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TEMPERATURE DISTRIBUTION ALONG A STRAIGHT BAR STICKING OUT FROM A HEATED PLANE SURFACE AND THE HEAT FLOW TRANSMITTED BY THIS BAR (I) – THEORETICAL APPROACH

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ABSTRACT: The author, based on the mentioned references, started to improve the now-today applied relations on the heat distribution problem. The paper offers a better approach concerning not only on the temperature distribution along a straight bar, heated in its lower end, but also on the output heat flow evaluation, too. Contrary to the general accepted (but unfortunately inaccurate in the most cases) hypothesis of the constant (invariable) value of the parameter m, defined in relation (7), the author propose a new and practical numerical evaluation of this parameter, based on an original experimental strategy. One other significant contribution of the present paper consists in an original methodology, concerning on the $\alpha_n(z)$ local heat transfer coefficient calculation for the non-isothermal bar. In the next period the author intend to focus his theoretical and experimental investigations on the improvement of the above-mentioned m parameter determination, both for small and large values of the so-called massivity, denoted by the ratio P/A.
Keywords: heat distribution, straight bar, heat flow, local heat transfer coefficient, massivity

1. THEORETICAL APPROACH

Let us consider (see Figure 1, a) a straight bar 2, having constant A cross-sectional area with perimeter/circumference P and length ℓ, connected - along the normal direction - to the plane surface S of the body B at the level Ao. The origin O of the axes z_1, z_2 represents the centre of gravity of the cross-sectional area A of the bar. The axis z_1 correspond to the negative direction of the gravitational acceleration vector $\vec{g}$, the z_2 represents the longitudinal direction of the bar and their angular disposition is denoted by α_g. The temperature of the body's surface S is t_S, of the surface Ao is t_O, respectively of the undisturbed ambient is t_a. The body B and the bar 2 are in thermal contact each to other and with the surrounding ambient, too.

Well-known facts are the following:

≡ when $t_S \rangle t_a$ than the body transfers heat to the ambient, and

≡ when $t_S \langle t_a$ than the body receives heat from the ambient.

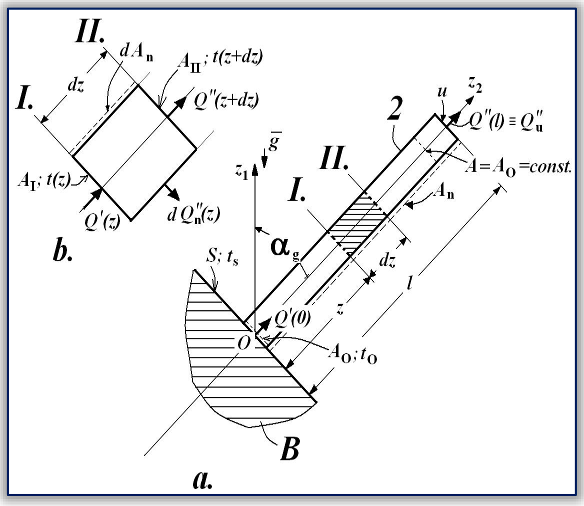

Figure 1. The conceptual schema

In the following, the first case ($t_s \rangle t_a$) will be considered, when the body B will transfer heat to the ambient in two ways:

≡ By a direct transfer through the free surface $S_f = S - A_O$ of the body, (having a free heat convection flow with given velocity and temperature fields), as well as

≡ By entering on the surface A_O of the lower end of the bar and after than by:

  ✓ Passing through of the bar and leaving/outrunning at its free upper end A_u, having magnitude of $Q''_u \equiv Q''(\ell)$ and

  ✓ Passing through the surface A_n of the bar's nappe, with a magnitude of Q''_n.

In the mentioned references one can find formulas concerning both the temperature and the output heat flow evaluation of the bar too, based on the following hypothesises:

≡ One has a long-enough bar;

≡ The temperature t of the bar, respectively of the heat transfer coefficient α_n of its nappe depend exclusively of the coordinate, i.e.: $t(z)$ and $\alpha_n(z)$.

Considering the elementary bar from Figure 1b located at distance z from the lower end of the bar and having length dz, one can obtain successively:

≡ the input heat flow $Q'(z)$ at the surface A_I

$$Q'(z) = -\lambda \cdot A \cdot \frac{dt(z)}{dz}\,; \tag{1}$$

≡ the outrunning heat flow $Q''(z+dz)$ through the surface A_{II}

$$Q''(z+dz) = -\lambda \cdot A \cdot \frac{dt(z+dz)}{dz}\,; \tag{2}$$

≡ the outrunning heat flow $dQ''(z)$ through the surface dA_n

$$dQ''_n(z) = \alpha_n \cdot dA_n \cdot \left[t(z) - t_a\right], \tag{3}$$

where: λ is the thermal conductivity of the bar; α_n - The heat transfer coefficient of the bar's nappe.

Based on the principle of the energy conservation ($\sum Q' = \sum Q''$), as well as on the above-mentioned equations, one can obtain successively

$$Q'(z) = Q''(z+dz) + dQ''_n(z);$$

$$-\lambda \cdot A \cdot \frac{dt(z)}{dz} = -\lambda \cdot A \cdot \frac{dt(z+dz)}{dz} + \alpha_n \cdot dA_n \cdot \left[t(z) - t_a\right]. \tag{4}$$

One can consider:

$$t(z+dz) = t(z) + \frac{dt(z)}{dz} \cdot dz\,,$$

From where:

$$\frac{dt(z+dz)}{dz} = \frac{dt(z)}{dz} + \frac{d^2t(z)}{dz^2} \cdot dz\,,$$

and:

$$dA_n = P \cdot dz\,. \tag{5}$$

By substituting them into the equation (4), one obtains

$$\frac{d^2t(z)}{dz^2} = \frac{P}{A} \cdot \frac{\alpha_n}{\lambda} \cdot \left[t(z) - t_a\right]. \tag{6}$$

Supplementary, by introducing the used symbol in literature (see the mentioned references)

$$m = \sqrt{\frac{P}{A} \cdot \frac{\alpha_n}{\lambda}}\,, \tag{7}$$

The equation (6) becomes

$$\frac{d^2t(z)}{dz^2} = m^2 \cdot \left[t(z) - t_a\right]. \tag{8}$$

Considering the relative temperature of the bar

$$t_r(z) = [t(z) - t_a], \quad (9)$$

The equation (8) becomes

$$\frac{d^2 t_r(z)}{dz^2} = m^2 \cdot t_r(z), \quad (10)$$

Because the derivative of the ambient temperature is $\frac{dt_a}{dz} = 0$.

Taking into the consideration the $m = const.$ hypothesis, the general solution of the equation (10) will be

$$t_r(z) = t_e(z) = c_1 \cdot e^{m \cdot z} + c_2 \cdot e^{-m \cdot z}, \quad (11)$$

Where: the sub-index e denote the estimated value and

c_1, c_2 are constants, which have to be determined by imposing adequate boundary conditions.

In the case of an ideal heat connection between the bar 2 and the body B, the above-mentioned conditions become:

$$\equiv \text{ for } z = 0: \; t(0) = t_s; \; t_e(0) = t(0) - t_a = t_s - t_a; \; t_{s,r} = t_s - t_a; \; c_1 + c_2 = t_{s,r}; \quad (12)$$

$$\equiv \text{ for } z = \ell: \; \lambda(\ell) \cdot \left.\frac{dt_e}{dz}\right|_{z=\ell} = \alpha_u \cdot t_e(\ell), \quad (13)$$

where: $\lambda(\ell)$ represents the thermal conductivity coefficient of the bar at its upper end;

α_u - The heat transfer coefficient of the bar at its upper end.

From the equation (11), by introducing the notation

$$c = e^{2 \cdot m \cdot \ell} \cdot \frac{\lambda(\ell) \cdot m + \alpha_u}{\lambda(\ell) \cdot m - \alpha_u}, \quad (14)$$

The boundary conditions (13) became

$$c_2 = c \cdot c_1. \quad (15)$$

Finally, from equations (12) and (15), one can obtain

$$c_1 = t_{s,r} \cdot \frac{1}{1+c}; \quad (16)$$

$$c_2 = t_{s,r} \cdot \frac{c}{1+c}. \quad (17)$$

2. METHODS

There are two methods in order to determine the outrunning heat flow from the bar.

a. The first one is based on the fact that the outrunning heat flow should be equal with the input one $Q'(0)$, which enters at the lower end level of the bar (through the section A_O):

$$Q'(0) = -\lambda(t_s) \cdot A \cdot \left.\frac{dt}{dz}\right|_{z=0}. \quad (18)$$

Considering

$$\left.\frac{dt(z)}{dz}\right|_{z=0} = \left.\frac{dt_e(z)}{dz}\right|_{z=0} = m \cdot (c_1 - c_2), \quad (19)$$

Relation (18) became

$$Q'(0) = m \cdot (c_2 - c_1) \cdot \lambda(t_s) \cdot A. \quad (20)$$

b. The second method requires that the outrunning heat flow from the bar, by heat transfer, can be established using the relation

$$Q'' = Q''_u + Q''_n, \quad (21)$$

where

$$Q''_u = \alpha_u \cdot A \cdot t_e(\ell) \quad (22)$$

Represents the amount of the outrunning heat flow by heat transfer at the upper end of the bar;

$$Q''_n = \iint_{A_n} \alpha_n(z) \cdot t_e(z) \cdot dA_n \tag{23}$$

Is the amount of the heat flow passing through the bar's nappe by heat transfer;
α_u - The local heat transfer coefficient at the upper end of the bar;
$\alpha_n(z)$ - The local heat transfer coefficient of the nappe of the bar.
Considering relation (5), relation (23) became

$$Q''_n = P \cdot \int_0^{\ell} \alpha_n(z) \cdot t_e(z) \cdot dz\ . \tag{24}$$

The author, based on his investigations in the literature, didn't find any result or references concerning on the $\alpha_n(z)$ local heat transfer coefficient calculation for the non-isothermal bar.
Supposing an $m = \text{const.}$, from equation (7) can be expressed

$$\alpha_n(z) = m^2 \cdot \frac{A}{P} \cdot \lambda, \tag{25}$$

And consequently the equation (24) became

$$Q''_n = m^2 \cdot A \cdot \int_0^{\ell} \lambda(z) \cdot t_e(z) \cdot dz\ . \tag{26}$$

Also, the $\lambda(z)$ thermal conductivity coefficient of the bar can be expressed

$$\lambda(z) = a' + b' \cdot t(z) + c' \cdot t^2(z)\ , \tag{27}$$

where the coefficients a', b', c' depend on the quality of the bar's material, which can be determined applying the Minimal Square Errors Method (MSEM) on a set of numerical data-pairs from Thermo-dynamical tables

$$\{t(i),\ \lambda(i)\},\ \text{for } i \in [1,\ n].$$

Introducing the function

$$f(z) = \lambda(z) \cdot t_e(z), \tag{28}$$

Relation (26) became

$$Q''_n = m^2 \cdot A \cdot \int_0^{\ell} f(z) \cdot dz\ . \tag{29}$$

Taking into the consideration that

$$t(z) = t_e(z) + t_a, \tag{30}$$

The relation (28) finally became

$$\begin{aligned} f(z) &= (a' + b' \cdot t_a + c' \cdot t_a^2) \cdot t_e(z) + (b' + 2 \cdot c' \cdot t_a) \cdot t_e^2(z) + c' \cdot t_e^3(z) = \\ &= a'_0 \cdot t_e(z) + b'_0 \cdot t_e^2(z) + c'_0 \cdot t_e^3(z). \end{aligned} \tag{31}$$

Also, one has to take into consideration that $t_e(z)$ is given by the relation (11) and its second and third powers can also be expressed, consequently, the function (31) finally will obtain the expression

$$\begin{aligned} f(z) = {} & c'_0 \cdot c_2^3 \cdot e^{-3 \cdot m \cdot z} + b'_0 \cdot c_2^2 \cdot e^{-2 \cdot m \cdot z} + \left(a'_0 \cdot c_2 + 3 \cdot c'_0 \cdot c_1 \cdot c_2^2\right) \cdot e^{-m \cdot z} + \\ & + 2 \cdot b'_0 \cdot c_1 \cdot c_2 + \left(a'_0 \cdot c_1 + 3 \cdot c'_0 \cdot c_1^2 \cdot c_2\right) \cdot e^{m \cdot z} + b'_0 \cdot c_1^2 \cdot e^{2 \cdot m \cdot z} + c'_0 \cdot c_1^3 \cdot e^{3 \cdot m \cdot z}. \end{aligned} \tag{31}$$

In order to establish the final expression of relation (29), one has to determine the integral

$$\int_0^{\ell} f(z) \cdot dz = a_1 \cdot \left(\frac{1}{\chi^3} - 1\right) + a_2 \cdot \left(\frac{1}{\chi^2} - 1\right) + a_3 \cdot \left(\frac{1}{\chi} - 1\right) + a_4 \cdot \ell + a_5 \cdot (\chi - 1) + a_6 \cdot (\chi^2 - 1) + a_7 \cdot (\chi^3 - 1), \tag{32}$$

where the introduced notations have the following expressions:

$$a_1 = -\frac{c'_0 \cdot c_2^3}{3 \cdot m};\ a_2 = -\frac{b'_0 \cdot c_2^2}{2 \cdot m};\ a_3 = -\frac{a'_0 \cdot c_2 + 3 \cdot c'_0 \cdot c_1 \cdot c_2^2}{m};\ a_4 = 2 \cdot b'_0 \cdot c_1 \cdot c_2\ ;$$

$$a_5 = \frac{a'_0 \cdot c_1 + 3 \cdot c'_0 \cdot c_1^2 \cdot c_2}{m};\ a_6 = \frac{b'_0 \cdot c_1^2}{2 \cdot m};\ a_7 = \frac{c'_0 \cdot c_1^3}{3 \cdot m};\ \chi = e^{m \cdot \ell}\ .$$

3. FINAL REMARKS AND CONCLUSIONS

In the mentioned literature, the relation (20) is applied, but the method, based on the relation (21), is missed.

Another missed item in the used literature is the numerical determination of the parameter m, which query definitively the practical use of the mentioned theoretical approach (20).

The hypothesis of $m = \text{const.}$ along the constant cross-sectional bars one can be accepted only for the relatively small values of the ratio $\frac{P}{A}$, namely for the solid/massive cross-sections (e.g.: circular cross-section).

Oppositely, for the tubular cross-sections, where the ratio $\frac{P}{A}$ presents much higher values, the above-mentioned hypothesis of $m = \text{const.}$ one has to be verified by further searching examination. This case is widely used in civil engineering structures and the fire-safety analysis of them represents a very important topic of the engineers.

Consequently, the further goals of the author consist in:

- ≡ the experimental establishing of the bar's temperature field (in several discrete points, along its longitudinal axis);
- ≡ deeply analysis even the condition $m = \text{const.}$ can be accepted in every particular case, namely if for every case one can find precise value for m, which can assure a suitable temperature estimation of the bar, based on the relation $t(z) = t_e(z) + t_a$, in comparison with the effective measured values;
- ≡ establishing useful correlations between the magnitudes of the ratio $\frac{P}{A}$ and of the parameter m along the bar;
- ≡ statistically accepted large number of temperature measurements, destined to estimate the probable law of the parameter m, corresponding to factual cases of bars and depending on several environmental parameters.

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