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TENSORIAL ANALYSIS OF AUTOMOTIVE ENGINE FUNCTIONING BASED ON EXPERIMENTAL DATA

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ABSTRACT: The paper highlights main possibilies for the study of automotive engine functioning by using algorithms of tensorial calculus, as a superior level of matrix calculus. Therefore, there are presented main theoretical aspects of tensorial calculus which can be applied to the study on automobile engine functioning. There are applied those algoritms which allow to study automotive engine functioning based on experimental data obtained from tests. Within the paper, these algorithms are applied in case of automotives equipped with electronic control engine, thus in case of data taken from automotive on-board computer. To that effect there are being used elements towards tensors, spectral analysis and modal analysis, as well as multivariate mathematical models of engine functioning like PARAFAC, TUCKER, PARALIND etc.
Keywords: tensor analysis, spectral analysis, modal analysis, tensor decomposition, automobile engine

1. INTRODUCTION

For the study of automobile engine functioning there are applied generally used algorithms from systems and signals theory, but also algorithms from mathematical statistics [3]. In specialty literature, this study uses value vectors, for example in case of a functional parameter obtained from experimental test, but also value matrix, in case of a functional parameter obtained from all experimental samples.

Tensorial analysis and tensorial calculus allow a more realistic study of automobile engine functioning, as well as for a quantitative estimation of engine dynamics and fuel saving based on experimental samples obtained from tests, because multiple sizes (eventual all of them) and multiple samples (eventual all of them) can simultaneously be taken into account.

2. THEORETICAL ASPECTS

Tensorial analysis, also called multimodal analysis or multivariate analysis, represents an extension of matrix analysis, which, in turn, represents an extension of vectorial analysis [6; 8; 9; 10].

Therefore, experimental data and those obtained by calculus based on it can be analysed in three ways. So, if only one size is considered (for example engine speed n) from a single experimental sample (for example P1), then there is a value vector, meaning a singlevariate picture (Figure 1a).

If two or more functional parameters are considered (for example engine speed n, throttle shutter's position ξ, intake air pressure p_a etc.) from one experimental sample (for example P1), then there is a matrix of values, meaning a bivariate picture (like in Figure 1b). Finally, if multiple functional parameters are targeted, from a couple of experimental samples (for example P1, P2, ..., P30), a tensor is obtained, in this case a tridimensional picture (Figure 1c).

Hence, a tensor represents a multivariate set of data, and the order of a tensor is equal with space size, in Figure 1c is a third oredr tensor N=3 (in tridimensional space). As a result, a scalar (a digit/value) represents a zero order tensor (meaning N=0), a vector represents a first order tensor (N=1), and a matrix represents a second order tensor (meaning N=2). The order of a tensor also leads to the concept of mode; for example, a third order tensor has 3 modes (mode 1, mode 2 and mode 3).

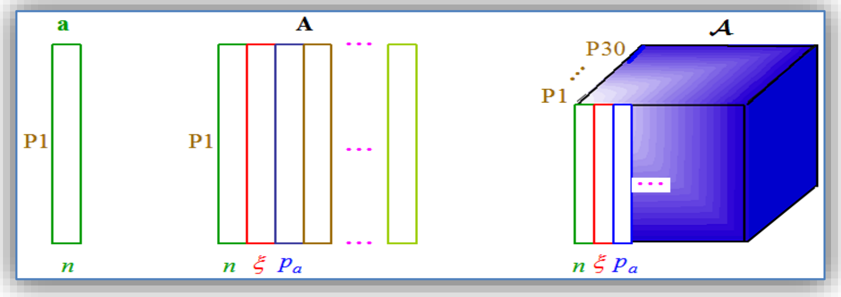

Figure 1: Board of experimental values

In Figure 2 is presented the tridimensional tensor A(481x3x4), whereat mode 1 is discrete time t_d with the 481 values of one experimental sample, mode 2 is given by three sizes (throttle shutter's position ξ, engine speed n and engine torque M_e), and mode 3 by the 4 samples mentioned in graph. As it can be seen from Figure 2, there are some blank spaces in tensor picture, which are associated with the lack of values on certain intervals from experimental time series of the mentioned functional parameters.

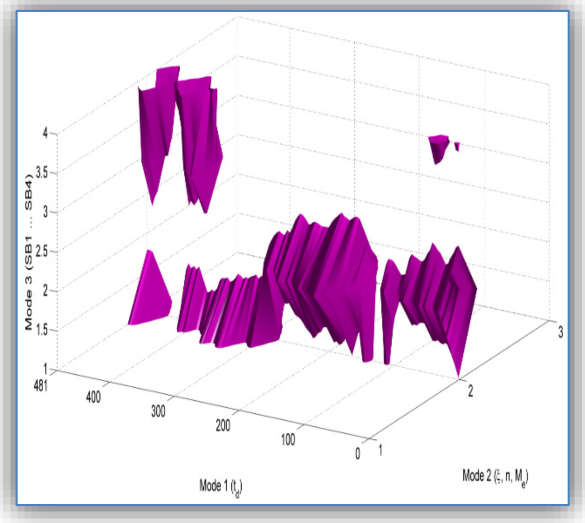

Figure 2: Graphical representation of a tridimensional tensor with 3 sizes (throttle shutter's position, engine speed and engine torque) from 4 samples (SB1, SB2, SB3, SB4)

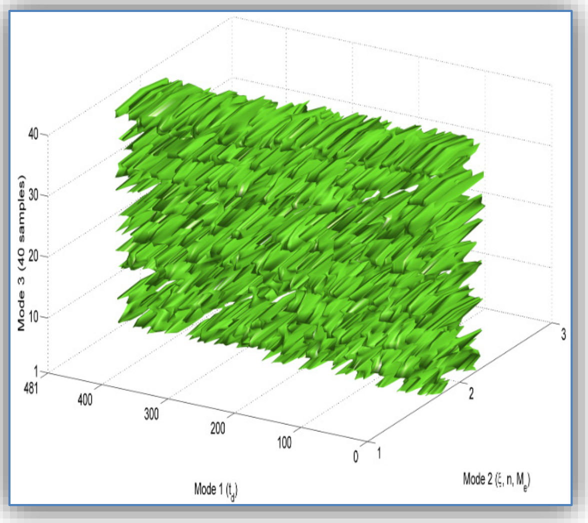

Figure 3: Graphical representation of the tridimensional tensor A(481x3x40) with 3 sizes (throttle shutter's position, engine speed and engine torque) from 40 samples with petrol

In Figure 3 is presented the tridimensional tensor A(481x3x40), whereat mode 1 is discrete time t_d with the 481 values of a sample, mode 2 is given by 3 parameters (throttle shutter's position ξ, engine speed n and engine torque M_e), and mode 3 by the 40 experimental samples.

Many times, in order to execute some operations, a tensor is transformed into matrix; this operation is called tensor matricization [1; 2]. The advantage of tensor matricization is the fact that matrix operations are still performed; so, tensor matricization means reducing it's order. Likewise, simplification can continue by using tensor vectorisation operation, meaning it's transformation into a vector.

Finally, like in case of matrix, it is established the order of a tensor [7], by using tensor matricization; the order of a tensor is equal to the minimum order of the matrix which are obtained through the matricization operation of the respective tensor.

3. TENSOR DECOMPOSITION

Like in case of matrixes, spectral analysis is based on decomposition (factorization) of a tensor, in order to obtain eigenvalues, eigenvector and it's singular values [5].

Generalizing, for a N order tensor, it is obtained the spectral decomposition:

$$A = S \times \boldsymbol{U}_1 \times \boldsymbol{U}_2 \times \boldsymbol{U}_3 \times \ldots \times \boldsymbol{U}_N \quad (1)$$

where inferior index marks the tensor mode (1, 2, 3, ..., N), and Σ is called nucleus and represents a tensor with the same size with A; tensor Σ is similar to the matrix that contains on main diagonal

the singular values of targeted matrix for decomposition (procedure SVD from matrix).
In addition, in formula (1) matrixes U_i are called modal matrixes (associated with module 1, 2, ..., N of tensor); these are orthogonal and contain main subunitary vectors (with unit infinite norm) of matrix $A_{(n)}$ obtained by A tensor matricization.
For example, for a tridimensional tensor, matrixes obtained from the matricization of tensor A are given by the ratios:

$$A_1 = U_1 S_1 V_1^T ;\ A_2 = U_2 S_2 V_2^T ;\ A_3 = U_3 S_3 V_3^T \tag{2}$$

each one of them having associated singular values.
The presented method, called HOSVD (Higher Order Singular Value Decomposition), proper to a tensor, it is also called SVD algorithm with N-modes, highlighting that there is a generalization of the one from matrix (where there is SVD - Singular Value Decomposition).
Presented formulas also allow the calculus of eigenvalues of A tensor:

$$\lambda_i = A \times U_{1_i}^T \times U_{2_i}^T \times \cdots \times U_{N_i}^T \tag{3}$$

In consequence, for the r eigenvalues λ_i, it is obtained an estimation of tensor A:

$$\hat{A} = \sum_{i=1}^{r} \lambda_i U_{1_i} \circ U_{2_i} \circ \cdots \circ U_{N_i} \tag{4}$$

case in which initial tensor A represents a sum of first order r tensors; in this case, r represents exactly the order of tensor A.
Like in case of matrixes, tensor data compression means to establish highest singular values (highest eigenvalues), which can assure a tensor estimation by a certain precision; in other words, in sum (4) there are only retained those singular values, from the total r, which can assure a required error, ordinarily of 2%.

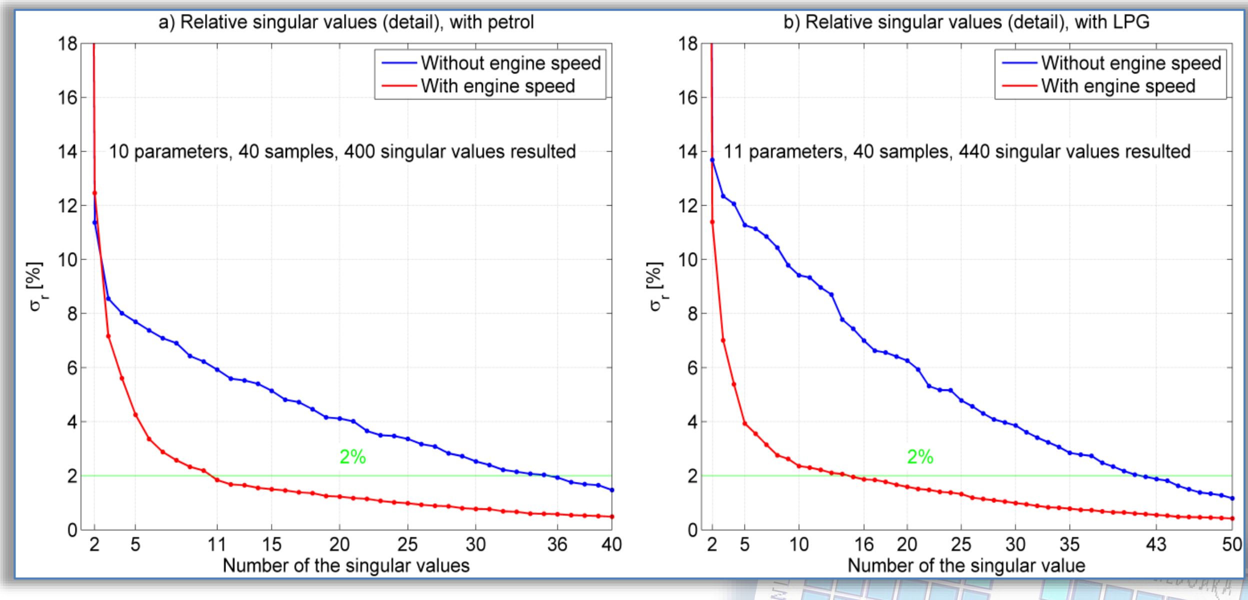

Figure 4: Setting singular relative values

For example, in Figure 4 there are established singular values for a tensor made by 10 parameters (whereby engine speed lacks) or 11 parameters (engine speed is added) from 40 experimental samples in case of running with petrol (Figure 4a) and from 40 experimental samples in case of running with LPG (Figure 4b); hence there are obtained 400 singular values in the first case and 440 singular values in the second case.
In order to establish each singular value contribution, in graphs are shown singular relative values, meaning current values divided by maximum value. To establish dominant singular values (with the highest contribution in engine functioning), an accuracy is imposed, in this case of 2%, as it can be seen from upper graphs where it is shown a detail at a time (y axis is not shown until the value of 100%).
Graphs from Figure 4 show different singular values for various groups of functional parameters; therefore, it is confirmed that various functional parameters have different contributions in engine functioning. In addition, in Figure 4 was also drawn the horizontal of a compulsory error of 2%.

As it can be seen from Figure 4a, in case of running with petrol, if it is adopted a calculus error of 2%, then in case of all parameters can only be retained 11 singular values; in other words, in this case it is enough to simply use the 11 relevant singular values for engine functioning, for example in case of establishing the mathematical model. If engine speed is not taken into account, then there have to be retained 36 relevant singular values (highest values).

Similarly, as it can be seen from Figure 4b, in case of running with LPG, if it is adopted an error of 2%, then in case of all parameters there can only be retained 16 singular values; if engine speed is left out, then 43 singular values must be retained.

Those presented show that data compression is best allowed by engine speed (the highest reduction of the number of values), with the advantage that follow-up the calculi will be made with fewer values.

4. MODAL MATHEMATICAL MODELS

The spectral analysis presented above, based on decomposition (factorization) and which uses eigenvalues, eigenvectors and singular values, has been expanded and enhanced, thus obtaining modal mathematical models, also called multivariate models [1; 2; 4].

Therefore, in initial phase, three models have been proposed, which are also based on decomposition:

- L.R. Tucker proposed in 1966 the model called TUCKER;
- J.D. Carroll and J. Chang have developed in 1970 the model called CANDECOMP(CANonical DECOMPosition);
- R.A. Harshman proposed in 1970 the model called PARAFAC (PARAllel FACtors).
- It must be mentioned that the last two models have been combined and resulted the model known today as CP (CANDECOMP-PARAFAC).
- Afterwards other versions have appeared, as well as other mathematical models, of what the most used are:
- models of PARAFAC kind: PARAFAC2 (in 1972), S-PARAFAC (in 2003), PARAFAC3W (in 2004), c – PARAFAC (in 2006), PARALIND (PARAllel factors with LINear Dependency) in 2005, PARATUCK (combination between PARAFAC and TUCKER);
- models of TUCKER kind: TUCKER1 (in 1980), TUCKER2 (in 1984), TUCKER3 (in 1992);
- alternative models: INDSCAL (INDividual Differences SCALing), in 1985; ME (Multilinear Engine) in 1999; STATIS in 2004; multi-block models in 2000.

PARAFAC model represents a factorization method (decomposition) which represents a generalization of PCA (Principal Component Analysis) from matrix analysis [4]; for example, in case of tridimensional analysis (third order tensor), decomposition leads to trilinear constituents.

In consequence, in this example the PARAFAC model has the structural form for tensor Ξ with x_{ijk} elements:

$$x_{ijk} = \sum_{f=1}^{F} a_{if} b_{jf} c_{kf} + e_{ijk} \tag{5}$$

in which F represents the number of factors (equal to the order of tensor Ξ), a_{if}, b_{jf} and c_{kf} elements of modal matrix A, B and C (also noted as U_1, U_2, U_3), and e_{ijk} simulation error (the residual).

In matrix form, the PARAFAC model is:

$$\underline{\boldsymbol{X}} = \boldsymbol{ABC} + \underline{\boldsymbol{E}} \tag{6}$$

where $\underline{\mathbf{E}}$ represents the residual tensor (the simulation error, meaning the difference between initial vector and the one obtained from simulation); as it can be observed, a tensor has two notations, for example, in this case $\underline{\mathbf{X}}$ or Ξ.

By using the Khatri-Rao product, the PARAFAC model can be written as:

$$\underline{\boldsymbol{X}} = (\boldsymbol{A} \circ \boldsymbol{B})\boldsymbol{C}^T + \underline{\boldsymbol{E}} \tag{7}$$

In consequence, estimating simulation error, let say by using norm 2 L_2 (Frobenius norm), leads to the minify of expression:

$$min(L_2) = min\left\| \underline{\boldsymbol{X}} - (\boldsymbol{A} \circ \boldsymbol{B})\boldsymbol{C}^T \right\|_F^2 \tag{8}$$

which represents the targeted function that needs to be minimized.

In Figure 5 there are presented values of modal matrix A, B and C in case of engine functioning with petrol (upper graphs) and in case of engine functioning with LPG (lower graphs). Graphs show that there is a tridimensional tensor Ξ (481x6x40) that contains 6 functional parameters (engine speed n, throttle shutter's position ξ, intake air pressure p_a, injection time t_i, ignition advance β and engine torque M_e), from 40 experimental samples with petrol and 40 samples with LPG, each sample having 481 values.

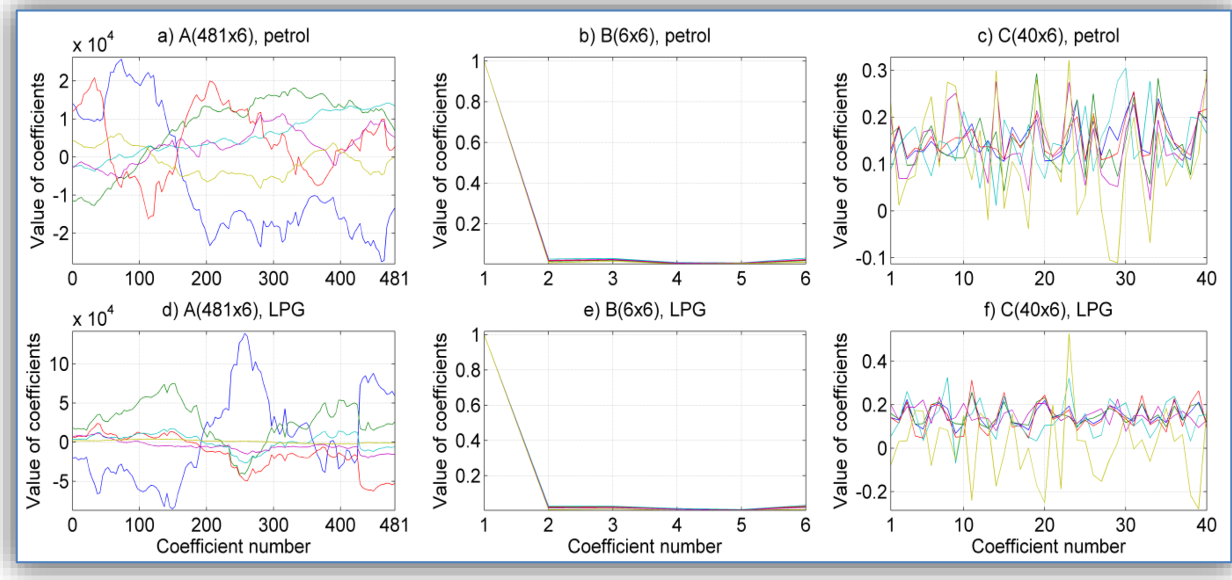

Figure 5: Modal matrixes, PARAFAC model, tensor X(481x6x40)=$(n,\xi,p_a,t_i,\beta,M_e)$

Likewise, it can be observed that the number of factors is F=6 (number of curves from graphs) meaning the order of tensor is 6, which, as it was mentioned above, is equal to the minimum order of the three modal matrixes.

PARALIND model represents the multidimensional model that is applied when factors are linear dependent; the other models previously listed are applied when factors (constituents) are linear independent, meaning that there are no singular solutions.

In matrix form, the PARALIND model is, for example, in case of a third order X tensor:

$$\underline{\boldsymbol{X}} = \boldsymbol{AH}(\boldsymbol{C} \circ \boldsymbol{BH})^T + \underline{\boldsymbol{E}} \tag{9}$$

in which matrix H is (for the targeted tridimensional model):

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

For this model, simulation error can be evaluated, for example, with Frobenius norm (norm 2):

$$min(L_2) = min\left(\underline{\boldsymbol{X}} - \boldsymbol{AH}(\boldsymbol{C} \circ \boldsymbol{BH})^T\right) \tag{11}$$

The plurality of modal mathematical models makes a detailed presentation to be difficult, that is why within the paper have been given basic elements of the PARAFAC and PARALIND models, for the first one with examples of modal matrixes; in specialty literature there are detailed displays of mentioned models, including with resolutions by using Matlab program, where there is also a toolbox dedicated to tensors [1; 2; 5].

5. CONCLUSIONS

Tensorial analysis assures a study of engine functioning of a more realistic and believable way, because it can simultaneously consider all experimetal data and all samples, meaning that it can operate with tensors of values; this aspect is very important for vehicles with electronic control, where there are inherent interactions and functional parameters are interdependent, and by eliminating some of them the results and conclusions can be affected.

Likewise, tensorial analysis can divide factors with the highest input in engine functioning, thus assuring the highlight of central tendencies of evolution and the diminish of used data number, last aspect is important mainly in case of vehicles with electronic control, where there are large sets of numbers, which are difficult to process and read off.

It can be concluded that algorithms specific to tensorial calculus can be used for the analysis of automobile engine functioning, aswell as for the study of engine dynamics and fuel saving.

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