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DAMAGE IDENTIFICATION IN BEAMS USING NONLINEAR REGRESSION ANALYSIS OF BENDING FREQUENCIES

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ABSTACT: This paper aims to present the technique for damage identification in beam-like structures using the numerical, regression and experimental values of the beam bending frequencies. The regression relations between the first four natural frequencies and damage parameters are established using the frequencies obtained numerically by undamaged and damaged beam FEA model in free-free state. The damage was modelled as a narrow open notch perpendicular to the beam axis. The efficiency of the proposed technique for damage identification is validated by seven free-free beam samples with four depths of the notch.
Keywords: damage identification, FEA, nonlinear regression

1. INTRODUCTION

Damage in a structure causes a change in its physical parameters and, consequently, in modal parameters such as its natural frequencies, mode shapes and modal damping. This phenomenon makes the base for vibration based non-destructive structural health monitoring procedures which have gained significant attention over the past few decades. The idea of using modal parameters for damage detecting and identification appeared during 1940s. From that time, great efforts have been put to develop more reliable and efficient identification techniques. Many of the available methods can be found in abundant literature, for instance in [1, 2, 7].

According to Rytter [6], the problem of damage identification refers to the following levels:

» Level 1 (Detection) - Indicating the occurrence of a damage in the structure,
» Level 2 (Localization) - Level 1 plus locating the damage,
» Level 3 (Assessment) - Level 2 plus estimation of the damage severity (depth),
» Level 4 (Prediction) - Level 3 plus prediction of the remaining structure life.

The first three levels of damage identification can be accomplished using only the changes in natural frequencies, as will be shown here.

The aim of the present paper is to evaluate the performance of the proposed technique for damage identification using the numerical, regression and experimental values of the beam bending frequencies.

2. DAMAGE IDENTIFICATION TECHNIQUE

The technique for damage identification is based on the fact that damage in a structure causes changes in its mass and stiffness and, consequently, its natural frequencies and other modal parameters.

To use the proposed identification criterion, it's necessary to perform numerical and regression analysis as well as the experimental tests.

2.1. Numerical analysis of a free-free beam

Using the numerical analysis the values of frequencies of undamaged and damaged beam-like structure can be calculated fast and easy.

A simple case of a free-free beam shown in Figure 1 was modelled using solid elements in software I-DEAS Master Modeler 9. The length of the beam was L_B=400 mm, height H=8,16 mm, width B=8.12 mm, modulus of elasticity E=2.068x10^{11} Pa, mass density 7820 kg/m^3, and Poisson's coefficient 0.29.

The damage was simulated as a narrow open notch perpendicular to the beam axis. The location of the damage is L_D, its depth is d, and the width of the notch is 1mm.

Using the numerical model the first four natural frequencies f_i^{NU} corresponding to the bending modes of the undamaged beam was calculated. Then, varying the relative location $L = L_D/L_B$ and relative depth of the damage $D = d/H$, the first four bending frequencies f_i^{ND} of damaged beam were calculated.

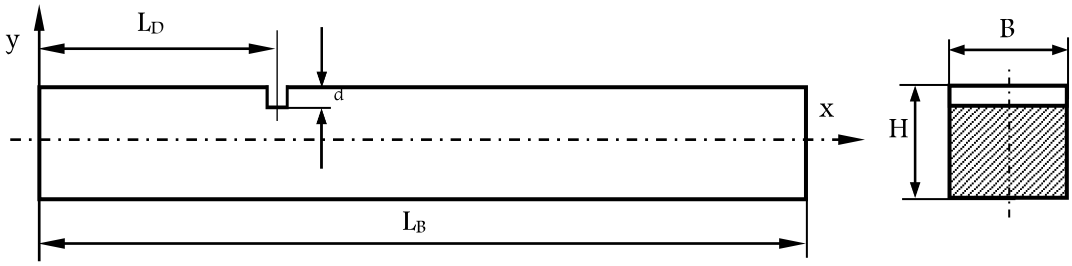


Figure 1. Free-free beam with a notch representing damage

Due to structural symmetry, the location of the notch L_D measured from the left end of the beam was varied from 10 mm to 200 mm in 10 mm increments. The depth *d* of the notch was varied from 1 mm to 4 mm in 1 mm increments. The values of frequencies f_i^{ND} are then used to find regression relations between the first four frequencies and damage parameters D and L, where D ranges from 0.125 to 0.5 and *L* from 0.01 to 0.5.

2.2. Regression analysis of the frequency changes

The numerically obtained values of the first four frequencies f_i^{ND} were used as the input data for software STATISTICA 6.0. After several attempts the best fit was chosen using the Nonlinear Estimation option. In case of the uniform beam here, the best results are obtained assuming the quadratic influence of the relative depth D and polynomial influence of the relative location L.

The following regression relations for the first four frequencies are obtained for the beam under consideration, Eqs.(1), (2), (3) and (4):

$$f_{1R}(D,L)=f_1^{NU}[1-0.177566D^2(-0.01948-0.85975L+6.32585L^2+47.5372L^3-83.912L^4)] \quad (1)$$

$$f_{2R}(D,L)=f_2^{NU}[1-0.42922\,D^2(0.065133-3.8651L+45.7407L^2-44.275L^3-267.41L^4+406.144L^5)] \quad (2)$$

$$f_{3R}(D,L)=f_3^{NU}[1-11.0353D^2(0.006469-0.37663L+5.74127L^2 -16.533L^3-32.81L^4+180.968L^5-177.36\,L^6)\,] \quad (3)$$

$$f_{4R}(D,L)=f_4^{NU}[1-69.938D^2(0.00252-0.17049L+3.30183L^2-18.862L^3 +21.3852L^4+121.863L^5-375.07L^6+298.339\,L^7)] \quad (4)$$

where f_i^{NU}, i=1,2,3,4, are numerically calculated frequencies of undamaged beam, which can be found in Table 1.

The largest differences between the values obtained numerically f_i^{ND} and those calculated by regression relations $f_{iR}(D,L)$ given by Eqs.(1)-(4) are at those locations of the beam where nodes and maximal amplitudes of the particular mode shapes occur due to the trend of these regression relations to smooth the data at extreme points. The corresponding coefficients of correlation were from 0.996 to 0.998. More on this topic can be found in [3].

2.3. Experimental measurements of a beam sample

Experimental measurements of beam frequencies were performed on seven beam samples using the experimental setup shown on Figure 2. The instruments used in this experimental study were: PC, frequency analyzer HP 3567A, interface HP82335A, accelerometers B&K 4394 and impact hammer B&K 8202 with load cell B&K 8200.

The beam samples were hung by two silicon ropes to attain a free-free state, Figure 3. The damage was made by a saw cut at one of the seven chosen locations. Firstly, a cut of 1mm depth (D=0.125) was made, then frequencies were measured. After that, the measurements were repeated for three other depths. The damage was made with a 1mm thin blade so the crack always remains open during the vibration test.

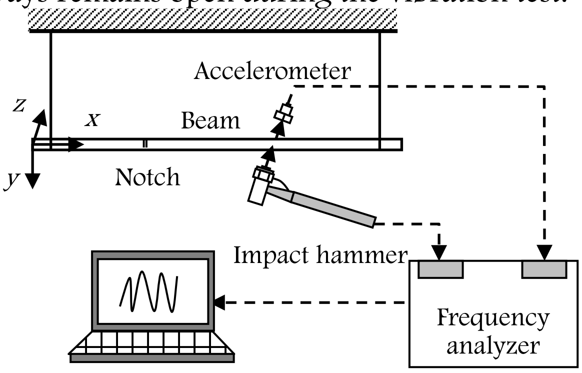


Figure 2. Experimental setup scheme

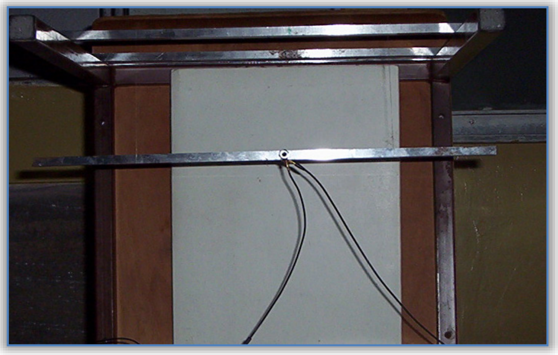


Figure 3. Photo of the beam sample

The numerical model of a beam was not updated to match all seven beam samples so there were slight discrepancies in the dimensions and frequency values in undeformed beams state. In a certain way this enabled to check the robustness of the used technique. In reality, however, the efforts should be made to establish the numerical model that is the best representation of the real beam, so the better identification results should be obtained.

The frequency measurement was performed twice for the undamaged beam sample and for each of the four crack depths. The mean value of frequencies obtained by two independent measurements of undamaged beams and each of damage scenarios, Figure 4, was used to calculate the frequencies f_i^{EU} and f_i^{ED}, i=1,2,3,4.

Also, the values of the measured frequencies were dependent on the frequency resolution and the number of hammer impacts. The number of impacts were chosen in dependence of the beginning of frequency value stabilization. The first frequencies were found with resolution 0.125 Hz and 5 impacts (frequency range 200-400 Hz), second and the third frequency with resolution 0.25 Hz and 10 impacts (range 600-1600 Hz,), and the fourth frequency with resolution 1 Hz and 10 impacts (range 1.5-5.5 kHz). More details on these experimental measurements can be found in [4].

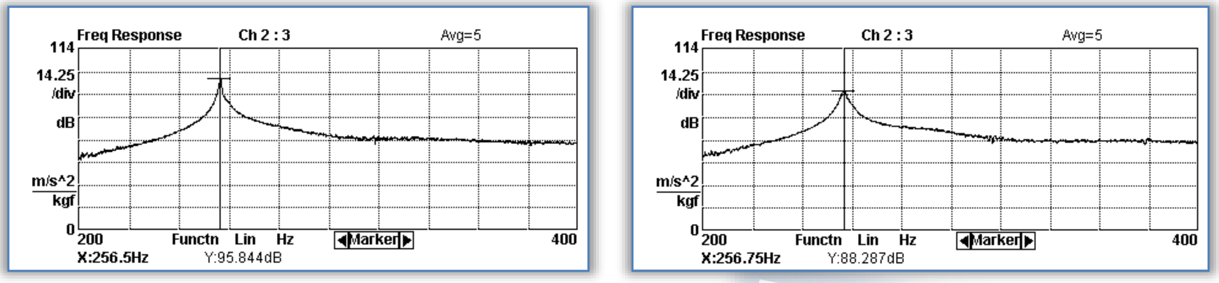


Figure 4. Two independent measurements of the first frequency of the damaged beam sample (damage case: L_{real}=0.35, D_{real}=0.375)

The values of frequencies obtained numerically and experimentally differ less or more due to modelling and measurement errors that are inevitable in reality. Also, it was impossible to cut accurately the nominal depth of the notches using the ordinary saw cut.

2.4. Identification criterion

To identify the location and the depth of a damage, the functional named FUN(D,L) is proposed in the form:

$$\mathrm{FUN}(D,L)=\sum_{i=1}^{4}\left[\frac{\left(f_i^{NU}\dfrac{f_i^{ED}}{f_i^{EU}}\right)^2-\left(f_{iR}(D,L)\right)^2}{\left(f_i^{NU}\dfrac{f_i^{ED}}{f_i^{EU}}\right)^2}\right]^2 \tag{5}$$

where are: f_i^{NU} - i^{th} natural frequency of undamaged beam numerically obtained, f_i^{EU}- i^{th} natural frequency of undamaged beam experimentally obtained, f_i^{ED} - i^{th} natural frequency of damaged beam experimentally obtained, $f_{iR}(D,L)$ - i^{th} natural frequency of damaged beam predicted by regression equations (1)-(4).

The estimation of damage location and depth is obtained by finding the minimum of the functional FUN(D,L) inside the reasonable bounds of D (from 0 to 0.5) and L (from 0 to 0.5).

The proposed functional is similar to that given in [5] and is based on the assumption that the frequency ratio $f_i^{NU}/f_{iR}(D,L)$ is close to the experimental ratio f_i^{EU}/f_i^{ED}.

3. AN EXAMPLE OF DAMAGE IDENTIFICATION

The values of calculated (numerical) and measured (experimental) characteristic frequencies for the undamaged beam which was aimed to be cut at location L_D=140 mm (with L_{real}=0.35) are presented in Table 1. The measured frequencies of the beam sample damaged at location L_{real}=0.35 with four values of damage depth are presented in Table 2.

Table 1. Frequencies of the undamaged beam (to be cut at L_{real}=0.35)

Undamaged beam	
Numerical values	Experimental values
f_1^{NU} = 264.2195	f_1^{EU} = 263.625
f_2^{NU} = 737.647	f_2^{EU} = 735
f_3^{NU} = 1416.678	f_3^{EU} = 1410
f_4^{NU} = 2354.085	f_4^{EU} = 2352

Table 2. Measured frequencies of the beam damaged at location L_{real}=0.35

Relative damage depth D			
D_{real}=0.125	D_{real}=0.25	D_{real}=0.375	D_{real}=0.5
f_1^{ED} = 262.68	f_1^{ED} = 260.875	f_1^{ED} = 256.62	f_1^{ED} = 248.875
f_2^{ED} = 732.12	f_2^{ED} = 728	f_2^{ED} = 713.12	f_2^{ED} = 693.12
f_3^{ED} = 1409.25	f_3^{ED} = 1408.75	f_3^{ED} = 1409.25	f_3^{ED} = 1407.88
f_4^{ED} = 2354.5	f_4^{ED} = 2337	f_4^{ED} = 2305	f_4^{ED} = 2260.5

Using the calculated and measured frequencies, the minimum of the identification functional FUN(D,L) was found in software Mathematica 5.1.

Figure 5 shows the results of damage identification (the area of FUN(D,L) minimum) obtained by use of Mathematica 5.1 for the beam sample with L_{real}=0.35 and four depths of the damage.

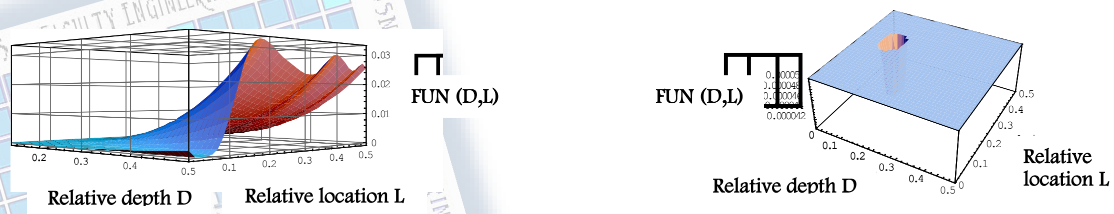
Finding the minimum of the functional FUN(D,L), the parameters of the damage D_{est} and L_{est} are estimated. The results of damage identification and relative identification errors for the specific case (L_{real}=0.35) are presented in Table 3. The relative identification errors are calculated by

$$ErrD(\%) = \frac{D_{est} - D_{real}}{D_{real}} \cdot 100\%, \qquad ErrL(\%) = \frac{L_{est} - L_{real}}{L_{real}} \cdot 100\% \ . \qquad (5)$$

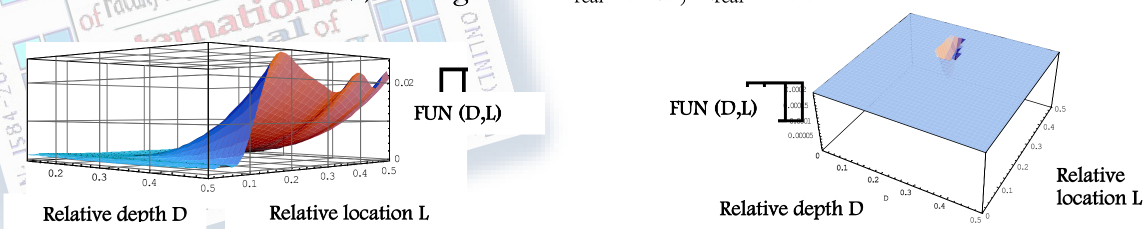
Table 3. Damage identification for the beam damaged at location L_{real}=0.35

Real and estimated damage parameters				Relative error (%)	
D_{real}	D_{est}	L_{real}	L_{est}	ErrD	ErrL
0.125	0.123219	0.35	0.307815	-1.4248	-12.285
0.25	0.216849	0.35	0.351392	-13.2604	0.397
0.375	0.361374	0.35	0.353335	-3.6336	0.952
0.5	0.509035	0.35	0.354808	1.807	1.373

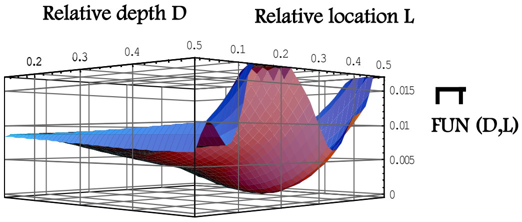
a) Damage case: D_{real}=0.125, L_{real}=0.35

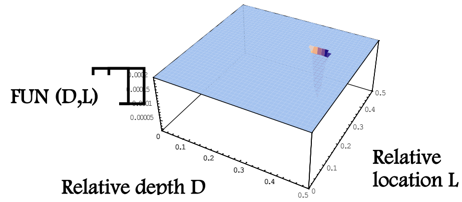


b) Damage case: D_{real}=0.25, L_{real}=0.35

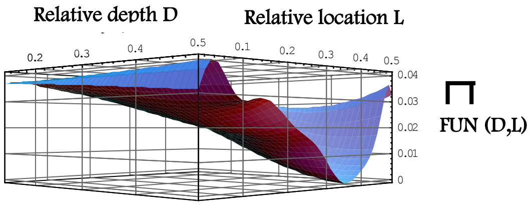


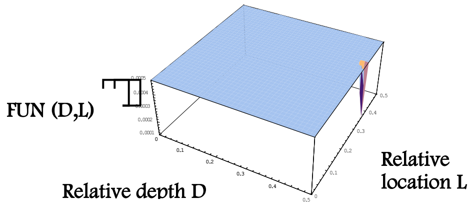
c) Damage case: $D_{real}=0.375$, $L_{real}=0.35$





d) Damage case: $D_{real}=0.5$, $L_{real}=0.35$





a) b)

Figure 5. Functional FUN(D,L) for 4 values of damage depth:
A) 3D view of FUN(D,L); B) zoomed view on area of functional minimum

4. THE RESULTS OF DAMAGE IDENTIFICATION

The same procedure was repeated for 4 cases of damage depths for another six beam samples. Figure 6 shows the results of damage identification for all 28 cases of damaged states (7 locations by 4 depths).

For four cases that refers to the location od $L_{real}=0.05$, the Mathematica software couldn't find the accurate numerical values of D_{est} and L_{est} for the functional minimum but using the appropriate diagrams, shown on Figure 7, the characteristic values of D_{est} and L_{est} have been defined approximately.

It should be noted that despite all modelling and measuring errors that appear here, the results of the used identification technique are quite satisfactory.

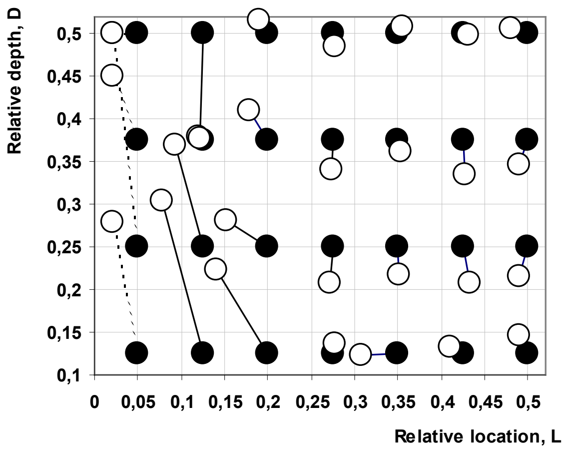
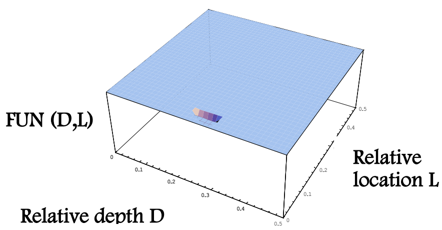
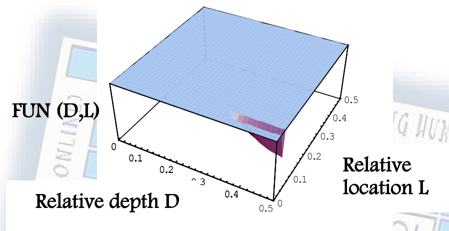


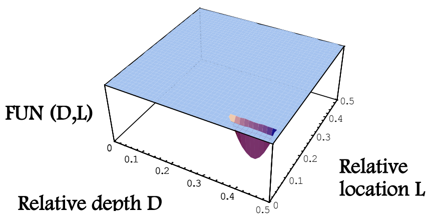
Figure 6. Results of damage identification:
● - point indicating L_{real} and D_{real},
○ - point indicating L_{est} and D_{est}



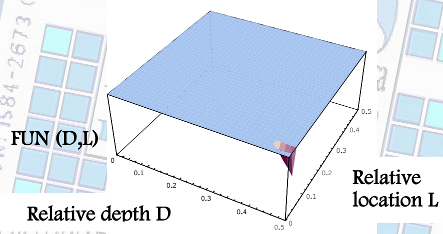
a) $D_{real}=0.125$, $L_{real}=0.05$



b) $D_{real}=0.25$, $L_{real}=0.05$



c) $D_{real}=0.375$, $L_{real}=0.05$



d) $D_{real}=0.5$, $L_{real}=0.05$

Figure 7. Zoomed view on area of functional FUN(D,L) minimum for 4 values of damage depth for damage relative location $L_{real}=0.05$

5. CONCLUSION

The paper shows how damage parameters can be estimated using bending frequencies of a beam obtained by numerical, regression and experimental way.

The accuracy of the technique depends on the quality of numerical model representing the real structure and estimation of regression relationships, as well as the quality of frequency measurements in reality.

Although the use of regression analysis makes this method approximate, the results are quite satisfactory. The identification results could be improved using better mesh refinement and higher number of numerical calculations to provide a sufficient number of points for establishing the regression relations. Also, special attention should be put on minimizing the measurement errors since there always exists a certain level of noise.

The presented procedure showed that regression analysis can be successfully used in non-destructive identification of damage parameters.

It would be interesting to explore if the presented method could be used in beams of different cross-sections or by use of axial or torsional frequencies.

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