ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering
Tome XIV [2016] – Fascicule 3 [August]
ISSN: 1584-2665 [print; online]
ISSN: 1584-2673 [CD-Rom; online]
a free-accessmultidisciplinarypublication
of theFaculty of Engineering Hunedoara

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SIMPLE METHOD OF IDENTIFICATION OF THE TWO INTEGRATION CONSTANTS FOR THE NUMERICAL DOUBLE INTEGRATION OF ACCELERATION

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ABSTRACT: This paper is intended to be a clarification, a technical support for those who might have difficulties when integrating twice the accelerations in order to obtain correct displacements, consistent with reality. Since the mounting of the displacement sensors on the dynamic system is not always easy, the possibility of obtaining the displacements from acceleration measurements which are usually much more affordable represents an interesting option. There are two possibilities: integrating analogically the accelerations using a performing data acquisition system (accelerometer, amplifier, oscilloscope), or the numerical double integration of the acceleration. This numerical integration is detailed here, presenting a simple searching method for determining the two a priori unknown integration constants $C_x(t_0)$ and $C_v(t_0)$ of the integration formula. Two practical case studies are illustrating the method for identifying these two integration constants. The second case study concerns the impact of a flat sample material with an impactor head.
Keywords: numerical integration of acceleration, displacement, integration constants, impact

1. INTRODUCTION

In practice, the displacements measurement and position determination using displacement sensors is not always easy to perform, since the mounting of these displacement sensors on the dynamic system is not always easy. Generally, in vibrations measurements the accelerations data are much more available, being provided by uniaxial or triaxial accelerometers. By using a data measurement/acquisition system capable to integrate analogically the accelerations, it is possible to obtain the velocities after a first analogical integration of the measured accelerations, then after a second analogical integration the displacements can finally obtained. The second possibility is to obtain numerically the displacement by double integration of the measured acceleration.

In the literature, this problem of analogical or numerical double integration of accelerations has been addressed in relation with various applications. For example in the field of seismological monitoring, Boore [1] showed how processing of strong-motion data can be in many cases "as straightforward as filtering the acceleration time series and integrating to obtain velocity and displacement. To avoid the introduction of spurious low-frequency noise in quantities derived from the filtered accelerations, however, care must be taken to append zero pads of adequate length to the beginning and end of the segment of recorded data. These padded sections of the filtered acceleration need to be retained when deriving velocities, displacements, Fourier spectra, and response spectra. In addition, these padded and filtered sections should also be included in the time series used in the dynamic analysis of structures and soils to ensure compatibility with the filtered accelerations" [1].

Huang, Liao and Zhao [2] presented a simple sensor-based pedestrian tracking technology, which uses the double integration of acceleration in the global coordinate system, after the acceleration of gravity component is filtered out, in order to track the position of a pedestrian. More precisely, the information about human walking is monitored by a sensor module composed of accelerometers, gyroscopes and magnetometers. Due to the fact that their low-cost sensors are susceptible to drift errors, an integral drift correction/reduction method is used, based on an algorithm for accurate fusion of acquired data.

In the field of monitoring structural deformation, in order to exploit the complementary characteristics of the GPS-RTK (Real Time Kinematic) and accelerometer sensors, Li, Rizos et al. [3] developed a new integration approach that employs the correlation signals directly detected from GPS-RTK and accelerometer to transform one form of measurement to another. More precisely, the forward transformation from displacement to acceleration is used, as well as the reverse transformation from acceleration to displacement (by applying the laws of motion through double integration). Their "methodology consists of Fast Fourier Transform for correlated signal identification, a filtering technique, delay compensation, and velocity linear trend estimation from both GPS and accelerometer measurements" [3].

2. PRACTICAL PROBLEM: DOUBLE INTEGRATION OF MEASURED ACCELERATIONS

In what concerns the determination of velocities and displacements from acceleration measurements, the data measurement/acquisition system, which comprises in our case a Brüel & Kjær accelerometer type 4368, a Brüel&Kjær amplifier type 2635 and a Velleman oscilloscope type PCSU1000, is usually able to integrate analogically the accelerations. Thus, starting from the measured accelerations, after a first analogical integration the velocities are obtained, then after a second analogical integration the displacements are finally obtained. As already mentioned, it is also possible to directly determine the displacements, experimentally, by using displacement sensors, but the mounting of these displacement sensors on the dynamic system is not always easy.

For the determination of velocities and displacements based only on measured accelerations, it exists also the possibility of numerical integration of the acceleration, in order to determine the velocities and then the displacements. The issue of numerical double integration of acceleration is addressed in this paper. Thus, when performing the double integration of the acceleration, the integration formula comprises two integration constants: a first integration constant $C_x(t_0)$ equal to the displacement at the initial time and a second integration constant $C_v(t_0)$ equal with the velocity at the initial time (it will appear in the displacement integral multiplied by $\Delta t = t - t_0$), as follows:

$$x(t) = \boldsymbol{C}_x(t_0) + \boldsymbol{C}_v(t_0) \cdot t + \int_{t_0}^{t} a(\tau) \cdot (t - \tau) \cdot \mathrm{d}\tau \cdot \qquad (1)$$

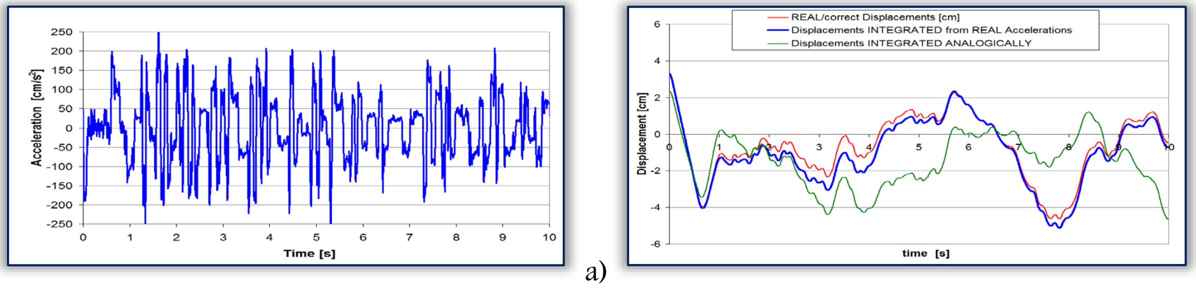
In the first case study presented in §3.1., based on some physical information concerning the velocities and displacements to be obtained, we will show how to reliably determine in practice these integration constants $C_x(t_0)$ and $C_v(t_0)$ from formula (1) of the double integration of acceleration.

3. CASE STUDIES

The significance of the two integration constants $C_x(t_0)$ and $C_v(t_0)$ in formula (1) is illustrated graphically in the first case study, while the second case study concerns the impact of a flat sample material with an impactor head, so in this case a more rapid dynamical evolution is subject to the numerical acceleration integration.

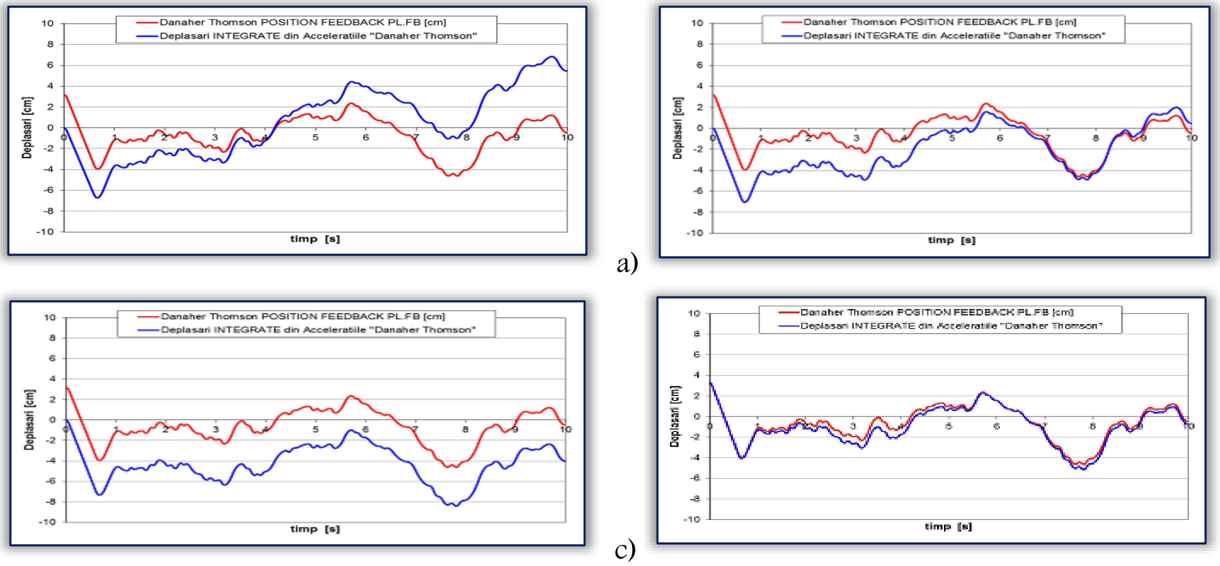
3.1. First integration of acceleration case study for the simulated vertical motion of the driver seat during a car ride [4]

Figure 1a shows the driver seat accelerations simulated using CARSIM [5], for a car riding on some normal random road profile [4] (there is no need of detailing here this random road profile). The same specialized CARSIM simulation software provides us also the vertical displacements of the driver seat corresponding to the accelerations from Figure 1a. Figure 1b shows in red the vertical displacements from the CARSIM simulation that will be called here "real displacements" even if we deal with a virtual CARSIM experiment/simulation and not with a real ride. Figure 1b shows also in blue the displacement obtained by the numerical double integration of the real accelerations (from Figure 1a), while in green is represented the displacement integrated analogically and provided by our data acquisition system (Brüel&Kjær accelerometer type 4368, a Brüel & Kjær amplifier type 2635 and a Velleman oscilloscope type PCSU1000). One can remark the good correspondence between the "real" displacement and the one obtained by our numerical double integration of "real" acceleration. The less good correspondence with the displacement integrated analogically by our data acquisition system can be explained by the fact that this data acquisition system is quite old and may work with inaccuracies.



a) b)

Figure 1: a) Driver seat accelerations for a car ride simulated using CARSIM [5]; b) Driver seat displacements: "real" ones, the ones integrated from the "real" accelerations and the ones integrated analogically by our data acquisition system



a) b) c) d)

Figure 2: Repeated attempts for identifying the two integration constants to obtain a correct displacement by double integration of acceleration: a) $C_x(t_0)=0$, $C_v(t_0)=0$ (step 1); b) $C_x(t_0)=0$, $C_v(t_0)=-0.5$ (step 2); c) $C_x(t_0)=0$, $C_v(t_0)=-0.95$ (step 2); d) $C_x(t_0)=3.3$, $C_v(t_0)=-0.95$ (step 4).

The question is how were determined the two integration constants $C_x(t_0)$ and $C_v(t_0)$ in formula (1), in order to obtain correctly the displacement by numerical double integration of acceleration? In our case, we tried several pairs of values until were found the correct two values of the integration constants. Of course, a more or less evolved searching algorithm can also be used for automatic computation of $C_x(t_0)$ and $C_v(t_0)$. In our case, the following four steps were performed when trying by repeated attempts to find the correct pair of $C_x(t_0)$ and $C_v(t_0)$ values:

- Step 1: simply initiate with $C_x(t_0)=0$, $C_v(t_0)=0$. Figure 2a shows the displacement numerically integrated for this case $C_x(t_0)=0$, $C_v(t_0)=0$;
- Step 2: firstly let us correct the "slope" of the obtained displacement curve, for which is responsible the constant $C_v(t_0)$. Figure 2b shows the displacement numerically integrated for $C_x(t_0)=0$, $C_v(t_0)=-0.5$. The slope starts to improve compared with Figure 2a, but still is not correct enough;
- Step 3: Figure 2c shows the displacement numerically integrated for $C_x(t_0)=0$, $C_v(t_0)=-0.95$. The slope of the displacement curve obtained by numerical double integration of acceleration is now quite correct;
- Step 4: finally let us correct also $C_x(t_0)$. Figure 2d shows the final correct displacement curve, resulting for the following integration constants: $C_x(t_0)=3.3$, $C_v(t_0)=-0.95$.

In conclusion of this first case study, the simple numerical double integration of acceleration has shown good performance and reliability. The result/obtained displacement is almost identical, as shown in Figure 2d, with the same displacement known from an independent measurement (in our case, reliable CARSIM simulation/virtual experiment).

So, by comparing/superposing the displacement obtained by the double integration of the acceleration and the displacement known from independent sources/measurements, one can adjust the first integration constant $C_x(t_0)$ as a measure of the offset on the ordinate axis between the averages of the two displacements. The second integration constant $C_v(t_0)$ can be adjusted as a measure of the slope of the median of the displacement curve. The graphical determination of the integration constants when computing the displacement by double integration of the acceleration, obviously based on some physical information concerning the displacement to be obtained, proved to be a reliable practical method. The graphical method presentation in this paper is intended to be a clarification, a technical support for those who might have difficulties when integrating twice the accelerations in order to obtain correct displacements, consistent with reality.

3.2. Second case study of numerical double integration of acceleration

The second case study concerns the impact of a flat sample material with an impactor head, during which the acceleration has been measured using a triaxial accelerometer, being shown in Figure 3a. The case study test corresponds to the impact of a spherical head with a mass of 4.6 kg, on a rubber mat used as damping protection for playgrounds.

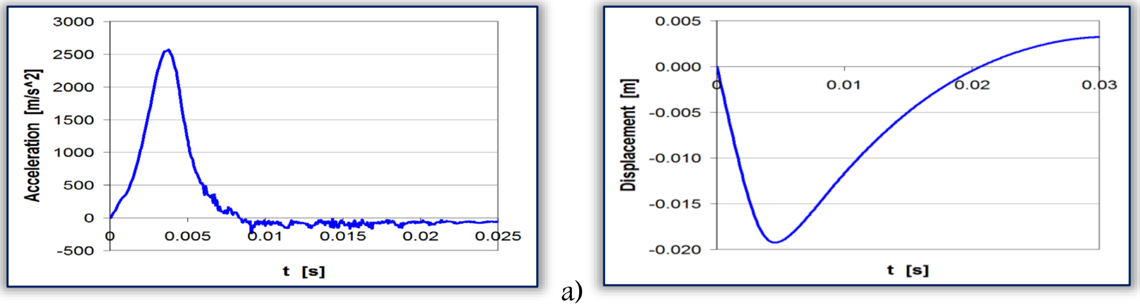


Figure 3: Impact of a flat rubber mat with a spherical impactor head: a) measured vertical acceleration; b) vertical displacement obtained by numerical double integration of the measured acceleration

Figure 3b shows the displacements obtained by double integration of the acceleration, using the integration method explained above. The force is also available as the mass of the impactor head multiplied by the acceleration. So, the force-displacement curve corresponding to an impact test can be obtained in this manner, only from measured accelerations data. Figure 4 shows this force-displacement curve for the impact of the rubber mat with the spherical impactor head, this force-displacement curve being obtained uniquely from the acceleration measurements.

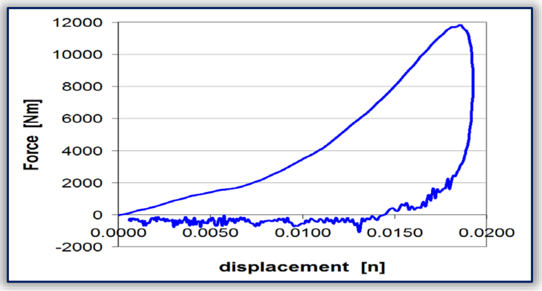


Figure 4: Force-displacement curve for the impact of a flat rubber mat with a spherical impactor head, obtained only from acceleration measurements

3. CONCLUSION

This paper presents a simple method of identification of the two integration constants for the numerical double integration of acceleration, which is exemplified on a first case study concerning the numerical integration of acceleration for the simulated vertical motion of the driver seat during a car ride. The searching method used is simple and reliable; it can be easily programmed for an automatic computation/determination of the two integration constants.

In the second case study, in order to obtain the force-displacement curve for an impact test of a flat sample material with an impactor head, theoretically both force transducer and displacement sensor measurements must be performed simultaneously, which is sometimes difficult to realize in practice.

But we have obtained this force-displacement curve only from acceleration measurements. More precisely, the displacements were obtained by double integration of the acceleration, while the force was also available as the mass of the impactor head multiplied by the vertical acceleration. So, the force-displacement curve corresponding to the impact of a rubber mat with a spherical impactor head has been obtained only from measured accelerations data.

This numerical method of double integration of measured accelerations for the case of impact tests or other rapid dynamical evolutions must be further confirmed in other case studies, before claiming its extended validity and reliability.

ACKNOWLEDGEMENT: Dr. Dan Dumitriu gratefully acknowledges the National Authority for Scientific Research (ANCS, UEFISCDI), for financial support through the PN-II-PT-PCCA-2011-3.1-0190 project, contract no. 149/2012.

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