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1. A. ADENIYAN, 2. J. A. ADIGUN

SIMILARITY SOLUTION OF HYDROMAGNETIC FLOW AND HEAT TRANSFER PAST AN EXPONENTIALLY STRETCHING PERMEABLE VERTICAL SHEET WITH VISCOUS DISSIPATION, JOULEAN AND VISCOUS HEATING EFFECTS

¹. Department of Mathematics, University of Lagos, NIGERIA

². Department of Physical Sciences, Bells University of Technology, Ota, Ogun State, NIGERIA

ABSTRACT: Analysis is conducted on natural convective flow and heat transfer of a boundary layer MHD flow over an exponentially stretching sheet encompassed in an expanse of an incompressible, radiating and electrically conducting fluid in the presence of viscous dissipation, viscous and Joulean heating along with suction/injection, the case of a steady two dimensional model. Using appropriate similarity variables, the governing coupled non-linear equations are reduced to a set of ODEs along with the considered BCs, which are solved numerically by shooting iteration technique alongside the fourth order Runge-Kutta integration scheme through a reliable software package. A comparison with previously published results on the similar special cases is made, and the results are found to be credible. The effect of Prandtl number, Eckert number, buoyancy force, Lorentz force, the thermal radiation, viscous dissipation, Joule heating, porous medium permeability and mass blowing or fluid withdrawal on the fluid behavior are illustrated in physical terms. Finally, numerical values of pertinent physical quantities, such as the skin-friction coefficient, the local Nusselt number are presented in tabular form while those of the fluid velocity and temperature are illustrated and discussed by graphs.

Keywords: exponentially stretching, mixed convection, hydromagnetics, viscous and Joulean heating, wall mass flux

1. INTRODUCTION

The boundary layer flow on a continuous stretching sheet has attracted considerable attention during the last few decades due to its numerous applications in industrial manufacturing processes such as hot rolling, wire drawing, glass-fiber and paper production, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products Magyari and Keller [1]. Crane [2] was one of the pioneering researchers to initiate and investigate the flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point and his work has been extended by several other researchers under different physical situations Cortel [3], Ishak et al [4] and Turkyilmazoglu [5]. Convection heat transfer has been a markedly important research area for more than five decades consequential upon its natural or forced occurrences in many scientific applications and technological devices. Mixed or free-forced convective heat transfer is driven by buoyancy force and pressure drop or any other external force. Srinivasacharya and Ram-Reddy [6] investigated mixed convection boundary-layer flow and heat transfer in the presence of Dufour and Soret effects using similarity transformation deficient of removing the axial variable explicitly. The MHD boundary layer flow and heat transfer over a stretching sheet for stagnation point with and without viscous dissipation and Joule heating was studied by Jat and Chaudary [7, 8]. Bidin and Nazar [9] investigated the Numerical solution of the boundary layer flow over an exponentially

stretching sheet with thermal radiation and viscous dissipation. Ishak [10] investigated the thermal radiation effects on hydro-magnetic flow due to an exponentially stretching sheet and reported that a retardation in the rate of heat transfer due to the presence of heat radiation effect. Mukhopadhyay [11] analyzed the effects of partial slip on MHD boundary layer flow over an exponentially stretching surface with suction or injection and the results obtained shows that the flow field was influenced by the embedded flow parameters. Lately, Mahmoud [12] examined the influence of variable fluid properties for a boundary layer flow past an exponentially stretching sheet with non-uniform magnetic field and, heat and mass fluxes.

On the control of the quality of final products from industrial processes such as polymer processing and “annealing and thinning of copper wires”, the effects of thermal radiation play an indispensable role in controlling how heat is transferred. The combined influence of buoyant forces and thermal radiation on boundary layer flow and heat transfer has received increasingly pertinent attention by many a researcher recently and in the past consequent upon the much needed engineering devices such as high performance space vehicles and production processes as in the aerodynamic production of polymer and some extrudates involving extrusion through a die (see Makinde and Olanrewaju [13], Olanrewaju and Adeniyani [14]). High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, magnetohydrodynamics (MHD) accelerators, power generation systems are some important applications of radiative heat transfer from a vertical wall to conductive gray fluids. Of recent, Ahmad et al. [15] examined the influence of thermal radiation, Lorentz and Darcy forces on an exponentially stretching sheet encompassed in a saturated porous medium, and bathed in an electrically and thermally conducting fluid for a forced convection considering cases of prescribed surface temperature and heat flux.

In the world of engineering procedures such as enhanced thermal oil recovery and design of thrust bearing, suction /injection is very germane. Also in chemical processes, suction is a power tool to remove reactants while injection add reactants, reduce friction drags, give some surface cooling and prevent metallic corrosion. Suction tends to increase the skin friction whereas injection acts in the opposite manner and both could appreciably change the flow field when applied on a fluid moving through the bounding surface. Rudraswamy and Giressha [16] examined the influence of thermophoresis and Brownian motion on a hydromagnetic boundary-layer flow of a chemically reactive nanofluid over a horizontal sheet stretching exponentially with surface mass flux using a shooting quadrature.

Jat and Chand [17] considered MHD Flow and heat transfer over an exponentially stretching sheet with viscous dissipation, Joulean heating and radiation effects. The present communication seeks to consider the combined influence of buoyant and magnetic forces, viscous heating in the presence of a heat source and fluid suction in addition to that which was already observed by Ref. [17].

2. PROBLEM FORMULATION

Consider a steady, incompressible, two-dimensional MHD flow of an electrically conducting viscous fluid towards an exponentially stretching sheet in a porous medium. The fluid occupies the space $y > 0$ and is flowing in the x direction and y -axis is normal to the flow. A constant magnetic field of strength B_0 is applied in the normal direction and the induced magnetic field is neglected which is a valid assumption when the magnetic Reynolds number is small. The boundary-layer equations that govern the present Boussinesq flow and heat transfer problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \gamma \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\gamma}{K_p} u + g\beta(T - T_\infty) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{\gamma}{\rho C_p} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{\rho C_p} u^2 + \frac{Q}{\rho C_p} (T - T_\infty) + \frac{\gamma}{\rho C_p K_p} u^2 \tag{3}$$

where u and v are the components of the velocity in x and y directions respectively, γ is the coefficient of kinematic viscosity, T is the fluid temperature, β is the thermal expansion coefficient, σ is the electrical conductivity, g is the acceleration due to gravity, ρ the density, C_p the specific heat at constant pressure, K the thermal conductivity, B_0 is the constant magnetic strength and q_r is the radiative heat flux. The boundary conditions are given by

$$y = 0, \quad u = U_w = U_0 e^{\frac{x}{L}}, \quad v(x, 0) = V_0(x), \quad T = T_w = T_\infty + T_0 e^{\frac{2x}{L}} \tag{4}$$

$$y \rightarrow \infty, \quad u \rightarrow 0, \quad T \rightarrow 0.$$

where $V_0(x)$, T_0 and L are the variable wall normal flux velocity, reference constant temperature and reference length respectively, T_∞ is the free stream temperature, and U_0 is a constant bearing the unit of speed for stretching sheet.

As the momentum eq. (2) must fulfill the inviscid conditions at the boundary edge, then the subsequent form of momentum balance equation is written as

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} - \left[\frac{\sigma B_0^2}{\rho} + \frac{\gamma}{K_p} \right] u + g\beta(T - T_\infty) \quad (5)$$

Simplifications that enhance good understanding of fluid radiation is attributed to the works carried out by Aboeldahab and El-Gendy [18]. As noted, Cogley et al. [19] assumed that the fluid is in the optically thin limit and consequently doesn't absorb its own radiation but that emitted by the boundaries. In the case of a gas which is optically thick, the gas self-absorption rises and results in some unprecedented complications. Nevertheless, following works done by Rosseland [20], Siegel and Howel [21] and Sparrow and Cess [22], we get a simplification for the radiative heat flux q_r as follows such that

$$q_r = -\frac{4\sigma^* \partial T^4}{3K^* \partial y} \quad (6)$$

where σ^* and K^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. Following Sajid and Hayat [23], we assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function, after expressing its Taylor series about the free stream temperature T_∞ and neglecting higher-order terms. This results in the following approximation:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4$$

and consequently

$$\frac{\partial q_r}{\partial y} \approx -\frac{16\sigma^* T_\infty^3}{3K^*} \frac{\partial^2 T}{\partial y^2} \quad (7)$$

Using (6) and (7), it follows that (3) is returned as

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\gamma}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \left[\frac{\sigma B_0^2}{\rho c_p} + \frac{\gamma}{\rho c_p K_p} \right] (u - u_e)^2 + \frac{Q}{\rho c_p} (T - T_\infty) - \frac{16\sigma^* T_\infty^3}{3K^* K} \alpha \frac{\partial^2 T}{\partial y^2} \quad (8)$$

where $\alpha = \frac{K}{\rho c_p}$ thermometric diffusivity.

The equation of continuity (1) is satisfied by introducing Stokes stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (9)$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by the following transformations Ref. [17]:

$$\psi(x, y) = \sqrt{2\gamma L U_0} e^{\frac{x}{2L}} f(\eta), \quad T = T_\infty + T_0 e^{\frac{2x}{L}} \theta(\eta), \quad \eta = y \sqrt{\frac{U_0}{2\gamma L}} e^{\frac{x}{2L}} \quad (10)$$

where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and primes denote differentiation with respect to η . The transformed ordinary equations are:

$$f'''' - 2f'^2 + ff'' - (M_x + Da_x)f' + Gr\theta = 0 \quad (11)$$

$$(1 + \frac{4}{3}Ra)\theta'' + Prf\theta' - 4Pr\theta f' + PrEc(f'')^2 + PrS_x\theta + PrEc(M_x + Da_x)(f')^2 = 0 \quad (12)$$

subject to the transformed boundary conditions

$$f(0) = F_w, \quad f'(0) = 1, \quad \theta(0) = 1, \quad f'(\infty) = 0, \quad \theta(\infty) = 0 \quad (13)$$

In the above equations,

$$\left. \begin{aligned} Pr &= \frac{\rho \gamma c_p}{k}, & F_w &= -V_w \sqrt{\frac{2L}{\gamma U_0}} e^{\frac{x}{2L}}, & M_x &= \frac{2\sigma B_0^2 L}{\rho U_0 e^{\frac{x}{2L}}}, & Gr &= \frac{2g\beta T_0 L}{u_0^2}, \\ S_x &= \frac{2QL}{\rho c_p U_0 e^{\frac{x}{2L}}}, & Ra &= \frac{4\sigma^* T_\infty^3}{kK'}, & Ec &= \frac{U_0^2}{c_p T_0}, & Da_x &= \frac{2\gamma L}{K_p U_0 e^{\frac{x}{2L}}} \end{aligned} \right\} \quad (14)$$

are respectively Prandtl number, wall mass flux (Suction/injection), magnetic parameter, Grashof number, heat generation/absorption parameter, radiation parameter, Eckert number and permeability parameter.

In order that both the momentum and thermal boundary layer equations have similarity solutions, the parameters V_w , Da_x and S_x are functions of x and must be made constants

Makinde [24]. This condition is met if we let $V_w = V_0 e^{-\frac{x}{L}}$, $M_x = M e^{\frac{x}{L}}$, $\sigma = \sigma_0 e^{\frac{x}{L}}$, $K_p = K_0 e^{\frac{x}{L}}$, $Q = Q_0 e^{\frac{x}{L}}$ and $S_x = S e^{\frac{x}{L}}$, where V_0 , M , σ_0 , K_0 and Q_0 are constants.

The physical quantities of pertinent interest are the skin-friction coefficient $f''(0)$, and the Nusselt number $-\theta'(0)$, which represent the wall shear stress and the heat transfer rate at the surface respectively. We seek to know how the values of $f''(0)$ and $-\theta'(0)$ as well as the dimensionless velocity $f'(\eta)$ and temperature $\theta(\eta)$ vary with all the embedded parameters.

3. RESULTS AND DISCUSSION

The system of ordinary differential equations (11) to (12) along with the boundary condition (13) has been solved numerically using the Fourth order Runge Kutta with shooting method. Comparison with the existing results from some literatures with open access shows a favourable agreement, as shown in Tables 1 and 2.

Table 1: Computations showing comparison with Magyari and Keller [1], El-Aziz [25], Mukhopadhyay [11] and Ishak [10] for $S = Gr = Ra = F_w = Ec = 0$.

Ra	Pr	M	Ref. [1]	Ref. [25]	Ref. [11]	Ref. [10]	Present result
0	1	0	0.954782	0.954785	0.95478	0.95478	0.95485201
	3		1.869075	1.869074	1.8691	1.8691	1.869061724
	5		2.500135	2.500132	-	2.5001	2.500122587
	10		3.660379	3.660372	-	3.6604	3.660365072
1	1		-	-	-	0.5312	0.537337859
1	1	1	-	-	0.4503	0.4505	0.466069713
0	1	1	-	-	0.8610	0.8611	0.861899580

Table 2: Computations showing comparison with Bindin Nazar [9] for $S = Gr = Ra = F_w = M = 0$.

Pr	Ec	Ra	Ref. [10]	Present result
1	0	0	0.9547	0.95485201
2	0.2	0.5	0.9654	0.96539043
3	0.9	1.0	0.6055	0.60546778
3	0.2	1.0	1.0067	1.00676014

A little adjustment was made in our codes to enable us have these particular cases Ref. [1, 10, 11, 12] and El-Aziz [25]. Strictly, for the sake of validation, the coefficient “4” in the term $\theta f'$ in equation (12) was replaced by “1” as this does not consequently distort authenticity needed for the Maple written script.

Table 3 gives a quantitative view of the influence of the embedded parameters on the skin-friction coefficient and Nusselt number. Each of the numerical values of the computed skin-friction coefficient demonstrates a negative sign. This is not unexpected as the impressed force of stretching surface exhibits dragging influence on the fluid unlike the usual case where a streaming fluid over a solid surface features the opposite, producing positive values of the shearing stress at the surface. Nonetheless the shear stress at the plate $f''(0)$, could be decimated gradually in absolute values when the values of Grashof number, radiation parameter, Eckert number and heat generation parameter were increased but the parameter of permeability, fluid suction and magnetic parameter were all seen to intensify it.

The rate of heat transfer at the plate surface as measured by the Nusselt number was reduced consistently by increasing values of the permeability parameter, magnetic parameter, radiation parameter, Eckert number and heat generation parameter. On the contrary, it had an increment when the values of the Grashof number and fluid suction were increased. As revealed on this table, the intensification of injection parameter leads to a diminution in both the Nusselt number as well as the skin-friction coefficient.

Figures 1-8 express the influence of the embedded controlling parameters on the velocity boundary layer and temperature distribution. Generally, the fluid attained the largest value of velocity at the plate surface and then decelerated sequentially to the zero free stream value satisfying the far field boundary condition. The dimensionless fluid temperatures as observed in the legend figures within the boundary layer unveiled similar trends. Starting from the highest value at the plate the temperature fell without bound to assume its zero value asymptotically into the far field.

Figure 1.a exhibits the nature of velocity for the variation of magnetic parameter M. With its increasing values, velocity is found to decrease but the temperature increases in this case (Figure 1.b). The transverse magnetic field opposes the motion of the fluid and the rate of transport is considerably reduced. This is because with the increase in the magnetic parameter (M), Lorentz

force increases and it produces more resistance to the flow. As it increases, the thermal boundary layer thickness increases but the momentum boundary layer get decreased.

Table 3: Values of Skin-friction coefficient $f''(0)$ and Nusselt number $-\theta'(0)$ for various parameter variations

M	Gr	Da	F_w	Pr	Ra	Ec	S	$f''(0)$	$-\theta'(0)$
0.2	0.1	0.2	0.2	0.72	0.1	0.1	0.1	-1.48660798	1.31076116
1.0								-1.74783053	1.20718745
5.0								-2.68723353	0.86188071
	0.4							-1.36817124	1.35684672
	0.7							-1.25814164	1.39057266
	1.0							-1.15329598	1.41838845
		0.5						-1.58992231	1.26966354
		1.5						-1.89200785	1.15081170
		3.0						-2.26838670	1.00863549
			-0.2					-1.30123041	1.22820926
			-0.1					-1.34480540	1.24780810
			0.1					-1.43746072	1.28901180
			0.2					-1.48660798	1.31076116
			0.4					-1.59064020	1.35689376
				4.00				-1.51029919	3.79505594
				5.00				-1.51221170	4.33270922
				7.10				-1.51484606	5.33529267
					0.5			-1.47946955	1.01316944
					0.8			-1.47563743	0.87595798
					1.0			-1.47356559	0.80669625
						0.5		-1.48472717	1.15056880
						1.0		-1.48240064	0.95148301
						2.0		-1.47782361	0.55702480
							0.01	-1.48788516	1.34441391
							0.20	-1.48471247	1.26866048
							0.30	-1.48188274	1.21787081

In Figure 2, velocity profiles are shown for different values of the Grashof number (Gr). The velocity curves show that the rate of transport increases with increasing values of the Grashof number. This is due to the temperature difference between the stretched wall and the surrounding fluid. In all cases, the velocity vanishes at some large distance from the sheet ($\eta = 6$) It is of a worthy note that the boundary layer thickness is increased as Gr intensifies.

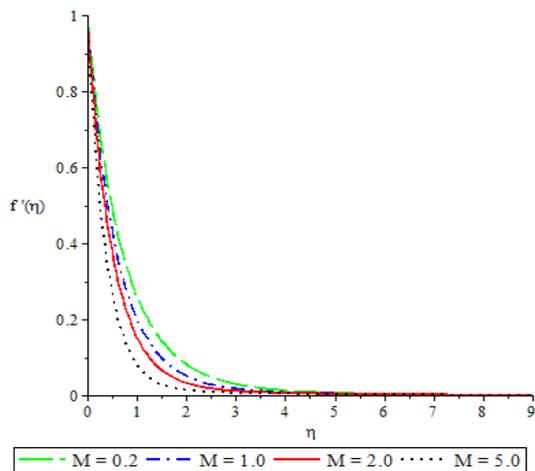


Figure 1.a: Variation of velocity $f'(\eta)$ with η for several values of magnetic parameter M

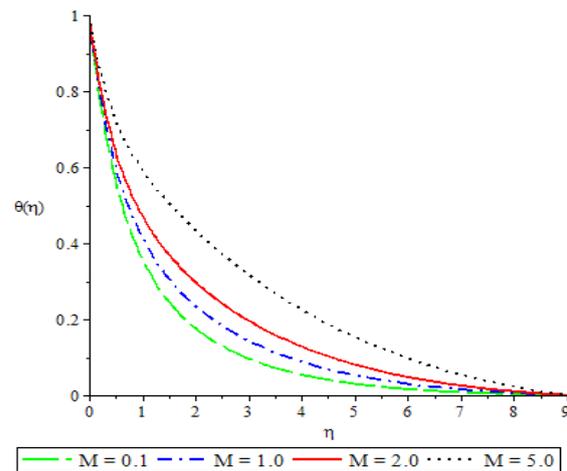


Figure 1.b: Variation of temperature $\theta(\eta)$ with η for several values of magnetic parameter M

Figure 3 presents the behavior of the fluid velocity when the Permeability parameter is increased. The fluid velocity within the boundary layer is decreased which is an indication that Permeability stabilizes the boundary layer growth.

Figure 4 shows the influence of Prandtl number on the fluid temperature. Physically, as the Prandtl number is a ratio of viscous to thermal diffusion, increase in Prandtl number indicates a

decrease in thermal diffusion and hence decreases the thermal boundary layer thickness. Figure 5 depicts the graph of temperature against spanwise coordinate (η) for various values of thermal radiation parameter. Due to radiation absorption, the thermal boundary layer thickness increases as the radiation parameter increases.

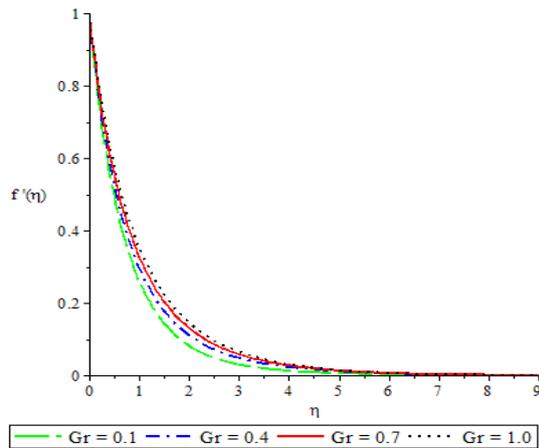


Figure 2: Variation of $f'(\eta)$ with η for several values of Grashof number

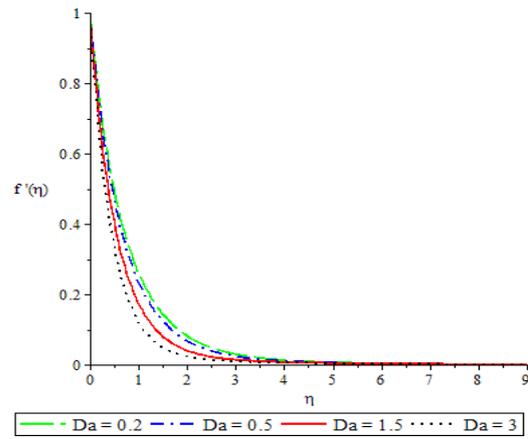


Figure 3: Variation of velocity $f'(\eta)$ with η for several values of Darcian parameter Da

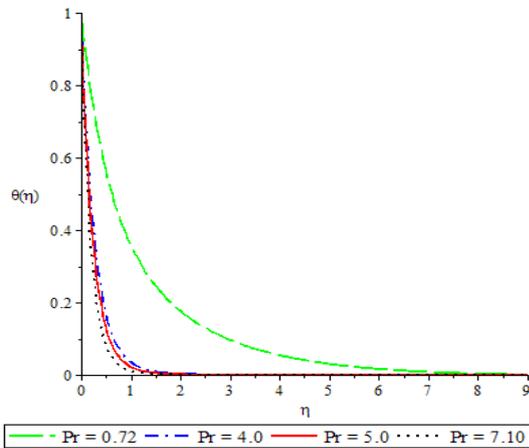


Figure 4: Variation of temperature $\theta(\eta)$ with η for several values of Prandtl number Pr

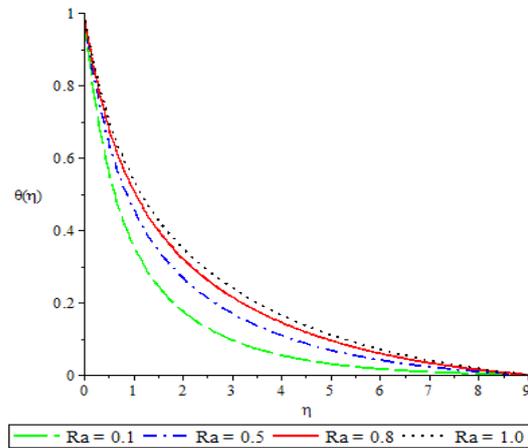


Figure 5: Variation of temperature $\theta(\eta)$ with η for several values of radiation parameter Ra

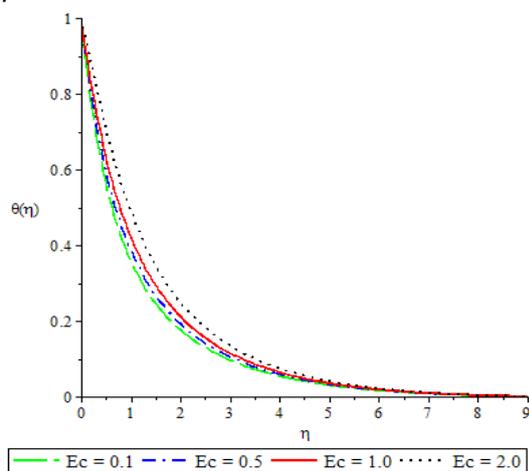


Figure 6: Variation of temperature $\theta(\eta)$ with η for several values of Eckert Number Ec

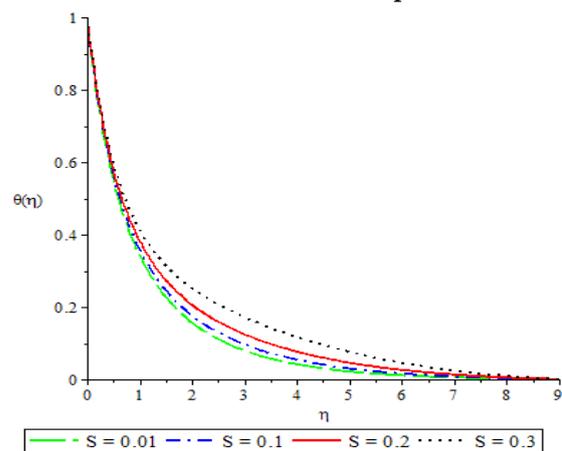


Figure 7: Variation of temperature $\theta(\eta)$ with η for several values of heat generation parameter S

Figure 6 illustrates the effect of Eckert number Ec in the thermal boundary layer. From this Figure, it is noticed that Eckert number increased the temperature profile. The heat generation parameter increased both the temperature and thermal boundary layer significantly. This is seen in Figure 7.

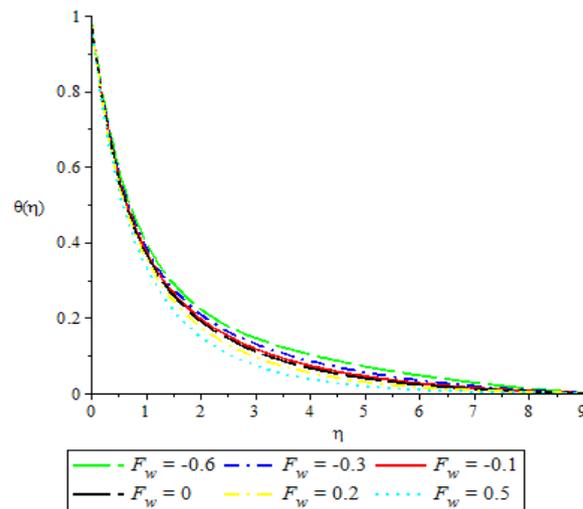
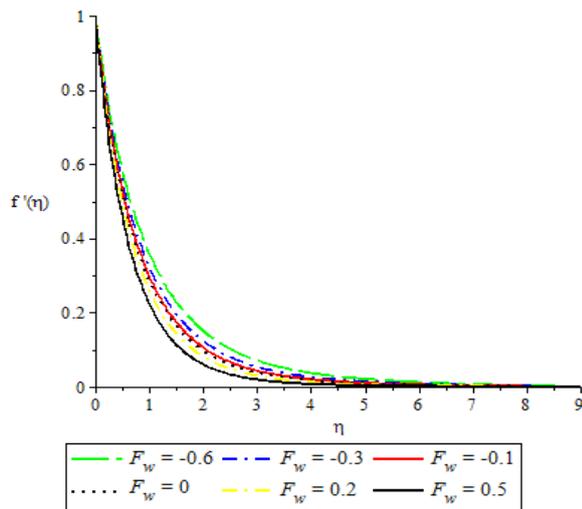


Figure 8.a: Variation of velocity $f'(\eta)$ with η for several values of suction/injection F_w

Figure 8.b: Variation of temperature $\theta(\eta)$ with η for several values of suction/injection F_w

Figures 8.a and 8.b depict the effects of suction ($F_w > 0$) and injection ($F_w < 0$) on both the velocity and temperature profiles respectively. The velocity was observed to decrease with increasing values of suction which also led to thinning of the momentum boundary layer thickness. However, an opposite trend was seen for fluid injection (Figure 8.a). The case for $F_w = 0$ indicates a non-porous stretching sheet. As it can be observed, its temperature profiles follow the same pattern as that of the velocity.

4. CONCLUSIONS

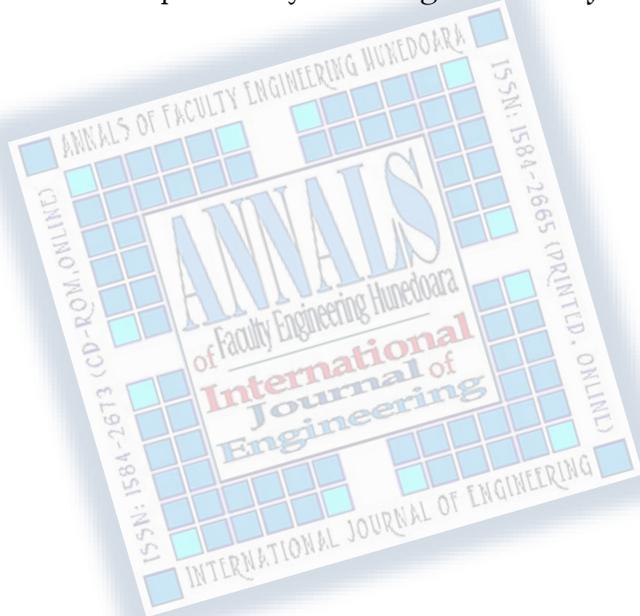
Magnetohydrodynamic boundary-layer flow and heat transfer characteristic due to exponentially stretching permeable vertical sheet with viscous dissipation, Joulean and viscous heating effects in the presence of thermal radiation, fluid suction/injection and heat generation/absorption has been investigated in this paper and the following were our discoveries:

- = The surface shear stress could be decimated gradually in absolute values when the values of Grashof number, radiation parameter, Eckert number and heat generation parameter were increased but the parameter of permeability, fluid suction and magnetic parameters intensify it.
- = An increase in the magnetic or Darcian parameter leads to increase in wall temperature but retardation in the momentum transport rate.
- = The effect of suction parameter on a viscous incompressible fluid is to suppress the velocity field and as a consequence, a reduction in the momentum boundary-layer thickness.
- = The thermal boundary layer thickens with increasing values of Eckert number, heat generation, thermal radiation and Magnetic parameters unlike the Prandtl number.

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