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THE MATHEMATICAL MODEL DESIGN FOR A MAAG CUTTING SIMULATION

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ABSTACT: Nowadays the gear presents of the most widespread part of the machines and equipment. There is needed to solve the machining process for a complicated involute profile of tooth face, because precision of the profile influences on the working of the parts. The paper deals with a mathematical model design for the Maag cutting of a gear profile, which is one of control mechanisms of the machining process. The mathematical model can be used for simulation of Maag cutting and analysis by the position of the gear rack profile towards involute gear profile. There is possible to determine final inaccuracy of gear rack machining which is caused by kinematical movement between work-piece and production tool. The results of the simulation in the reverse realization determine accurate profile of gear rack.
Keywords: Simulation, gear rack, machining, mathematical model

1. INTRODUCTION

The Maag gear cutting machine normally operates on the generating principle, the workpiece performing the traverse generating motion and the tool, the cutting motion. The generating motion is the resultant of combined translation and rotation. The tool used is a rack type cutter. The gear cutting is affected by rolling the workpiece clamped with vertical axis of the work table across the rack type of cutter from its starting position lateral to the tool. The involute shape of the tooth flanks is generated by the enveloping planes swept by the straight tool edge. [1]

Cutting usually takes place during the downward motion of the cutter while the generating mechanism is stand still. For the upward motion, the cutter is withdraw a radially clear of the workpiece. The traverse rolling generating motion serves as the feed motion, occurring intermittently after each working stroke, i.e. during the upward motion of the tool.

Since the cutter has generally fewer teeth than the gear to be cut, the gear blank gas to be returned to its original position after reaching the end of the cutter, and the generating motion resumed. At the end of the generating path, the cutter is therefore stopped above the gear blank and the work table is returned to its initial position. This return motion is combined with indexing, in that the rotary motion of the work table is interrupted for a distance equivalent to the number of pitches

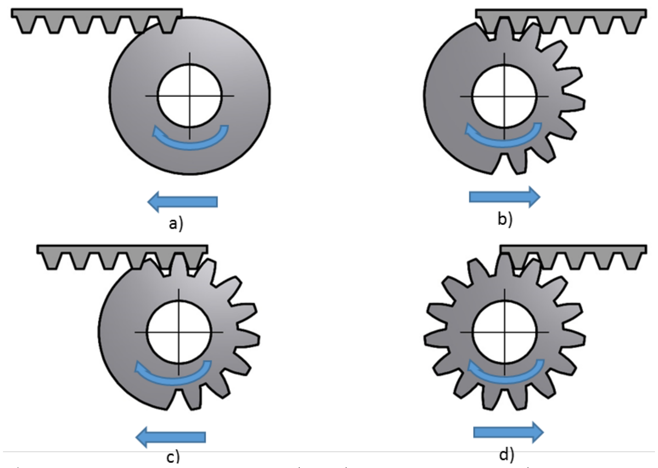

Figure 1. Generating and indexing motion during gear cutting on Maag machines

cut during the preceding generating pass. The generating and return motions are repeated until all the teeth in the gear are cut (Figure 1.). For the finishing cut, the generating path is limited to one pitch so that each tooth of the gear is cut under identical conditions. One feature of the Maag gear cutting method is that the finishing tool does not normally cut the root of the tooth space but only the tooth flanks. The root is previously finished in the roughing operation. [1]

2. MATERIALS AND METHODS

2.1 Methodology of mathematical simulation model

Relative motion of rack tool and gear wheel is created by combination of rack tool translation and gear wheel rotation. The we can establish that the trajectory size s_h of rack tool coordinate system of translation in the stationary coordinate system is depended on an angle of rotation of gear wheel coordinate system.

$$s_h = \frac{2\pi r\varphi}{360} \tag{1}$$

For the determination of transformation of indifferent point on rack tool profile there is needed to investigate relative motion in the three coordinate systems (Figure 2):

» Rack tool coordinate system O_h (x_h, y_h, z_h).
» Gear wheel coordinate system O_k (x_k, y_k, z_k).
» Stationary coordinate system O_p (x_p, y_p, z_p).

The motion of rack tool in the stationary coordinate system O_p (x_p, y_p, z_p) we can describe like motion rack tool geometry which is depended on angle of rotation φ based on equation ((1)). The parametrical translation equation in stationary coordinate system will be:

$$x_p = \frac{2\pi r\varphi}{360} + x_h, \quad y_p = r + y_h. \tag{1}$$

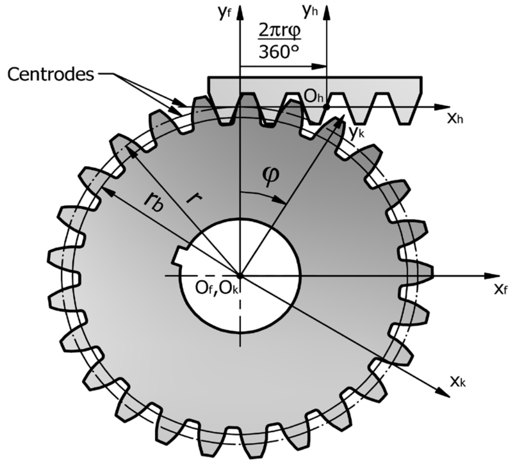

Figure 2. Description of relative motion of rack tool and gear wheel

The next step which is needed for the final equation is transformation of stationary coordinate system with parametrical equations to the gear wheel rotary coordinate system. Coordinate transformation is based on theory, which was described in Gear geometry and Applied theory (LITVIN L.F. – FUENTES. A. 2004).

Consider two coordinate system $S_p(x_p, y_p, z_p)$ a $S_k(x_k, y_k, z_k)$ (Figure 3). Point *A* is represented in coordinate system S_p by the position vector

$$r_p = \begin{bmatrix} x_p & y_p & z_p & 1 \end{bmatrix}^T \tag{3}$$

The same point *M* can be determined in coordinate system S_k by the position vector

$$r_k = \begin{bmatrix} x_k & y_k & z_k & 1 \end{bmatrix}^T \tag{4}$$

with the matrix equation

$$r_k = M_{kp} r_p \tag{5}$$

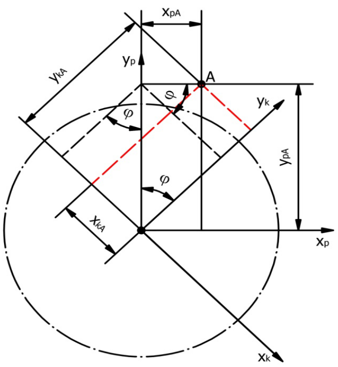

Figure 3. Coordinate transformation

Matrix $\mathbf{M}_{kp}$ is represented by

$$M_{kp} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} \cos(\widehat{x_k, x_p}) & \cos(\widehat{x_k, y_p}) & \cos(\widehat{x_k, z_p}) & x_{kA} \\ \cos(\widehat{y_k, x_p}) & \cos(\widehat{y_k, y_p}) & \cos(\widehat{y_k, z_p}) & y_{kA} \\ \cos(\widehat{z_k, x_p}) & \cos(\widehat{z_k, y_p}) & \cos(\widehat{z_k, z_p}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The determination of elements a_{lk} (k = 1,2,3; l= 1, 2, 3) of matrix M_{kp} is based on following rules:

a) Elements of the 3 x 3 submatrix

$$L_{kp} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{7}$$

represent the direction cosine of the "old" unit vectors in the "new" coordinate systems S_k. For instance, $a_{21} = \cos(\widehat{y_k, x_p})$, $a_{32} = \cos(\widehat{z_k, y_p})$, and so on. The subscripts of elements a_{kl} in matrix L_{kp}, indicate the number l of the "old" coordinate axis and the number k of the "new" coordinate axis. Axes x, y, z are given numbers 1, 2, and 3 respectively.

b) Elements a_{14}, a_{24}, and a_{34} represent the "new" coordinate' sx_{kA}, y_{kA}, z_{kA}, of the "old" origin O_h.

To determine the new coordinates (x_k, y_k, z_k, 1) of point A, we have to use the rule of multiplication of a square matrix (4 x 4) and a column matrix (4 x 1). The number of rows in the column matrix is equal to the number of columns in matrix M_{kp}.

$$\begin{aligned} x_k &= a_{11}x_p + a_{12}y_p + a_{13}z_p + a_{14} \\ y_k &= a_{21}x_p + a_{22}y_p + a_{23}z_p + a_{24} \\ z_k &= a_{31}x_p + a_{32}y_p + a_{33}z_p + a_{34} \end{aligned} \tag{8}$$

The final parametrical equations of rack profile translation motion in the stationary coordinate system are the next:

$$x_p = \frac{2\pi r\varphi}{360} + x_h, \qquad y_p = r + y_h. \tag{9}$$

Based on previous analysis we can determine final parametrical equations for rack tool translation motion in the gear wheel coordinate system as follow:

$$\begin{aligned} x_k &= x_h\cos\varphi - y_h\sin\varphi + r\left(\frac{2\pi\varphi}{360}\cos\varphi - \sin\varphi\right), \\ y_k &= x_h\sin\varphi + y_h\cos\varphi + r\left(\frac{2\pi\varphi}{360}\sin\varphi + \cos\varphi\right), \\ z_k &= 0. \end{aligned} \tag{10}$$

2.2 Determination of rack tool profile

Coordinates x_h, y_h, z_h in the equation (10) represent point of rack tool profile. This profile is not linear in the point $M_{2h}=P_{2h}$ it means, that it cannot describe by on equation. There is needed to divide to two parts which influence on final gear profile. Section is illustrated on Figure 4.

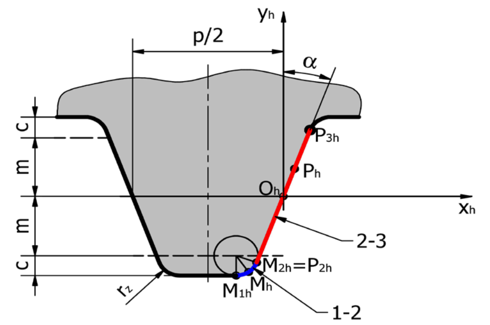

Figure 4. Characterize of rack tool section

» **Analyse Section 1-2**

Section 1-2, limited nodal point $|M_{1h}, M_{2h}|$ creates convex section of rack tool profile Figure 5. The section represent part of circle which is created by point M_h motion around circle centre M_{sh}, with angle of rotation Θ_M.

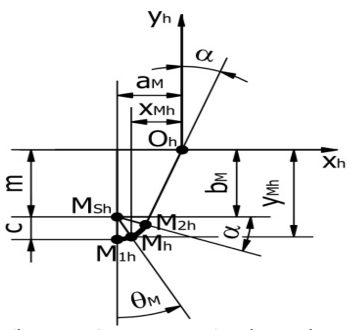

Figure 5. Geometrical analyse Section 1-2

In the rack tool coordinate system O_h (x_h, y_h, z_h) there are parametrical equation as follow:

$$\begin{aligned} x_{Mh} &= a_M + c.\sin\theta_M, \\ y_{Mh} &= -m - c.\cos\theta_M, \end{aligned} \tag{11}$$

where coordinate a_M presents distance between centre of circle M_{sh} and centre of rack tool coordinate system O_h.

$$a_M = -m.\mathrm{tg}\alpha - \frac{c}{\cos\alpha} \tag{12}$$

Final parametrical equation of circle on interval of rotation angle Θ_M will be

$$\begin{aligned} x_{Mh} &= -m.\mathrm{tg}\alpha - \frac{c}{\cos\alpha} + c.\sin\theta_M, \quad \theta_M \in \langle 0; 90-\alpha \rangle \\ y_{Mh} &= -m - c.\cos\theta_M, \end{aligned} \tag{13}$$

» **Analyse Section 2-3**

Section 1-2, limited nodal point $|P_{2h}, P_{3h}|$ creates linear section of rack tool profile Figure 6. Straight line is rotated in angle α. There is determine parametrical equations like point P_h, position which is situated on straight line.

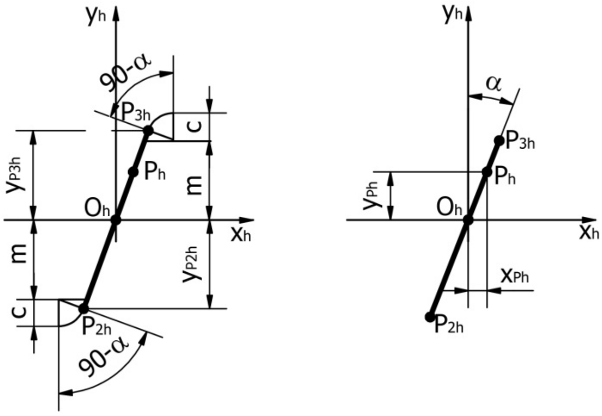

Figure 6. Geometrical analyse Section 2-3

Parametrical equation of linear part will be:

$$\begin{aligned} x_{Ph} &= y_{Ph}.\mathrm{tg}\alpha, \\ y_{Ph} &= \text{constant}, \end{aligned} \quad y_{Ph} \in \langle \pm (m + c.\sin\alpha) \rangle \tag{14}$$

3. RESULTS AND DISCUSION

3.1 Influence simulation of rack tool profile

For determination of tool profile influence on final gear profile there will based on from trajectories of singular point of rack profile. We consider, rack tool profile is created of infinitely large number of points. If the points move and we investigate their movement in the gear wheel coordinate system we will have equations of the point trajectories. Because the rack tool profile is discontinuous, there is needed to describe for two section which influence on gear profile during machining.

» For Section 1-2 trajectories of the profile point movements will be combination of equations (10) and (13):

$$\begin{aligned} x_{k(M)} &= \left(-m.\mathrm{tg}\alpha - \frac{c}{\cos\alpha} + c.\sin\theta_M \right)\cos\varphi - \left(-m - c.\cos\theta_M \right)\sin\varphi + r\left(\frac{2\pi\varphi}{360}\cos\varphi - \sin\varphi \right), \\ y_{k(M)} &= \left(-m.\mathrm{tg}\alpha - \frac{c}{\cos\alpha} + c.\sin\theta_M \right)\sin\varphi + \left(-m - c.\cos\theta_M \right)\cos\varphi + r\left(\frac{2\pi\varphi}{360}\sin\varphi + \cos\varphi \right), \\ z_k &= 0. \end{aligned} \tag{15}$$

Final profile which is created by section 1-2 of rack tool profile we determine as follow:

$$\begin{vmatrix} \dfrac{\partial x_{k(M)}}{\partial \theta_{(M)}} & \dfrac{\partial x_{k(M)}}{\partial \varphi} \\ \dfrac{\partial y_{k(M)}}{\partial \theta_{(M)}} & \dfrac{\partial y_{k(M)}}{\partial \varphi} \end{vmatrix} = 0, \tag{16}$$

where partial derivation are:

$$\begin{aligned} \frac{\partial x_{k(M)}}{\partial \theta_{(M)}} &= c\cos\theta_{(M)}\cos\varphi - c\sin\theta_{(M)}\sin\varphi, \\ \frac{\partial y_{k(M)}}{\partial \varphi} &= \left(-m.\mathrm{tg}\alpha - \frac{c}{\cos\alpha} + c.\sin\theta_M \right)\cos\varphi - \left(-m - c.\cos\theta_M \right)\sin\varphi + \frac{2\pi r}{360}\sin\varphi + \frac{2\pi r\varphi}{360}\cos\varphi - r\sin\varphi, \\ \frac{\partial y_{k(M)}}{\partial \theta_{(M)}} &= c\cos\theta_{(M)}\sin\varphi + c\sin\theta_{(M)}\cos\varphi, \\ \frac{\partial x_{k(M)}}{\partial \varphi} &= \left(-m.\mathrm{tg}\alpha - \frac{c}{\cos\alpha} + c.\sin\theta_M \right)(-\sin\varphi) - \left(-m - c.\cos\theta_M \right)\cos\varphi + \frac{2\pi r}{360}\cos\varphi - \frac{2\pi r\varphi}{360}\sin\varphi - r\sin\varphi. \end{aligned} \tag{17}$$

Result of the solving equations is envelope equations for the section:

$$m.\mathrm{tg}\alpha + \frac{c}{\cos\alpha} + m.\mathrm{tg}\theta_{(M)} - \frac{2\pi r}{360}\mathrm{tg}\theta_{(M)} - \varphi = 0 \Rightarrow \theta_{(M)} \tag{18}$$

Combination of equation (15) and (18) we get final profile which is generated by convex section of rack tool profile. (Figure 7)

» For Section 2-3 trajectories of the profile point movements will be combination of equations (10) and (14):

$$x_{k(P)} = y_h \mathrm{tg}\alpha.\cos\varphi - y_h \sin\varphi + r\left(\frac{2\pi\varphi}{360}\cos\varphi - \sin\varphi\right),$$
$$y_{k(P)} = y_h \mathrm{tg}\alpha.\sin\varphi + y_h \cos\varphi + r\left(\frac{2\pi\varphi}{360}\sin\varphi + \cos\varphi\right), \tag{19}$$
$$z_k = 0.$$

Final profile which is created by section 2-3 of rack tool profile we determine as follow:

$$\begin{vmatrix} \dfrac{\partial x_{k(P)}}{\partial y_h} & \dfrac{\partial x_{k(P)}}{\partial \varphi} \\ \dfrac{\partial y_{k(P)}}{\partial y_h} & \dfrac{\partial y_{k(P)}}{\partial \varphi} \end{vmatrix} = 0 \tag{20}$$

where partial derivation are:

$$\frac{\partial x_{k(P)}}{\partial y_h} = \mathrm{tg}\alpha\cos\varphi - \sin\varphi,$$
$$\frac{\partial y_{k(P)}}{\partial \varphi} = y_h \mathrm{tg}\alpha\cos\varphi - y_h \sin\varphi + \frac{2\pi r}{360}\sin\varphi + \frac{2\pi r\varphi}{360}\cos\varphi - r\sin\varphi, \tag{21}$$
$$\frac{\partial y_{k(P)}}{\partial y_h} = \mathrm{tg}\alpha\sin\varphi + \cos\varphi,$$
$$\frac{\partial x_{k(P)}}{\partial \varphi} = y_h \mathrm{tg}\alpha(-\sin\varphi) - y_h \cos\varphi + \frac{2\pi r}{360}\cos\varphi - \frac{2\pi r\varphi}{360}\sin\varphi - r\sin\varphi.$$

Result of the solving equations is envelope equations for the section:

$$2\pi.r.\varphi.\mathrm{tg}\alpha - 360 y_h + 360 y_h \mathrm{tg}^2\alpha = 0 \Rightarrow y_h \tag{22}$$

Combination of equation (19) and (22) we get final profile which is generated by convex section of rack tool profile.

The equation solving results for input parameters of the wheel r = 125 mm, m = 10 mm, c = 2.5 mm, 20°, which copy the motion of the basic rack in the coordinate system of the wheel are shown in Figure 7. The initial position of the rack is recorded at the angle of wheel rotation - 30° and final position of the rack at the angle of rotation + 30°. The motion trajectories of individual sections are displayed in different colors so that it would be able to see the change of motion of the individual parts and their influence upon the gear profile. The transitive curve envelope formed by the convex part of the gear profile and the envelope of main profile formed by the straight line part of the gear profile are highlighted with bold lines.

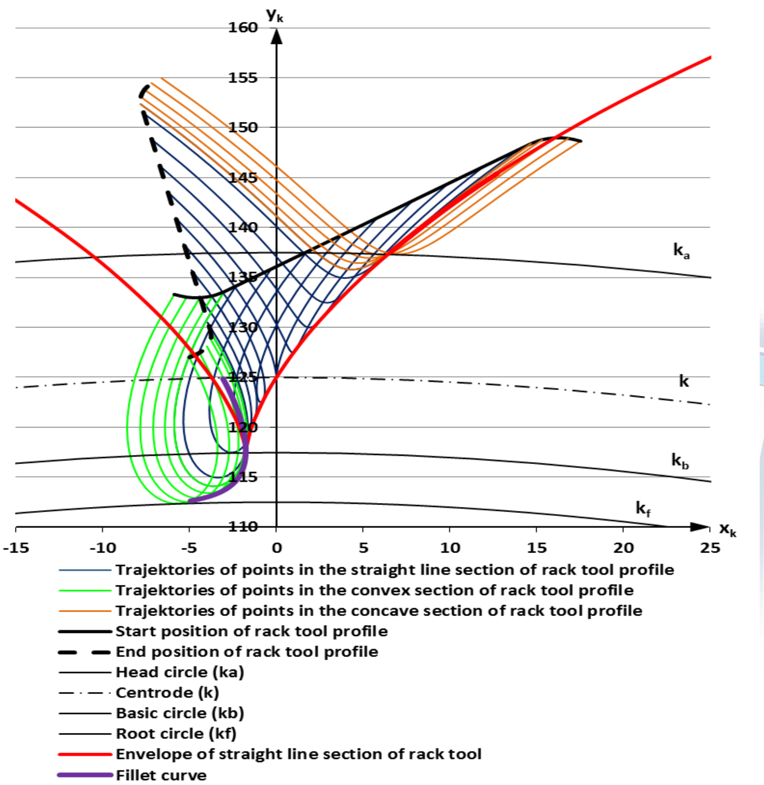

Figure 7. Simulation of influence rack tool profile for Maag machining

4. CONCLUSION

In the paper we made mathematical simulation of relative motion between rack tool profile and its influence on final profile of gear which is produced by Maag technology. The simulation is very important for other research and development of the tool in the gear production. Every modification of rack tool can influence to final produced profile. There were analysed mathematical models of inputs outputs profiles in the one planar for spur gear. It means that this process could investigate in the planar x,y. By analogy we can investigate in the 3D space. The simulation refer to possibility to investigate relative influence between profile generated by rack tool profile and theoretical involute profile. The inaccuracy could by implemented to the rack profile

References

[1.] Da-Ren, W. – Jia-Shu, L. “A Geometric Theory of Conjugate Tooth Surfaces”. World Scientific. The USA.1992.

[2.] Eisenhart, P. L. “Differential Geometry of Curves and Surfaces”. Dover Publications. The USA.2004.

[3.] H M T Bangalore. “Production Technology”. Tata McGraw-Hill. New Delhi. 1980.

[4.] Juttler B. – Piene R. – Dokken, T. “Geometric Modeling and Algebric Geometry”. Springer. 2008.

[5.] Litvin L.F. – Fuentes A. “Gear Geometry and Applied Theory“. Cambridge University Press. United Kingdom.2004.

[6.] Davis, J. R. “Gear Materials, Properties and Manufacture”. ASM International. The USA.2006.

[7.] Radzevich, S. P. “Kinematic Geometry of Surface machining”. CRC Press. The USA.2008.

[8.] Pavlenko, S. – Litecka, J. – Bicejova, L.. “On Gear Hobs Profilling”.RAM-Verlag. Germany.2013

[9.] Pavlenko, S. “K profilovaniuodvalovacichfrez”. FVT TU. Kosice. 2006.

[10.] Rasa, J. et al. “Vypocetnimetody v konstrukcireznychnastroju”. SNTL. Praha.1986.

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