.,x_{i+k}]=\dfrac{f[x_{i+1},x_{i+2}..x_{i+k}]-f[x_i,\ldots x_{i+k-1}]}{x_{i+k}-x_i}$ $i=0,1,\ldots n-k$ is the relative difference of *k*-order nodes $x_i,\ldots x_{i+k}$, for any $k=1,2......n$.

Example the five nodal points for Lagrange interpolation polynomial and Newton interpolating polynomial are in table 1.

Table1. Coordinates for the five nodal points

i	0	1	2	3	4
x_i	0	1	3	5	6
y_i	2	4	5	4	2

Lagrange interpolating polynomial:

$$\equiv\quad l_0(x)=\frac{1}{90}\left(x^4-15x^3+77x^2-153x+90\right)$$

$$\equiv\quad l_1(x)=-\frac{1}{40}\left(x^4-14x^3+66x^2-90x\right)$$

$$\equiv\quad l_2(x)=\frac{1}{36}\left(x^4-12x^3+41x^2-30x\right)$$

$$\equiv\quad l_3(x)=-\frac{1}{40}\left(x^4-10x^3+27x^2-18x\right)$$

$$\equiv\quad l_4(x)=\frac{1}{90}\left(x^4-9x^3+23x^2-15x\right)$$

$$L_4(x)=-\frac{1}{60}x^4+\frac{1}{5}x^3-\frac{13}{12}x^2+2{,}9x+2$$

Newton interpolating polynomial:

$$N_4(x)=2+2x-0{,}5x(x-1)+0{,}05x(x-1)(x-3)-0{,}0167x(x-1)(x-3)(x-5),$$

$$N_4(x)=-\frac{1}{60}x^4+\frac{1}{5}x^3-\frac{13}{12}x^2+2{,}9x+2$$

Table2. Newton interpolating polynomial

i	x_i	$f[x_i]$	$f[x_i,x_{i+1}]$	$f[x_i,x_{i+1},x_{i+2}]$	$f[x_i,x_{i+1},.,x_{i+3}]$	$f[x_i,x_{i+1},.,x_{i+4}]$
0	0	2	2	-0.5	0.05	-0.0167
1	1	4	0.5	-0.25	-0.05	
2	3	5	-0.5	-0.5		
3	5	4	-2			
4	6	2				

The resulting interpolating polynomial does not depend on the method by which was can find. It is clearly designated by points through which passes its graph.

4. CONCLUSION

Analysis of trajectory robots enable us to choose from all possible trajectories, which are based on the mathematical model and subsequent graphical display, optimal and most appropriate for the particular case of manual assembly. In the case that the position is not satisfactory, it is necessary to make changes and identify new feature trajectory data gloves. After implementing the changes, it is possible to re-do the analysis and determine whether there has been improvement. Thus, it is possible to interactively proceed until it finds such a position and the trajectory of the hand, which is satisfactory from an ergonomic point of view.

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