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# EXPERIMENTAL EVALUATION OF NONLINEAR BEHAVIOR OF THE SOIL NATURAL PERIOD BY RESONANT COLUMN TESTS

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**ABSTRACT**: This paper proposes an evaluation method for nonlinear behavior of the site natural period assuming the site materials as nonlinear visco-elastic materials modeled with a nonlinear Kelvin-Voigt model (which includes dynamic stiffness degradation and increasing damping). By using resonant column tests we can quantify the nonlinear dependence of the site natural period in the normalized form $T_n = T_n(PGA)$ where $T_n = T_g / T_0$ and PGA is peak ground acceleration. Then, from "in situ" information we can obtain the normalization value $T_0$ and finally, the nonlinear site natural function result in the form: $T_g(PGA) = T_0 \cdot T_n(PGA)$ .

**Keywords**: resonant column, soil nonlinearity, soil dynamic degradation, nonlinear site natural period

## 1. INTRODUCTION

The site materials are nonlinear materials with a dynamic behavior strongly dependent of strain or loading level and this behavior affects the completely dynamic response including the system natural period values. [2, 4, 5, 6, 8, 11]. As a result, the site natural period $T_g$ becomes dependent on the earthquakes amplitude ( $M_{GR}$ - moment Gutenberg-Richter or PGA), and this dependence can be observed in the seismic records (Figure 1) [11, 12].

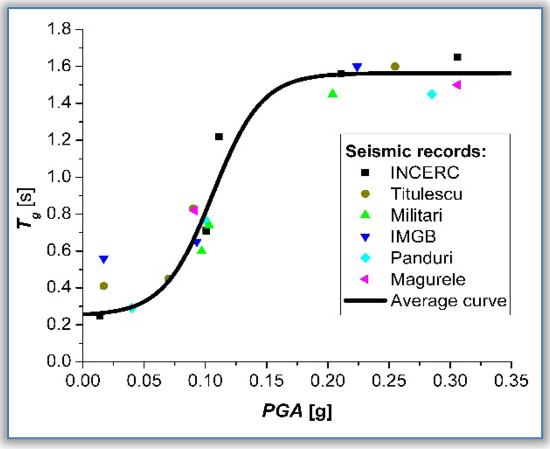


Figure 1: Nonlinear tendency of site natural periods

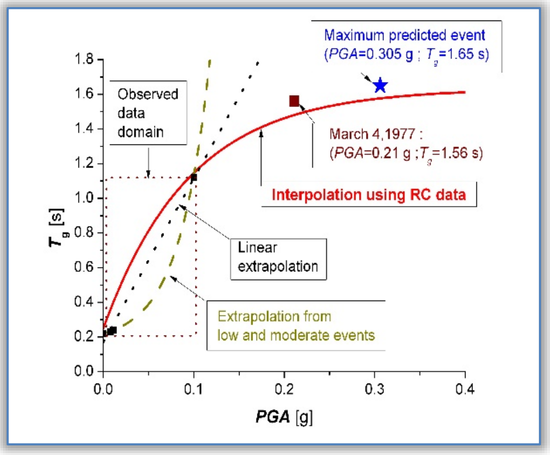


Figure 2 : Extrapolation versus interpolation

The direct evaluation of the nonlinear natural period functions in the form $T_g = T_g(M_{GR})$ or $T_g = T_g(PGA)$ is an adequate method but is not always possible. The seismic station network is not so expanded and only in a few stations, the recorded events are enough for determination of natural period functions with a reasonable precision. For a large majority of the usual sites only seismic recording of the low and moderate events are available. In these cases, the evaluation of the dominant period for strong earthquakes using only seismic low and moderate data presume an extrapolation, procedure with inherent large errors (Figure 2) [4, 7, 11].

The resonant column device can charge the soil specimen to a loading range equivalent to low until strong earthquakes and the nonlinear natural period dependence $T_n = T_n(PGA)$ can be obtained by means of an interpolation process with an upper accuracy. In case of only low and moderate seismic data are available, the resonant column determination by the interpolation of the nonlinear variations in normalized form: $T_n = T_n(PGA)$ together with the determination of the normalization value $T_0$ from seismic recording can leads to a better approximation of the natural periods for large *PGA* values (Figure 2) [4, 13].

To facilitate the connection between RC results and seismic records it is necessary to determine the normalized natural period in function of loading amplitude usually described by *peak ground acceleration* (*PGA*). For this conversion - $T_n = T_n(\gamma)$ into $T_n = T_n(PGA)$ - we use the numerical simulation of the RC sample behavior, modeled as nonlinear Kelvin-Voigt model subjected to abutment motion $\ddot{x}_g(t) = \ddot{x}_g^0 \sin\omega t$ with different acceleration amplitude $\ddot{x}_g^0$ [6, 7, 9].

The initial value $T_0$ must be obtained from seismic measurements. When $T_0$ value is difficult to obtain from processing of seismic records one can use any known pair of values $\left(T_g^{kn}, PGA^{kn}\right)$. In this case, the $T_0$ is obtained by "translational motion" of the normalized RC curve $T_n = T_n(PGA)$ in this known "point" of the $\left(T_g, PGA\right)$ space [6].

## 2. RESONANT COLUMN TEST

The resonant column apparatus was designed for laboratory determination of the dynamic response of soils by the means of the propagating steady-harmonic shear or longitudinal waves in a cylindrical soil specimen (*column*) under *resonant* frequency conditions [4, 13]. Usually, under harmonic torsional inputs with different amplitudes $M^i = M_0^i \sin\omega t$ we can obtain the corresponding strain level $\gamma_i$, the modulus-function value $G^i$ and damping value $\zeta^i$. The shear-modulus value $G^i$ is obtained using the relationship:

$$G^i = \rho\left(v_s^i\right)^2 = \rho\left(\frac{\omega_0^i h}{\Psi}\right)^2 \quad \text{with} \quad \Psi = \sqrt{R - \frac{1}{3}R^2 + \frac{4}{45}R^3} \tag{1}$$

where ρ is the mass density of specimen, $v_s^i$ and $\omega_0^i$ are the shear wave velocity and the specimen natural frequency at level *i*, *h* is the specimen height and $\Psi$ is the root of torsional frequency equation with analytical form in terms of the ratio *R* between torsional inertia of the specimen and the torsional inertia of the vibrator: $R = J / J_v$. After several tests with different strain level $\gamma_i$ $(i = 1, 2 \ldots n)$ the shear-modulus function $G = G(\gamma)$ and the damping function $\zeta = \zeta(\gamma)$ can be obtained [4, 8, 13].

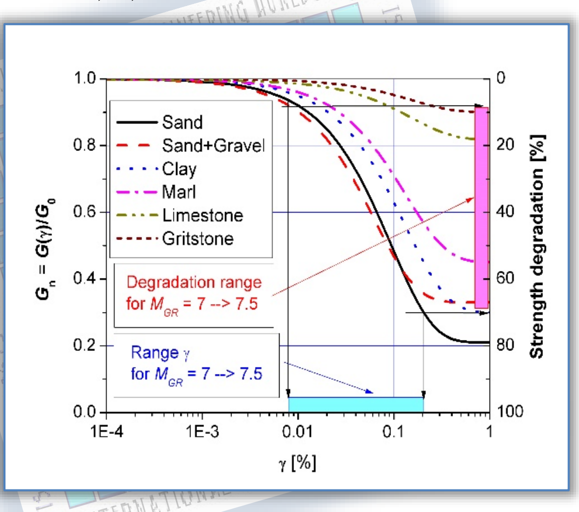


Figure 3 : Strength degradation

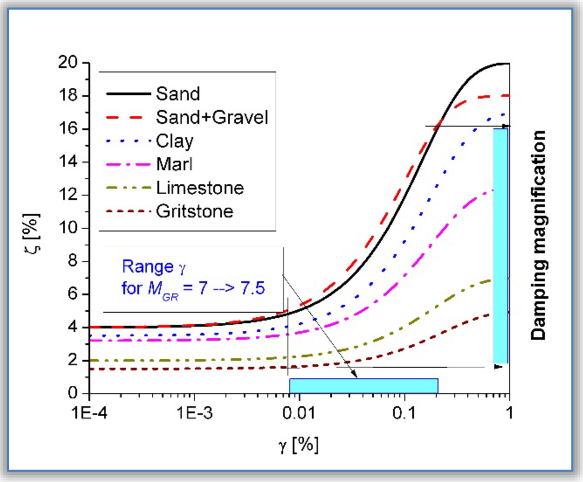


Figure 4 : Damping magnification

In order to exemplify the nonlinear behavior of the usual site materials, in Figures 3 and 4 some nonlinear material functions obtained from RC tests performed upon sand, sand+gravel, marl, limestone and gritstone sample are given. It is observed that when the external loads are increasing the rigidity is reduced, due to the dynamic degradation effect and the material damping increases [4, 6].

## 3. NATURAL PERIOD OF RESONANT COLUMN SPECIMEN

As can be seen from Figure 1, dynamic degradation leads to a significant reduction in stiffness of site materials. So, we expect that natural period to grow in the same proportion and soil-structure system becomes a nonlinear oscillating system dependent on the level of deformation, stress or loading input. The specimen natural period for a level $\gamma_i$ is:

$$T^i = \frac{2\pi}{\omega_0^i} = \frac{2\pi h\sqrt{\rho}}{\Psi} \cdot \frac{1}{\sqrt{G^i}} \tag{2}$$

and using eq.(1), *the nonlinear natural period function* of the soil specimen can results in the form:

$$T_g(\gamma) = T_0 \cdot T_n(\gamma) \quad \text{with:} \quad T_0 = T_g(\gamma)\big|_{\gamma=0} = \frac{2\pi h\sqrt{\rho}}{\Psi} \cdot \frac{1}{\sqrt{G_0}} \quad \text{and} \quad T_n(\gamma) = \frac{T_g(\gamma)}{T_0} = \frac{1}{\sqrt{G_n(\gamma)}} \tag{3}$$

We mention that the natural periods obtained by RC test in the form (3) is the natural periods of the single degree of freedom oscillating system composed by a single mass (the vibration device) supported by a spring and a damper represented by the specimen. This system is much different in comparison with site-structure system. But, as can see from eq. (3) the physical and geometrical sample properties ($h$, $\rho$, $J$, $J_V$) are included only in the initial value $T_0$. Thus, the resonant column tests can offer accurate data for obtaining only the nonlinear dependence of the normalized natural period $T_n = T_n(\gamma)$ and the normalization values $T_0$ must be determined using seismic recordings.

## 4. THE NATURAL PERIOD OF THE SAMPLE DEPENDING ON LOADING

To facilitate the connection with seismic records it is necessary to determine the normalized natural period in function of loading amplitude usually described by *PGA*. For this conversion - $T_n = T_n(\gamma)$ into $T_n = T_n(PGA)$ - we can use the numerical simulation of the RC sample behavior, modeled as nonlinear Kelvin-Voigt model subjected to abutment motion $\ddot{x}_g(t) = \ddot{x}_g^0 \sin\omega t$ with different acceleration amplitude $\ddot{x}_g^0$. In this loading case, the motion equation reads as [2, 3, 4, 6, 9]:

$$\ddot{x} + 2\omega_0\zeta(x)\cdot\dot{x} + \omega_0^2 G_n(x)\cdot x = -\ddot{x}_g^0 \sin\omega t \tag{4}$$

Using the change of variable $\tau = \omega_0 t$ and introducing a new time function $\varphi(\tau) = x(t) = x(\tau/\omega_0)$ we can obtain the dimensionless form of the eq. (4) [3]:

$$\varphi'' + C(\varphi)\cdot\varphi' + K(\varphi)\cdot\varphi = \mu \sin\upsilon\tau \tag{5}$$

where the superscript accent denotes the time derivative with respect to $\tau$, and:

$$C(\varphi) = \frac{c(x)}{m\omega_0} = 2\zeta(x) \quad ; \quad K(\varphi) = \frac{k(x)}{m\omega_0^2} = G_n(x) \quad ; \quad \mu = \frac{\ddot{x}_g^0}{\omega_0^2} = x_{static} \quad ; \quad \upsilon = \frac{\omega}{\omega_0} \tag{6}$$

The steady-state solution of the equation (5) can be numerically obtained using a computer program based on Newmark algorithm [3, 7].

The solution can be written in the form: $\varphi(\tau,\upsilon,\mu) = \mu\Phi(\upsilon,\mu)\sin(\upsilon\tau-\psi)$, where $\Phi(\upsilon,\mu)$ is *the nonlinear magnification function:*

$$\Phi(\upsilon,\mu) = \frac{\max_{\tau}\left[\varphi(\tau,\upsilon,\mu)\right]}{\mu} = \frac{x_{dynamic}}{x_{static}} \tag{7}$$

a ratio of maximum dynamic amplitude $\varphi_{\max} \equiv x_{dynamic}$ to static displacement $\mu = x_{static}$.

By numerical simulations with different values of normalized loading amplitudes $\mu$ we can obtain a set of nonlinear magnification functions $\Phi(\upsilon) = \Phi(\upsilon;\mu)\big|_{\mu=ct.}$ Because $\mu = \ddot{x}_g^0/\omega_0^2 = (g\cdot PGA)/\omega_0^2$, $\mu$ can be replaced by *PGA* and because $\upsilon = \omega/\omega_0 = T_0/T = 1/T_n$, $\upsilon$ can be replaced by $T_n$ (Figure 5).

It follows a series of values $(T_n - PGA)$ from which we obtain a relationship $T_n = T_n(PGA)$ (Figure 6) [2, 4, 6].

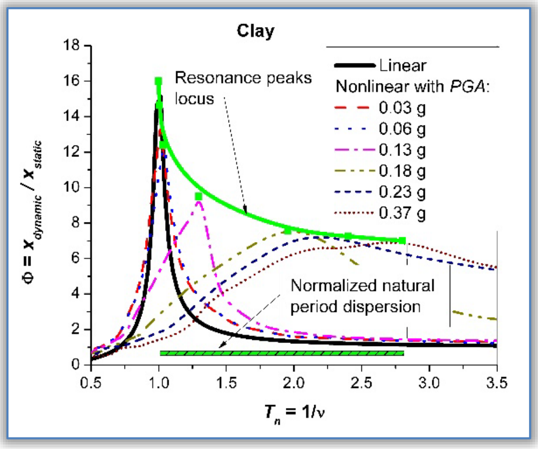


Figure 5 : Nonlinear magnification functions in terms of normalized periods (for a clay specimen)

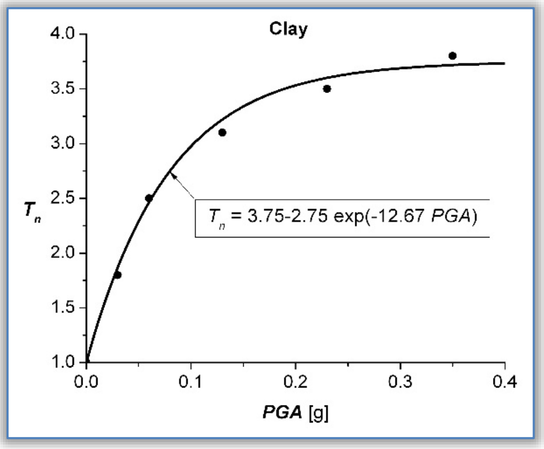


Figure 6 : Relationship $T_n = T_n(PGA)$

## 5. NONLINEAR NORMALIZED NATURAL PERIOD OF THE SITE

For evaluation of the normalized natural periods for entire site first one must determine by RC tests the nonlinear variation $T_n^i$ for each site stratum, and then one can obtain the average natural period variation for the entire site layers $T_n^{av}$ as the average of the strata normalized natural period $T_n^i$ weighted with its thickness $h_i$ [6, 12]:

$$T_n^{av} = \left(\sum T_n^i \times h_i\right) / \sum h_i \tag{8}$$

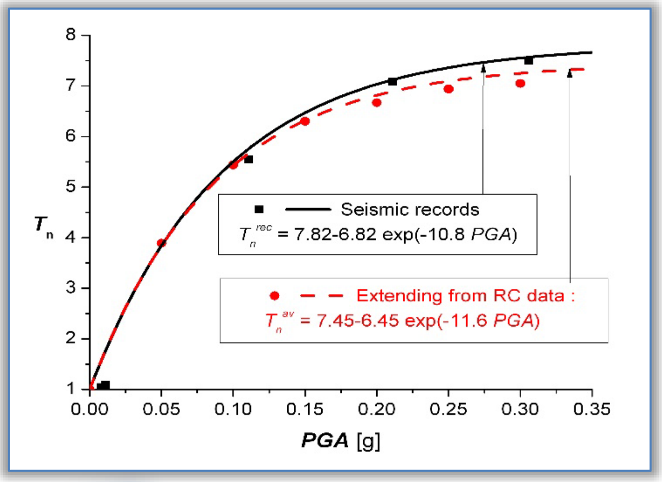


Figure 7: Dependence $T_n - PGA$ provided both RC data and seismic records

This method was validated using the site emplacement of the seismic station INCERC with known stratification [1]. First, for each constituent layer the material functions $G = G(\gamma)$ and $\zeta = \zeta(\gamma)$ were estimated and by numerical simulation, some functions $T_n^i = T_n^i(PGA)$ one for each stratum $i$ was obtained.

Then, for some $PGA$ values (0.05, 0.10, 0.15, 0.20, 0.25 and 0.30 g) using eq. (8) the site natural period values $T_n^{av}\big|_{PGA=ct.}$ was obtained and by statistical fit from these period values a normalized averaged natural period function $T_n^{av} = T_n^{av}(PGA)$ results [6].

In Figure 7 the validation result is given, by comparison between nonlinear function $T_n^{av} = T_n^{av}(PGA)$ obtained with the aid of RC data and the same function directly obtained from seismic measurements $T_n^{rec} = T_n^{rec}(PGA)$ [11]. As can see from this figure the differences between seismic records and resonant column simulations are acceptable.

## 6. $T_0$ ESTIMATION FROM SEISMIC RECORDS

We remember that only RC data are not enough for complete determination of the natural period function $T_g(PGA) = T_0 \cdot T_n(PGA)$ and besides of the normalized function $T_n^{av} = T_n^{av}(PGA)$ given by the RC data, the initial value $T_0$ obtained from seismic measurements it is necessary.

When $T_0$ value is not available or is too difficult to obtain from processing of seismic records one can use any known pair of values $\left(T_g^{kn}, PGA^{kn}\right)$. In this case, the $T_0$ can be obtained by "translational motion" of the normalized resonant column curve $T_n^{av} = T_n^{av}(PGA)$ in any known "point" $\left(T_g^{kn}, PGA^{kn}\right)$ of the $\left(T_g, PGA\right)$ space:

$$T_0 = \frac{T_g^{kn}\Big|_{PGA=PGA^{kn}}}{T_n^{av}\Big|_{PGA=PGA^{kn}}} \tag{9}$$

Finally, the calculated form of the site natural period function becomes:

$$T_g^{calc}(PGA) = T_0 \cdot T_n^{av}(PGA) \tag{10}$$

The validation of this method can be done by comparison between $T_g^{calc}$ and $T_g^{rec}$ curves both obtained from the same site. Thus, in Figure 8 such comparison is given using the laboratory and in situ data for INCERC site. The calculated curve $T_g^{calc} = T_g^{calc}(PGA)$ was obtained by translational motion of the normalized curve $T_n^{av} = T_n^{av}(PGA)$ into the 1977 earthquake "point" $\left(PGA = 0.21\text{g} \ ; \ T_g = 1.56\text{s}\right)$ and the recorded curve $T_g^{rec} = T_g^{rec}(PGA)$ was obtained directly from seismic measurements processing [6].

In addition, the translational motion can be done in any measurement points. Thus, in Figure 9 the normalized curve $T_n^{av} = T_n^{av}(PGA)$ was moved in three known points of the strong events: 1977 point $\left(PGA = 0.21\text{g} \ ; \ T_g = 1.56\text{s}\right)$, 1986 point $\left(PGA = 0.11\text{g} \ ; \ T_g = 1.22\text{s}\right)$ and in the point $\left(PGA = 0.306\text{g} \ ; \ T_g = 1.65\text{s}\right)$ corresponding to maximum predicted event. [ ]. In all these cases the differences between calculated and measured curves was reasonable: $\Delta T_g = \left|T_g^{rec} - T_g^{calc}\right| \le 0.1$ s [6] .

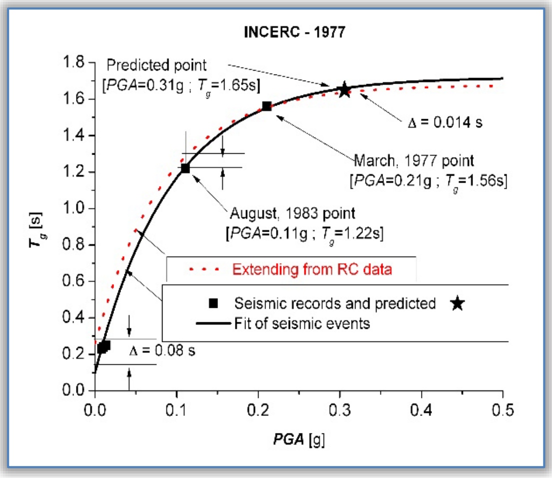


Figure 8 : Translational motion in the 1977 point

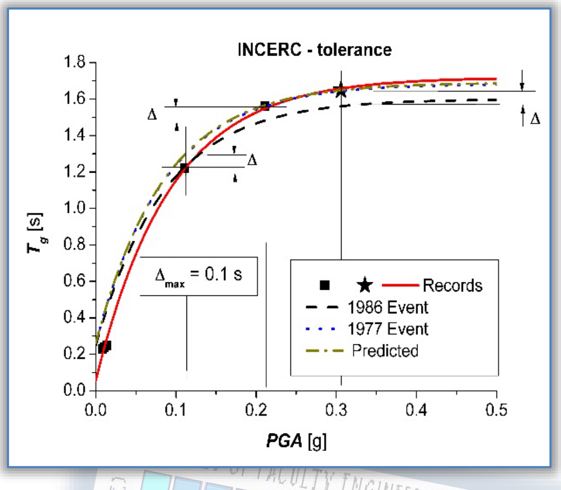


Figure 9 : Translational motion in some strong seismic points

## 7. CONCLUDING REMARKS

» The usual method for determining by calculus of the natural period consist by using "the quarter length formula" [12] ($T_g = 4H/v_s$) where $H$ is the total depth of layers and $v_s$ is the shear wave velocity. However, this formula treats the site as linear elastic space in contradiction with mechanical reality and gives a unique natural period value in contradiction with the earthquake recordings.

» The nonlinear natural site period - $T_g(PGA) = T_0 \cdot T_n(PGA)$ - can be obtained from recorded seismic data if these data cover the entire expected *PGA* value range.

» In default of complete and reliable site information, the site nonlinear natural period can be done by a combination of *in situ* data - $T_0$ - and RC data processing - $T_n(PGA)$.

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