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MATHEMATICAL MODEL OF DYNAMIC VIBRATION ABSORBER-RESPONSE PREDICTION AND REDUCTION

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ABSTACT: This paper is dealing with procedures for mathematical modeling of DVA. Usually the problems of optimization are converted to equivalent single degree of freedom (SDOF) structure at particular mode in order to optimize the damper. There are three main parameters in a DVA system: DVA mass, DVA stiffness coefficient and DVA damping ratio. Consequently, the objective is to find the optimum value of these parameters. This paper is investigating also the effect of relative speed of primary structure and its influence on response.
Keywords: structural dynamics, DVA, relative speed, response, state-space model

1. INTRODUCTION

In engineering applications, many systems can be modeled as single degree-of-freedom systems [1]. For example, a machine mounted on a structure can be modeled using a mass-spring-damper system, in which the machine is considered to be rigid with mass m and the supporting structure is equivalent to a spring k and a damper c, as shown in Figure 1. The machine is subjected to a sinusoidal force $F_0 \sin\Omega t$, which can be an externally applied load or due to imbalance in the machine.

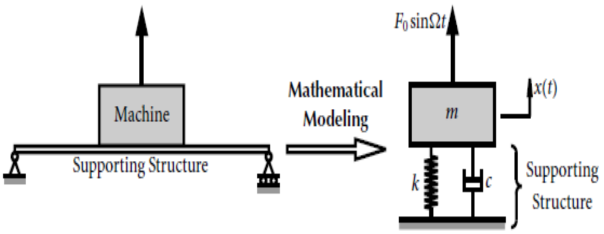

Figure 1. A machine mounted on a structure

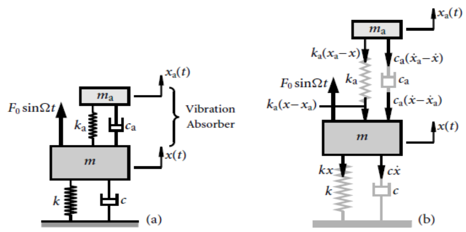

Figure 2. Vibration absorber installed

It is well known that when the excitation frequency Ω is close to the natural frequency of the system $\omega_0 = \sqrt{k/m}$, vibration of large amplitude occurs. In particular, when the system is undamped, i.e., $c = 0$, resonance occurs when $\Omega = \omega_0$, in which the amplitude of the response grows linearly with time.

To reduce the vibration of the system, a vibration absorber (DVA) or a tuned mass damper (TMD), which is an auxiliary mass-spring-damper system, is mounted on the main system [2,3] as shown in Figure 2. The mass, spring stiffness, and damping coefficient of the viscous damper are m_a, k_a and c_a, respectively, where the subscript "a" stands for "auxiliary".

2. EQUATION OF MOTION AND DMF

To derive the equation of motion of the main mass m, consider its free-body diagram as shown in Figure 2(b). Since mass m moves upward, spring k is extended and spring k_a is compressed. Considering Figure 2(b) and Newton's Second Law we get the following equitation's:

$$m\ddot{x} = \sum F: m\ddot{x} = -kx - c\dot{x} - k_a(x - x_a) - c_a(\dot{x} - \dot{x}_a) + F_0 \sin\Omega t \qquad 1$$

or

$$m\ddot{x} + (c + c_a)\dot{x} + (k + k_a)x - c_a\dot{x}_a - k_a x_a = F_0 \sin\Omega t \qquad 2$$

Similarly, consider the free-body diagram of mass m_a. Since mass m_a moves upward a distance $x_a(t)$, spring k_a is extended. The net extension of spring k_a is $x_a - x$. Hence, the spring k_a and damper c_a exert downward forces $k_a(x_a - x)$ and $c_a(\dot{x}_a - \dot{x})$, respectively. Applying Newton's Second Law gives:

$$m_a\ddot{x}_a = \sum F: m_a\ddot{x}_a = -k_a(x_a - x) - c_a(\dot{x}_a - \dot{x}) \qquad 3$$

or

$$m_a\ddot{x}_a + c_a\dot{x}_a + k_a x_a - c_a\dot{x} - k_a x = 0 \qquad 4$$

The Dynamic Magnification Factor (DMF) for mass m is equal to:

$$DMF = \frac{|x_P(t)|_{max}}{x_{static}} \qquad 5$$

Adopting the following notations:

$$\omega_0^2 = \frac{k}{m}, c = 2m\zeta\omega_0, r = \frac{\Omega}{\omega_0}, \mu = \frac{m_a}{m}, \omega_a^2 = \frac{k_a}{m_a}, c_a = 2m_a\zeta_a\omega_0, r_a = \frac{\omega_a}{\omega_0} \qquad 6$$

The Dynamic Magnification Factor becomes:

$$DMF = \sqrt{\left\{\frac{({r_a}^2 - r^2)^2 + (2\zeta_a r)^2}{[(1 - r^2)(r_a^2 - r^2) - \mu r_a^2 r^2 - 4\zeta_a\zeta\, r^2]^2 + 4r^2[\zeta_a(1 - r^2 - \mu r^2) + \zeta\,(r_a^2 - r^2)]^2}\right\}} \qquad 7$$

For the special case when $\mu = 0$, $r_a = 0$, $\zeta_a = 0$, the Dynamic Magnification Factor reduces to:

$$DMF = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \qquad 8$$

which recovers the DMF of a single degree-of-freedom system, i.e., the main system without the auxiliary vibration absorber or TMD.

The Dynamic Magnification Factors for an undamped main system, i.e., $\zeta = 0$,are shown in Figure 3. Without the vibration absorber or TMD, the single degree-of-freedom system is in resonance when $r = 1$ or $\Omega = \omega_0$, where the amplitude of the response grows linearly with time or DMF approaches infinite.

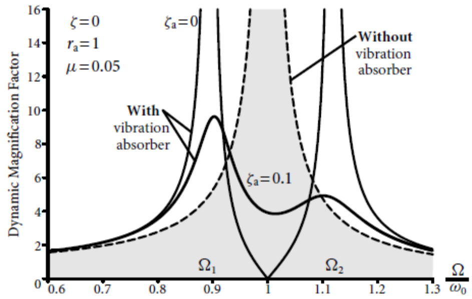

Figure 3. DMF for $\zeta = 0$

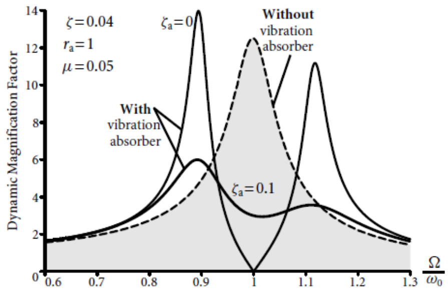

Figure 4. DMF for $\zeta = 0.04$

In order to reduce the vibration of the main system at resonance, a vibration absorber or TMD is attached to the main mass m. The vibration absorber is usually tuned [4] so that $\omega_a = \omega_0$ or $r_a = 1$, hence the name tuned mass damper. In practice, the mass of the vibration absorber or TMD is normally much smaller than that of the main mass, i.e., $m_a \ll m$ or $\mu \ll 1$; in Figure 3 and 4, μ is taken as 1/20=0.05.

If the vibration absorber or TMD is undamped, i.e., $\zeta_a = 0$, then DMF=0 when $\Omega = \omega_0$, meaning that the vibration absorber eliminates vibration of the main mass m at the resonant frequency $\Omega = \omega_0$. However, it is seen that the vibration absorber or TMD introduces two resonant frequencies Ω_1 and Ω_2, at which the amplitude of vibration of the main mass m is infinite. In practice, the excitation frequency Ω must be kept away from the frequencies Ω_1 and Ω_2.

In order not to introduce extra resonant frequencies, vibration absorbers or TMD are usually damped [4]. A typical result of DMF is shown in Figure 3 for $\zeta_a = 0.1$. It is seen that the vibration of the main mass m is effectively suppressed for all excitation frequencies. By varying the value of ζ_a, an optimal vibration absorber can be designed.

When the main system is also damped, typical results of DMF are shown in Figure 4. Similar conclusions can also be drawn.

3. MATHEMATICAL MODEL OF DYNAMIC VIBRATION ABSORBER WITH MATLAB & SIMULINK

Governing equitation's of motion for the system on Figure 2 can be written as:

$$m\ddot{x} + (c + c_a)\dot{x} + (k + k_a)x - c_a\dot{x}_a - k_a x_a = F_0 \sin\Omega t \quad 9$$

$$m_a\ddot{x}_a + c_a\dot{x}_a + k_a x_a - c_a\dot{x} - k_a x = 0 \quad 10$$

or in matrix for:

$$\begin{bmatrix} m & 0 \\ 0 & m_a \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{x}_a \end{bmatrix} + \begin{bmatrix} (c + c_a) & -c_a \\ -c_a & c_a \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{x}_a \end{bmatrix} + \begin{bmatrix} (k + k_a) & -k_a \\ -k_a & k_a \end{bmatrix}\begin{bmatrix} x \\ x_a \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} F_0 \sin\Omega t \quad 11$$

Hence

$$M\ddot{X} + C\dot{X} + KX = I \cdot F_0 \sin\Omega t \quad 12$$

In the following step equitation of motion is re-written in State Space system [5]:

$$\ddot{X} = [-M^{-1}K]X + [-M^{-1}C]\dot{X} + [-M^{-1}I] \cdot F_0 \sin\Omega t \quad 13$$

$$\dot{X} = [0]X + [1]\dot{X} + [0] \cdot F_0 \sin\Omega t \quad 14$$

or in matrix form:

$$\begin{Bmatrix} \dot{X} \\ \ddot{X} \end{Bmatrix} = \begin{bmatrix} 0 & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}\begin{Bmatrix} X \\ \dot{X} \end{Bmatrix} + \begin{bmatrix} 0 \\ -M^{-1}I \end{bmatrix} \cdot F_0 \sin\Omega t \quad 15$$

Hence

$$\dot{X} = A \cdot X + B \cdot u \quad 16$$

Second equitation of the system is:

$$X = C \cdot X + D \cdot u \quad 17$$

where

$$C = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \quad 18$$

Above equitation's conclude the final State-Space form:

$$\dot{X} = A \cdot X + B \cdot u \quad 19$$

$$X = C \cdot X + D \cdot u \quad 20$$

which is one way for solving the system of differential equitation's by using the command State-Space system in Matlab Simulink.

Figure 5 shows the structural scheme in Simulink. The second equitation with coefficients C and D is called equitation of output values.

On the left side of Figure5 are given input values as column vectors (time of sampling and external force $F_0\sin\Omega t$). The initial conditions are also listed here as x and x_a at t=0, as well as $\dot{x}$ and $\dot{x}_a$ at $t = 0$. For the studied case it is adopted initial values to be equal to zero.

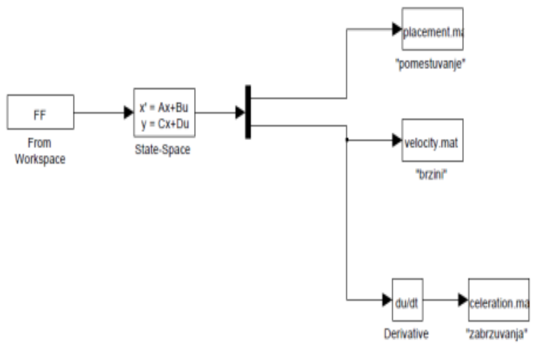

Figure 5. State-Space model in Matlab Simulink for solving system of ODE's

Another way of modeling the system in Matlab Simulink is with block diagrams using function ODE45 for solving differential equitation's. For this purpose we need to re-write the governing equitation's:

$$m\ddot{x} + (c + c_a)\dot{x} + (k + k_a)x - c_a\dot{x}_a - k_a x_a = F_0 \sin\Omega t \quad 9$$

$$m_a\ddot{x}_a + c_a\dot{x}_a + k_a x_a - c_a\dot{x} - k_a x = 0 \quad 10$$

in the following order:

$$\ddot{x} = \frac{F_0}{m}\sin\Omega t + \frac{c_a}{m}\dot{x}_a + \frac{k_a}{m}x_a - \frac{(c + c_a)}{m}\dot{x} - \frac{(k + k_a)}{m}x \quad 21$$

$$\ddot{x} = A\sin\Omega t + B\,\dot{x}_a + C\,x_a - D\,\dot{x} - E\,x \quad 22$$

and

$$\ddot{x}_a = \frac{c_a}{m_a}\dot{x} + \frac{k_a}{m_a}x - \frac{c_a}{m_a}\dot{x}_a - \frac{k_a}{m_a}x_a \quad 23$$

$$\ddot{x}_a = K\,\dot{x} + L\,x - M\,\dot{x}_a - N\,x_a \quad 24$$

if adopted variables:

$$\omega_0^2 = \frac{k}{m}, c = 2m\zeta\omega_0, r = \frac{\Omega}{\omega_0}, \mu = \frac{m_a}{m}, \omega_a^2 = \frac{k_a}{m_a}, c_a = 2m_a\zeta_a\omega_0, r_a = \frac{\omega_a}{\omega_0} \quad 25$$

the coefficients A, B, C, D, E, K, L, M, N become:

$$B = 2\mu\zeta_a\omega_0, C = \mu\omega_a^2, D = 2\omega_0(\zeta + \mu\zeta_a), E = \omega_0^2 + \mu\omega_a^2, K = M = 2\zeta_a\omega_0, L = N = \omega_a^2 \quad 26$$

Following equitation's:

$$\ddot{x} = A\sin\Omega t + B\,\dot{x}_a + C\,x_a - D\,\dot{x} - E\,x \quad 27$$

$$\ddot{x}_a = K\,\dot{x} + L\,x - M\,\dot{x}_a - N\,x_a \quad 28$$

we make the block scheme in Matlab Simulink shown on Figure 6.

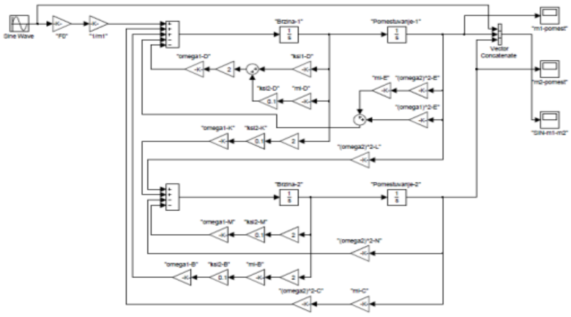

Figure 6. Block representation in Matlab Simulink for solving ODE's

Table 1. Selected values for main parameters

m [kg]	200	ζ_a	0.1
μ	0.05	c[N·(s/m)]	502.4
m_a[kg]	10	k[N/m]	197192
f[Hz]	5	c_a [N·(s/m)]	62.8
ω_0[rad/s]	31.4	k_a [N/m]	9859.6
ω_a [rad/s]	31.4	F_0[N]	5000
ζ	0.04	Ω [rad/s]	31

In this type of representation, initial conditions are given by clicking on each block in the scheme. The solutions obtained with block scheme representation and State-Space system must be identical.

The analysis of the system on Figure 2, 5 and 6 is carried out with the values of the main parameters given in Table 1.

With this input parameters we can calculate the governing equitation's of motion for 10 sec. and sampling time of 0.01 s. The results of the response are shown on Figure 7 and 8.

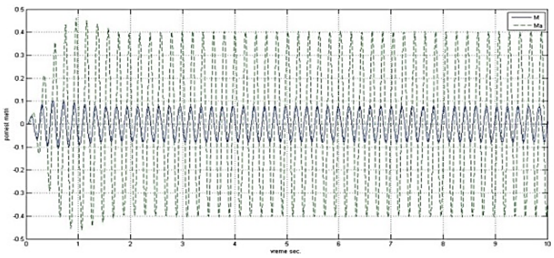

Figure 7. Displacement of main mass with TMD

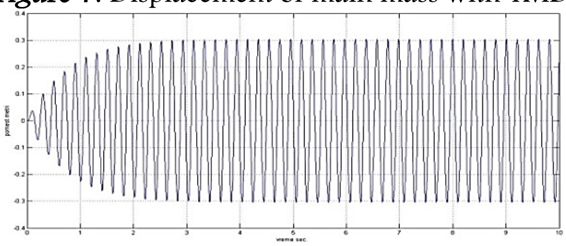

Figure 8. Displacement of main mass without TMD

Table 2. Comparison of amplitudes

Max. displace [m] without TMD for t=5s	Max. displace [m] with TMD for t=5s
0.3038	0.07892

If we calculate the maximal value of the displacement at same time of the main (primary) mass for case with and without TMD it can be seen that displacements are larger in the case without the auxiliary mass. Comparison of amplitudes from Figure 7 and 8 is given in table 2. It shows that TMD decreased the vibrations for 74% of the value without TMD.

Another important detail for review is the displacement of the auxiliary mass which can be seen on Figure 7. It is clear that the displacement is significantly larger than the response of the primary mass. In our case, the amplitude of the auxiliary mass is 0.4 compared with 0.07892 of the main mass. This is one of the problems with application of TMD. In order to do its function, it is necessary to be provided with large space for the auxiliary mass so it can oscillate without any obstacles. Considering that these devices are installed on top of the building roofs, this space is usually limited.

Figure 9 represents the influence of the ratio μ (auxiliary mass/main mass). It is clear that increasing the auxiliary mass m_a is widening the area of impaired oscillations, but as it is mentioned before, this is limited with the ration of less than 15%. Usually large masses lead to big unpractical structures.

Figure 10 illustrates the weakening effect caused by shifting frequency ratio $r_a = \omega_a/\omega_0 \neq 1$ over size of the area of impaired oscillations. Figure 11 shows the DMF as function of the normalized frequency r and the damping ration of the auxiliary mass ζ_a. It is clear that with increasing the damping two major maximums intend to joint in one, which has much lower value. It is evident from the diagram that TMD has optimal value [6].

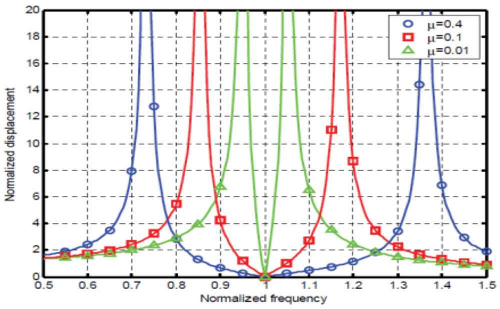

Figure 9. Influence of the mass ratio μ

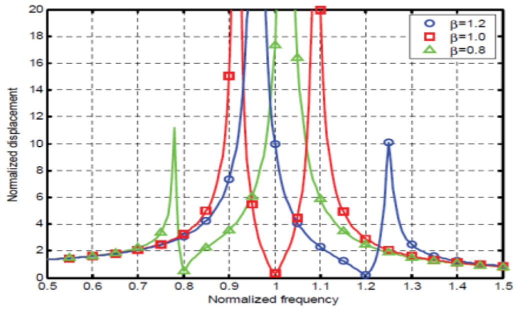

Figure 10. Influence of the ratio $r_a = \omega_a/\omega_0$

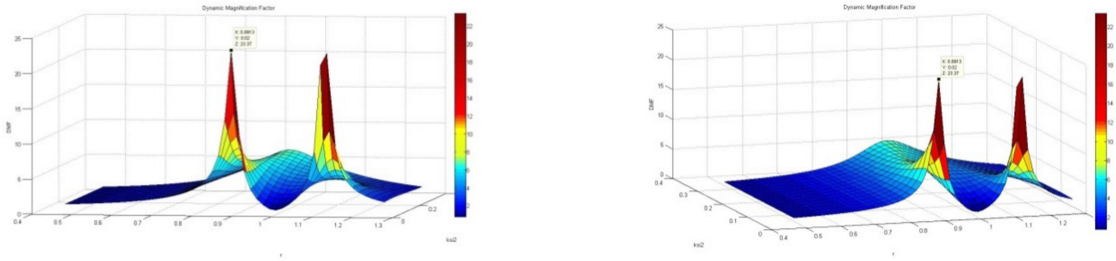

Figure 11. DMF as function of normalized freq. r and the damping ration ζ_a

4. MODELING FORCE AS FUNCTION OF RELATIVE VELOCITY

Wind force is dependent on wind velocity and also type of airflow. This is described with the equitation [7,8]:

$$F = C_F \cdot A \cdot q \qquad 29$$

where C_F is coefficient of shape, A is building surface and q is wind pressure.
Wind pressure is equal to:

$$q = \frac{1}{2} \cdot \rho \cdot V^2 \qquad 30$$

where ρ is air density and V is wind velocity.
The C_F is dimensionless number which depends on Reynolds number, and for normal wind velocities can be considered as constant. Therefore the equitation for wind force can be written as:

$$F = C_F \cdot A \cdot q = C_F \cdot A \cdot \frac{1}{2} \cdot \rho \cdot V^2 = C_W \cdot V^2 \qquad 31$$

where C_W is wind constant.
From the analysis in previous section Figure7 and 8, we can make following statement:

$$F = \mathrm{sgn}(V) \cdot C_W \cdot V^2 = F_0 \sin\Omega t \qquad 32$$

Wind force is always in the direction of wind velocity and therefore:

$$\mathrm{sgn}(V) = \mathrm{sgn}(\sin\Omega t) \qquad 33$$

Hence, wind velocity and force for analyzed model can be calculated as:

$$V = \mathrm{sgn}(\sin\Omega t) \cdot \sqrt{\frac{F_0}{C_W} \cdot |\sin\Omega t|} \qquad 34$$

$$F = \mathrm{sgn}(\sin\Omega t) \cdot C_W \cdot \left(\sqrt{\frac{F_0}{C_W} \cdot |\sin\Omega t|}\right)^2 \qquad 35$$

Next analysis calculates differential equitation's (ODE's) with external wind force that will depend on the relative speed between wind and structure velocity [9, 10]:

$$F = \mathrm{sgn}(V - \dot{x}) \cdot C_W \cdot (V - \dot{x})^2 \qquad 36$$

the differential equitation's will take shape:

$$\ddot{x} = \frac{A}{F_0}\,\mathrm{sgn}(V - \dot{x}) \cdot C_W \cdot (V - \dot{x})^2 + B\,\dot{x}_a + C\,x_a - D\,\dot{x} - E\,x \qquad 37$$

$$\ddot{x}_a = K\,\dot{x} + L\,x - M\,\dot{x}_a - N\,x_a \qquad 38$$

For easier calculation we will adopt$C_W = 1$. Next, we lower the order of ODE's:

$$x_1 = x,\ x_2 = \dot{x},\ x_3 = x_a,\ x_4 = \dot{x}_a \qquad 39$$

and $A/F_0 = A'$ and we have:

$$\dot{x}_1 = x_2 \qquad 40$$

$$\dot{x}_2 = A'\,\mathrm{sgn}(V - x_2) \cdot (V - x_2)^2 + B\,x_4 + C\,x_3 - D\,x_2 - E\,x_1 \qquad 41$$

$$\dot{x}_3 = x_4 \quad 42$$

$$\dot{x}_4 = K\,x_2 + L\,x_1 - M\,x_4 - N\,x_3 \quad 43$$

Using ODE45 (an explicit Runge-Kutta method) in Matlab for solving non-stiff differential equations, we obtain solution for the system above. The results are presented graphically with Figure 12.

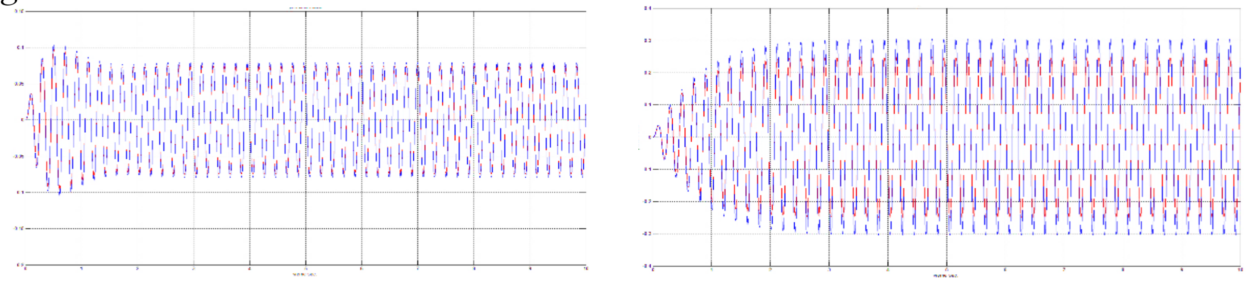

Figure 12. Comparison between calculation with and without relative speed included

The results show general trend for smaller amplitudes of displacement of the primary and secondary mass when relative speed is included (Table 3).

Table 3. Comparison of amplitudes

Amplitude of primary mass for t=5s [m]	
without TMD (relative speed included)	without TMD (relative speed not included)
0.25	0.3038
with TMD (relative speed included)	with TMD (relative speed not included)
0.07467	0.07929

5. CONCLUSION

This paper is analyzing the mathematical approach for modeling dynamic vibration absorber with s.d.o.f. and m.d.o.f. mass. The paper presents the technique for modeling the differential equitation's with state-space form and block diagrams using Matlab Simulink.

Also, it shows how the relative speed affects the primary mass response under external excitation. It can be concluded that when the primary mass vibrates with frequency close to the natural frequency and it is without TMD the relative speed is causing smaller amplitudes of oscillations.

If TMD is installed, the relative speed is not affecting the response of the primary mass and the difference between the amplitudes of oscillations is insignificant.

If relative speed is included, the response of primary mass is smaller than the case without it.

Note

This paper is based on the paper presented at The 12th International Conference on Accomplishments in Electrical and Mechanical Engineering and Information Technology – DEMI 2015, organized by the University of Banja Luka, Faculty of Mechanical Engineering and Faculty of Electrical Engineering, in Banja Luka, BOSNIA & HERZEGOVINA (29th – 30th of May, 2015), referred here as[11].

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