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## EXAMINATION OF TEMPERATURE CHANGE OF GREEN ROOF AND FLAT ROOF IN FREQUENCY RANGE

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**ABSTRACT:** Importance of energy efficiency has increased over the last decades, as the cost of energy resources has an increasing tendency. Designing and constructing a new building the profession has to consider this issue with outstanding significance. The green roof can be a solution of the ecological approach. Advantage of the green roof is that this layer can be an additional top structure of an existing flat-roof building, just as thermal insulation. In the green roof topic, there are far-gone researches for the water balance, but there are few results available as for thermal balance of the structure. In the following, a series of measurement and evaluation is presented in case of an existing green roof building. The rooftop of this nursery school building is installed half-green roof, half-paved terrace roof. During the evaluation properties of temperature change inside the roof structure has been analyzed both in summer and winter conditions. Primarily the daily periodicity is presented and discussed in this article by transforming time-functions into frequency-functions.

**Keywords:** energy efficiency, energy resources, green roof, measurement and evaluation

### 1. INTRODUCTION

Energy-saving structures and buildings have increasing importance by the growing costs of energy resources. This helps provide such solutions, which enable to create buildings with minimalized maintenance energy and cost. Definitely green roof is this kind of structure. Today, there is real opportunity by greening the top of the buildings, to compensate the lost natural surfaces in cities. Furthermore, green roof can be install not only on newly built roofs, but also even on existing buildings. It only requires a structural calculation considering the additional weight.

Several researches has been proceeded about energetic properties of green roofs in the last decades. A few examples are mentioned in the literature. There were steps in order to standardize green roof structure with publishing guidelines. The German Landscape Research, Development and Construction Society (FLL) compiled first guideline, in the late 1970's.

### 2. OBJECT OF RESEARCH

There had been chosen a nursery school building, which was built in the 1970's, and an experimental green roof was installed on the top in the early 1990's. This layer lies only on the half of the roof, while the rest is covered with concrete pavement. This situation provided the opportunity to examine both types of roof, and to make a comparison between them. A photo of this roof is shown on Figure 1 with the extensive green roof on the left, and behind the railings the terraced roof. Under the tile, the thermometer and data collector unit can be seen, which will be presented later. Temperature sensors had been installed



Figure 1. Green roof and terraced roof with the measuring unit

inside the roof structure, between each layer. Temperature values of every minute have been collected during a whole year period. Therefore, representative data has been set for both summer and winter conditions. Structure of the roof is shown on Figure 2, with only the main layers and their thickness values in cm. The only difference is the upper layers between green roof and paved roof.

Basic layers from inside: scooped reinforced concrete slab panel, concrete slope layer, vapor proof layer, thermal insulation, waterproof layer.

Green roof layers: plastic drainage layer with geotextile, vegetation layer  
Terraced roof layers: gravel ballast, concrete pavement.

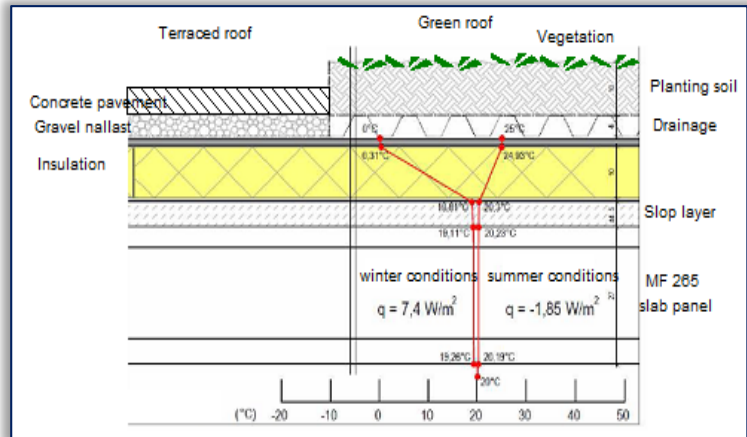


Figure 2. Layers of the green roof and terraced roof with the green roof temperature distribution in stationary state

Temperature distribution of the green roof layers is presented on Figure 2, both winter and summer conditions, which show a stationary heat transport. Real measured values are far away from this stationary state, which is displayed in the following.

**3. MEASURING UNIT**

The collector unit has been placed under a tile to be covered and protect from rain and sun radiation (Figure 1). This unit also includes the sensor for the ambient air temperature.

There has been installed the same unit on both parts of the roof with 8 sensors of each, measuring the temperature of different layers of the roofs, the air, the surface and the inside of the building. Each temperature value is calculated as an average of 20 different measured values and saved to the memory of the unit every minute. Accuracy is +/- 0.3°C. The measuring unit is specified in article [1].

**4. SUMMER AND WINTER MEASURING RESULTS**

First of all on Figure 3 a series of measured values are presented in summer conditions during a weekly period. Values of all sensors are listed and marked on the diagram.

Several important statements can be settled by the diagram of Figure 3, comparing green and terraced roof.

Surface temperature of each roof is parallel with ambient air temperature. Top layer of green roof has a much higher temperature in daytime then air, while at night the same value is about 10°C lower then air temperature, which is caused by evaporation of the vegetation. The same sensor on terraced roof, shows higher temperature values then air both day and night.

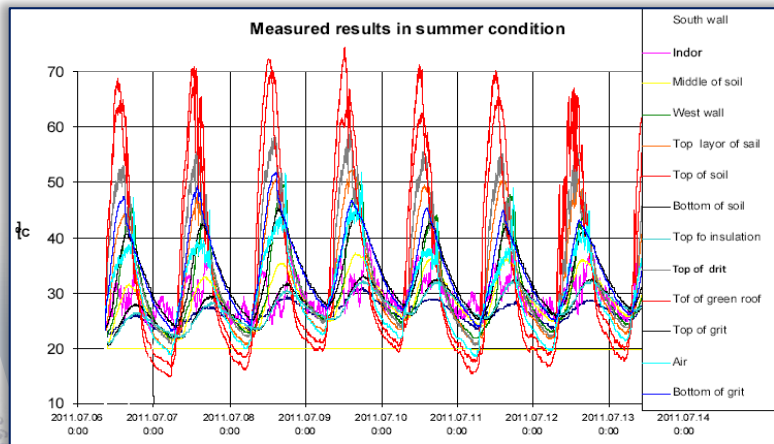


Figure 3. Overview diagram with all sensors (green roof and terraced roof), summer conditions

On Figure 3 daily temperature fluctuation diagram highlights the difference between bottom temperatures of the vegetation and the ballast layers (both marked with black line). In summer conditions, there is an average 10°C difference in daytime, which means that the green roof layers have much better thermal insulating properties.

The reason of this difference is on one hand because of the heat capacity of soil, which is changing depending the water content. The other hand there is the significant effect of the evaporation of the vegetation, which means a notable cooling on the structure. The difference can

be noticed also in night period, which value is about 3-5°C. These facts from the measurement all prove our assumption about the thermal advantages of the green roof structure.

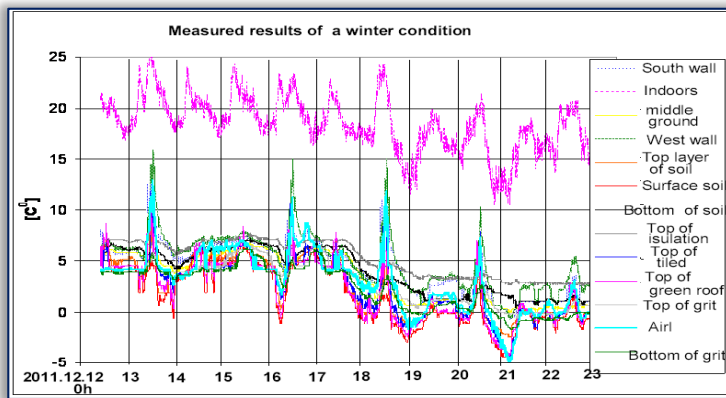


Figure 4. Overview diagram with all sensors (green roof and terraced roof), winter conditions

hour delay. On the contrary, the same layer of the terraced roof has 1-1.5°C higher temperature than outside air. This is caused by the heat flux from inside the building, which is higher in case of the terraced roof as it has a lower heat insulation value.

The same conclusions follow from the comparison of the bottom layer temperatures, as well. The difference is 1.5-2°C, which shows that the green roof has higher heat attenuation; however its thickness is almost the double of the gravel and pavement of the other roof.

### 5. EXAMINATION OF MEASURED TEMPERATURE VALUES IN FREQUENCY RANGE

The most important difference between summer and winter diagrams is in periodicity. While the summer curve has a significant daily period, the winter diagram does not show any regularity, which is caused by the narrow range of temperature fluctuation between day and night. In the following only green roof temperature data series are presented, but the other temperature functions can also be examined similar way in frequency range.

Periodic functions can be examined with Fourier analysis. The function (generally time function) is approximated by a series of sinus and cosines functions. General equation is the following.

$$f(t) \cong \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos nt + b_n \cdot \sin nt$$

or written by complex numbers  $f(t) = \sum_{k=-N}^N C_k e^{i(k \cdot \omega t)}$ , where  $i = \sqrt{-1}$  is the imaginary unit,  $k$  is the

running index,  $\omega_k$  is the changing angular frequency,  $C_k$  is complex coefficient.

If  $N$  is large enough, the curve approaches the signal. The frequency components are called harmonic component. A given function  $f(t)$  is characterized by a sense of the Fourier coefficients. On this basis, it is possible that the normally complex function  $f$  (time varying signal) is in many ways easier to enter a series of numbers. This operation is called a Fourier analysis: a transition from the time to the frequency domain. The reverse process, when the reconstructing coefficients of the function - called Fourier synthesis.

It can be used for functions composed by discrete values, such as time diagram set from measured values. In the following Fourier analysis of a winter and summer series of temperature is presented, without the details of the theory. More details are in literature [3].

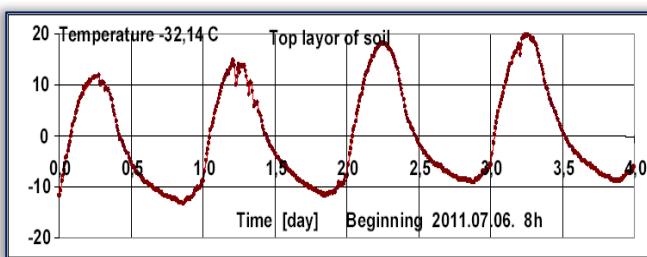


Figure 5. Temperature diagram in time for summer

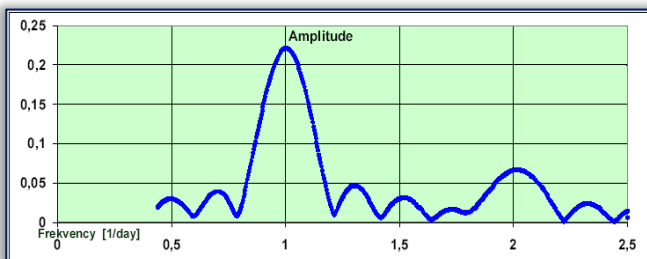


Figure 6. Temperature diagram in frequency range for a summer day (Average value subtracted from function values)

First, a summer diagram analysis is presented on Figure 5 and 6. It is important to note, that the average value is subtracted from the measured temperatures, so that only the periodic part of the function is plotted. Whole examined period is 4 days.

The analysis has been performed with adapting a program from „excelunusual.com” by George Lungu [3]. Result is presented on Figure 6. Signal with 1 day periodic time has significant amplitude in the analysis (0.22). Further frequencies has also higher amplitudes, but much lower than 1 1/day. This dominant frequency value is showing the periodic time of the function. Coefficients have been calculated with the following equations.

$$a_n = \frac{2}{T} \cdot \int_0^T f(x) \cdot \cos\left(n \cdot \frac{2 \cdot \pi}{T} t\right) \cdot dt \qquad b_n = \frac{2}{T} \cdot \int_0^T f(x) \cdot \sin\left(n \cdot \frac{2 \cdot \pi}{T} t\right) \cdot dt$$

where  $a_n$  and  $b_n$  are Fourier coefficients. In this case four periods has been analyzed, so discretised formula is the following.

$$a_n = \frac{1}{2} \cdot \sum_{j=0}^{576} f(t_j) \cdot \cos\left(n \cdot \frac{2 \cdot \pi}{1} t_j\right) \cdot \Delta t \qquad b_n = \frac{1}{2} \cdot \sum_{j=0}^{576} f(t_j) \cdot \sin\left(n \cdot \frac{2 \cdot \pi}{1} t_j\right) \cdot \Delta t$$

Running index of time steps is  $j$ , time difference,  $\Delta t$  is permanent between time steps, namely  $t_{j+1} = t_j + \Delta t$ . Calculation of sum value has been performed for four periods, that is why  $1/2$  stands

before the sum symbol. Circle frequency of a certain Fourier member is  $\omega_n = n \cdot \frac{2 \cdot \pi}{1} = \frac{2 \cdot \pi}{1/n} = \frac{2 \cdot \pi}{T}$ .

Dimension is 1/day.  $T$  means periodic time of each harmonic member, while 1 means the 1 day periodic time.

Circle frequency of the first sinus and cosign member is  $\omega_1 = 1 \cdot \frac{2 \cdot \pi}{1} = \frac{2 \cdot \pi}{1}$ ,  $T_1 = 1$  day.

One of the second members is  $\omega_2 = 2 \cdot \frac{2 \cdot \pi}{1} = \frac{2 \cdot \pi}{1/2}$ ,

$T_2 = \frac{1}{2}$  day. And for the third member this is

$$\omega_3 = 3 \cdot \frac{2 \cdot \pi}{1} = \frac{2 \cdot \pi}{1/3} \quad T_3 = \frac{1}{3} \text{ day.}$$

This means that a certain sinus and cosign function gives 1, 2 or 3 periods in 1 day in case of  $n=1, 2$  or  $3$ .

Adequacy of approximation is presented for  $n=1, 2$  and  $3$  values. Results are plotted on diagrams.

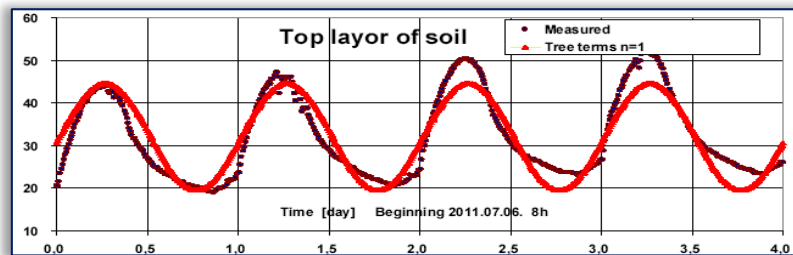


Figure 7. Temperature function and approximation with  $f(t)$

$$f(t) \cong 32,14 - 1,45 \cdot \cos\left(\frac{2 \cdot \pi}{1} \cdot t\right) + 12,43 \cdot \sin\left(\frac{2 \cdot \pi}{1} \cdot t\right)$$

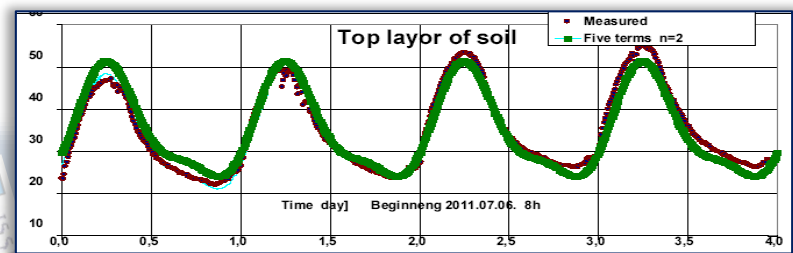


Figure 8. Temperature function and approximation with  $f(t)$

$$f(t) \cong 32,14 - 1,45 \cdot \cos\left(\frac{2 \cdot \pi}{1} \cdot t\right) + 12,43 \cdot \sin\left(\frac{2 \cdot \pi}{1} \cdot t\right) - 3,83 \cdot \cos\left(\frac{4 \cdot \pi}{1} \cdot t\right) + 1,193 \cdot \sin\left(\frac{4 \cdot \pi}{1} \cdot t\right)$$

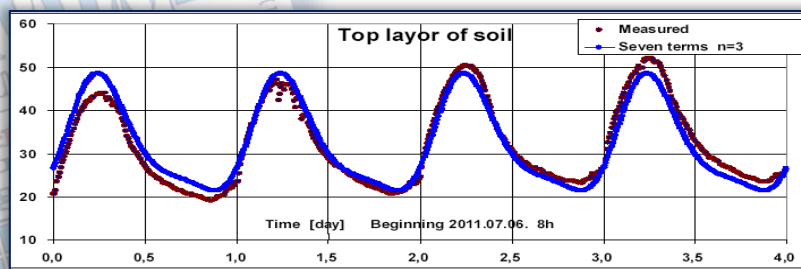


Figure 9. Temperature function and approximation with  $f(t)$

$$f(t) \cong 32,14 - 1,45 \cdot \cos\left(\frac{2 \cdot \pi}{1} \cdot t\right) + 12,43 \cdot \sin\left(\frac{2 \cdot \pi}{1} \cdot t\right) - 3,83 \cdot \cos\left(\frac{4 \cdot \pi}{1} \cdot t\right) + 1,193 \cdot \sin\left(\frac{4 \cdot \pi}{1} \cdot t\right) - 0,32 \cdot \cos\left(\frac{6 \cdot \pi}{1} \cdot t\right) - 0,14 \cdot \sin\left(\frac{6 \cdot \pi}{1} \cdot t\right)$$

Change of the approximation temperature function (T) in time (t) from the analysis can be described. By the help of Fourier analysis, difference of the phase angle of each temperature function is demonstrable. Details are not put here, because it is noticeable from time signals.

However, approximation of measured time signal with sinus Fourier series and phase shift is the following. Periodicity of time signal is so regular that only a 2-member Fourier series is good enough to make the approximation.

$$T(t) = 12 \cdot \sin[2\pi \cdot t] + 3 \cdot \sin[4\pi \cdot t - 1]$$

Approximation function diagram and measured data is presented on Figure 10. Further improvements can be reached by including other elements of Fourier analysis.

The bottom layer of green roof vegetation has been chosen from winter measurements. Time signal is shown on Figure 11. Average value is subtracted from temperatures this time, as well.

Result of Fourier analysis is on Figure 12. There is no dominant frequency value on the diagram, which means, there is not significant periodic time of the signal. Therefore, Fourier analysis also proved the theoretical assumption. In this case, phase angle analysis would have no sense, because heat flux cannot be identified through the layers.

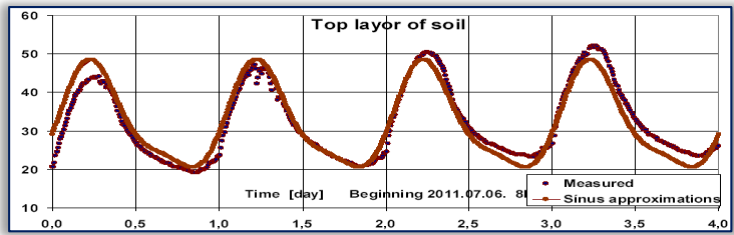


Figure 10. Temperature function of summer conditions and approximation with only sinus members

$$f(t) \cong 32,14 + 13,12 \cdot \sin\left[\frac{2 \cdot \pi}{1} \cdot t\right] + 3,6 \cdot \sin\left[\frac{4 \cdot \pi}{1} t + 1\right]$$

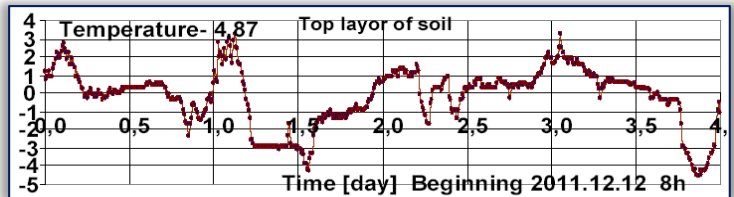


Figure 11. Temperature diagram in time for winter (Average value subtracted from function values)

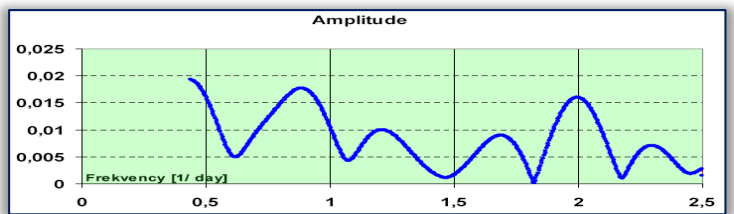


Figure 12. Temperature diagram in frequency range for a winter day

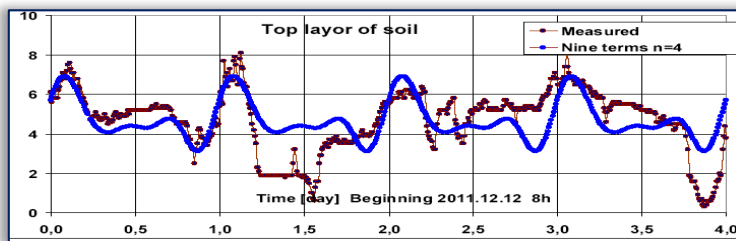


Figure 13. Approximation function of temperature values in winter

$$f(t) \cong 4,69 + 0,536 \cdot \cos\left(\frac{2 \cdot \pi}{1} \cdot t\right) + 0,582 \cdot \sin\left(\frac{2 \cdot \pi}{1} \cdot t\right) + 0,245 \cdot \cos\left(\frac{4 \cdot \pi}{1} \cdot t\right) + 0,960 \cdot \sin\left(\frac{4 \cdot \pi}{1} \cdot t\right) + 0,117 \cdot \cos\left(\frac{6 \cdot \pi}{1} \cdot t\right) + 0,557 \cdot \sin\left(\frac{6 \cdot \pi}{1} \cdot t\right) + 0,109 \cdot \cos\left(\frac{8 \cdot \pi}{1} \cdot t\right) + 0,033 \cdot \sin\left(\frac{8 \cdot \pi}{1} \cdot t\right)$$

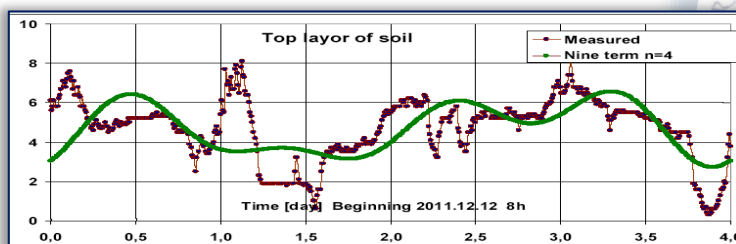


Figure 14. Temperature function approximated by f(t)

$$f(t) \cong 4,69 + 0,215 \cdot \cos\left(\frac{2 \cdot \pi}{4} \cdot t\right) - 0,726 \cdot \sin\left(\frac{2 \cdot \pi}{4} \cdot t\right) - 0,497 \cos\left(\frac{4 \cdot \pi}{4} \cdot t\right) + 0,784 \cdot \sin\left(\frac{4 \cdot \pi}{4} \cdot t\right) - 0,683 \cdot \cos\left(\frac{6 \cdot \pi}{4} \cdot t\right) + 0,156 \cdot \sin\left(\frac{6 \cdot \pi}{4} \cdot t\right) - 0,691 \cdot \cos\left(\frac{8 \cdot \pi}{4} \cdot t\right) + 0,595 \cdot \sin\left(\frac{8 \cdot \pi}{4} \cdot t\right)$$

Fourier series approximation has been performed for winter diagram, as well. Only 9-member case is presented on Figure 13, which still do not cover the original function of measured values.

Diagram shows that approximation is inaccurate for both the values and the periodic time. For proper approximation much higher number of members are needed. Figure 14 shows the approximation function considering the periodic time 4 days, but the result is not nearer to measured values.

## 6. SUMMARY

Detailed analysis of temperature field of the roof structure has been performed both with measurements and calculation. Aspects of the examination of this temperature field were the periodicity and the usefulness of average values of longer periods for energetic calculations. By Fourier transformation, periodicity of temperature signals can be settled. By the results, summer time periodicity can be shown significantly, because of relatively wide range of temperature. Measured and calculated temperature diagrams can be approximated quite well with first 2 members of Fourier series. Daily period is significant. However, in winter conditions, when there is only a narrow temperature fluctuation, there was no way to get the daily periodicity as a result. These temperature functions could not be approximated even with 4-5 members of Fourier series.

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