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VIBRATION CHARACTERISTICS OF A FREELY VIBRATING SSSC RECTANGULAR THIN ORTHOTROPIC PLATE

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Abstract: In this article, a truncated Taylor–Maclaurin series was used in Rayleigh–Ritz method to analyze the free vibration of a rectangular thin orthotropic plate bounded by three simply supported edges and one clamped edge (i.e. SSSC plate). The total potential energy functional (which is a function of the strain energy and kinetic energy of a free vibrating plate), was derived from the theory of elasticity. Taylor–Maclaurin series truncated at the fourth term was used to obtain a shape function which satisfied all the boundary conditions of an SSSC plate under free vibration. The shape function was substituted into the total potential energy functional, and the resulting equation was eventually minimized. The equation for the fundamental frequency was then derived from the minimized equation, and fundamental frequencies computed for different aspect ratios, p (varying from 0.1 to 2.0 in steps of 0.1) and different flexural rigidity ratios, φ . The results show that the average percentage differences in the values of the fundamental frequency for flexural rigidity ratios, φ_1 , φ_2 , and φ_3 , are -3.404%, -2.029%, and -2.456%. Hence, the displacement function obtained for the SSSC plate is a very good approximation of the exact shape function for the plate.

Keywords: Orthotropic Plate, Rayleigh-Ritz Method, Rectangular Plate, Taylor-Maclaurin Series, Free Vibration

INTRODUCTION

Thin rectangular plate elements used in engineering structures are often subject to free vibration. Thus, it is important to determine the vibration characteristics of thin rectangular plates undergoing free vibration.

Many researchers have carried out investigations on vibration of thin orthotropic plates. Hearmon [3] proposed an approximate general solution based on Rayleigh method for the free vibrations of orthotropic plates. Leissa [6] presented the accurate analytical results for free vibrations of orthotropic plates for cases having two opposite sides simply supported and others with possible combinations of clamped, simply supported, and free edge conditions. According to Meirovitch [7], the Rayleigh–Ritz method is one of the most popular methods used for obtaining approximate solutions for the fundamental frequencies of an orthotropic rectangular plate due to its high versatility and simplicity. Wu et al. [9] proposed a novel Bessel function method for deriving exact solutions to free vibration problems of rectangular thin plates. Chakraverty [2] gave higher modes of vibrations for plates of various shapes and boundary conditions. Xing and Liu [10] used separation of variables for obtaining exact solutions for the free vibration of thin orthotropic rectangular plates, with all combinations of simply supported and clamped boundary conditions.

Although a lot of researches have been done on plates, none of the researchers used Taylor's series function in Rayleigh–Ritz method to obtain the fundamental frequencies of thin orthotropic rectangular SSSC plate subjected to free vibration. This work used Taylor's series function in Rayleigh–Ritz method in formulating the deflection function, which in turn was used to obtain total potential energy function. Then, the total potential energy was used to obtain the fundamental frequency of rectangular orthotropic thin plate bounded by three simply supported edges and one clamped edge (i.e. SSSC plate) under free vibration.





MATHEMATICAL FORMULATIONS

Shape Function

Taylor-Maclaurin series can be used to obtain the shape function of a rectangular plate. Ibearugbulem [4] expressed the shape function as:

$$w = w(x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{F^{(m)}(x_0) \cdot F^{(n)}(y_0)}{m! n!} (x - x_0)^m \cdot (y - y_0)^n$$
(1)

where $F^{(m)}(x_0)$ is the mth partial derivative of the function with respect to x, $F^{(n)}(y_0)$ is the nth partial derivative of the deflection, w, with respect to y. The terms, m! and n!, are the factorials of m and n, respectively, while x_0 and y_0 are the co-ordinates of the origin. By truncating the infinite series at m = n = 4, the Eq. (2) is obtained.

$$w = \sum_{m=0}^{4} \sum_{n=0}^{4} I_m J_n x^m y^n$$
(2)

Transforming the x-y coordinate system to R-Q coordinate system yields:

$$R = x/a; Q = y/b$$
(3)

where R and Q are dimensionless quantities, and 'a' and 'b' are the dimensions of the plate along the xand y-axes, respectively. Therefore:

$$x = aR; \ y = bQ \tag{4}$$

$$a_m = I_m a^m; \ b_n = J_n b^n \tag{5}$$

 $w = \sum_{m=0}^{4} \sum_{n=0}^{4} a_m b_n R^m Q^n$ The shape function given by Eq. (6) can be expanded further in the following form:

► X

$$w(R,Q) = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4)(b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4)$$
(7)
a_i and b_i (i = 0, 1, 2, 3, 4) are unknown constants of the polynomial series shape function.

Boundary Conditions of SSSC Plate

where

S

The boundary conditions of the rectangular thin orthotropic plate bounded by three simply supported edges and one clamped edge (SSSC plate) shown in Figure 1, have to be taken into consideration in the mathematical formulation. Four boundary conditions along the x-axis and four boundary conditions along the y-axis are required to obtain a distinct solution for the SSSC rectangular plate. The moments at the corners of the simply supported edge are equal to zero since simply supported edges are free to rotate. The slope disappears along the corners of the clamped edge because clamped edges are assumed to resist rotation. For all the edges of the plate, the deflection at the corners must be zero. The boundary conditions of the SSSC plate are expressed as:

$$w(R = 0) = w''^{R}(R = 0) = 0$$
(8)

$$w(R = 1) = w''^{R}(R = 1) = 0$$
(9)

$$w(Q = 0) = w'^{Q}(Q = 0) = 0$$
(10)

$$w(Q = 1) = w^{nQ}(Q = 1) = 0$$
(11)

Substituting one after the other, w(R = 1) = 0 and $w''^{R}(R = 1) = 0$ into the Eq. (7), and solving the resulting simultaneous equations, yields:

$$a_1 = a_4; a_3 = -2a_4$$

Similarly, substituting successively, w(Q = 1) = 0 and $w''^Q(Q = 1) = 0$, into the Eq. (7) and solving the resulting simultaneous equations gives:

$$b_2 = 1.5b_4; \ b_3 = -2.5b_4$$

Substituting these parameters into Eq. (7) yields:
 $w = AH$ (12)

Figure 1: Boundary conditions of SSSC plate

H =

С

$$A = a_4 b_4 \tag{13a}$$

$$(1.5R2 - 2.5R3 - R4)(Q - 2Q3 - Q4)$$
(13b)

The term 'A' in Eqs. (12) and (13a) is the amplitude of the deflected shape, while H in Eqs. (12) and (13b) is the expression for buckling curve.

where:

FUNDAMENTAL FREQUENCY EQUATION OF FREELY VIBRATING ORTHOTROPIC PLATE **Total Potential Energy Functional**

Using the technique proposed by Ibearugbulem et al. [5], the equation for the fundamental frequency of the vibrating continuum, was derived by Abamara [1] by employing the principle of conservation of energy which allows the strain and kinetic energies of the continuum to be derived from first principles using the theory of elasticity. The resulting expressions, are subsequently substituted into the potential



(6)

S

S



energy functional and then minimized to determine the fundamental frequency for the first mode, i.e. M = N = 1. Then, the square fundamental frequency, λ^2 , was made the subject of equation after substituting the aspect ratios, p = a/b or p = b/a as the case may be. In the derivation of fundamental frequency, the strain energy equation, U, is given as:

$$U = \frac{D_x}{2b^2} \int_0^1 \int_0^1 \left[\frac{\phi_1}{p^3} \left(\frac{\partial^2 w}{\partial R^2} \right)^2 + 2 \frac{\phi_2}{p} \left(\frac{\partial^2 w}{\partial R \partial Q} \right)^2 + p \phi_3 \left(\frac{\partial^2 w}{\partial Q^2} \right)^2 \right] \partial R \, \partial Q \tag{14}$$

where U is the strain energy, ϕ_1 , ϕ_2 , and ϕ_3 are the flexural rigidities of the plate, and p is the aspect ratio. The kinetic energy equation is given in Eq. (15) as:

K. E. =
$$\frac{pb^2\lambda^2\rho h}{2}\int_0^1\int_0^1w^2\,\partial R\,\partial Q$$
 (15)

where K.E. is the kinetic energy, p is the aspect ratio, λ is the natural fundamental frequency, h is the thickness of the plate, and ρ is the mass density of the material. The total potential energy functional, Π , is expressed as follows:

$$\Pi = U - K. E. \tag{16}$$

From Eqs. (14) and (15), Eq. (16) can be re-written as:

$$\Pi = \frac{D_x}{2b^2} \int_0^1 \int_0^1 \left[\frac{\phi_1}{p^3} \left(\frac{\partial^2 w}{\partial R^2} \right)^2 + 2 \frac{\phi_2}{p} \left(\frac{\partial^2 w}{\partial R \partial Q} \right)^2 + p \phi_3 \left(\frac{\partial^2 w}{\partial Q^2} \right)^2 \right] \partial R \, \partial Q - \frac{p b^2 \lambda^2 \rho h}{2} \int_0^1 \int_0^1 w^2 \, \partial R \, \partial Q \qquad (17)$$

Let the flexural rigidities be expressed as:

$$\phi_1 = \frac{D_x}{D_x} = 1 \tag{18a}$$

(18c)

where:

$$B = \mu_{x}D_{y} + 2D_{xy} = \mu_{y}D_{x} + 2D_{xy}$$
(19)

Substituting Eqs. (18a) - (18c) into Eq. (17) give

$$\Pi = \frac{p_{x}A^{2}}{2b^{2}} \int_{0}^{1} \int_{0}^{1} \left[\frac{\phi_{1}}{p^{3}} \left(\frac{\partial^{2}H}{\partial R^{2}} \right)^{2} + 2 \frac{\phi_{2}}{p} \left(\frac{\partial^{2}H}{\partial R \partial Q} \right)^{2} + p \phi_{3} \left(\frac{\partial^{2}H}{\partial Q^{2}} \right)^{2} \right] \partial R \partial Q - \frac{pA^{2}b^{2}\lambda^{2}\rho h}{2} \int_{0}^{1} \int_{0}^{1} H^{2} \partial R \partial Q$$
(20)
Inimizing the Eq. (20) yields:

Minimizing the Eq. (20) yields:

$$\frac{\partial \Pi}{\partial A} = \frac{D_{x}A}{b^{2}} \int_{0}^{1} \int_{0}^{1} \left[\frac{\phi_{1}}{p^{3}} \left(\frac{\partial^{2}H}{\partial R^{2}} \right)^{2} + \frac{2\phi_{2}}{p} \left(\frac{\partial^{2}H}{\partial R \partial Q} \right)^{2} + p\phi_{3} \left(\frac{\partial^{2}H}{\partial Q^{2}} \right)^{2} \right] \partial R \partial Q - pAb^{2}\lambda^{2}\rho h \int_{0}^{1} \int_{0}^{1} H^{2} \partial R \partial Q \quad (21)$$

It should be noted that the minimized equation as given by Eq. (21), is equal to zero. At this point, the fundamental frequency, λ^2 , is made the subject of the equation. The fundamental frequency can be determined for aspect ratios, p = a/b and/or p = b/a. In terms of p and b, the square of the fundamental frequency becomes:

$$\lambda^{2} = \frac{\frac{D_{X}}{b^{4}\rho h} \int_{0}^{1} \int_{0}^{1} \left[\frac{\phi_{1}}{p^{4}} \left(\frac{\partial^{2}H}{\partial R^{2}} \right)^{2} + \frac{2\phi_{2}}{p^{2}} \left(\frac{\partial^{2}H}{\partial R \partial Q} \right)^{2} + \phi_{3} \left(\frac{\partial^{2}H}{\partial Q^{2}} \right)^{2} \right] \partial R \partial Q}{\int_{0}^{1} \int_{0}^{1} H^{2} \partial R \partial Q}$$
(22)

In terms of a and b, the square of the fundamental frequency is given by the Eq. (23).

$$\lambda^{2} = \frac{\frac{D_{x}}{a^{4}\rho h} \int_{0}^{1} \int_{0}^{1} \left[\phi_{1} \left(\frac{\partial^{2}H}{\partial R^{2}} \right)^{2} + \frac{2\phi_{2}a^{2}}{b^{2}} \left(\frac{\partial^{2}H}{\partial R \partial Q} \right)^{2} + \frac{\phi_{3}a^{4}}{b^{4}} \left(\frac{\partial^{2}H}{\partial Q^{2}} \right)^{2} \right] \partial R \partial Q}{\int_{0}^{1} \int_{0}^{1} H^{2} \partial R \partial Q}$$
(23)

In terms of p and 'a', the equation for fundamental frequency squared is as follows:

$$\lambda^{2} = \frac{\frac{D_{X}}{a^{4}\rho h} \int_{0}^{1} \int_{0}^{1} \left[\phi_{1} \left(\frac{\partial^{2} H}{\partial R^{2}} \right)^{2} + 2\phi_{2} p^{2} \left(\frac{\partial^{2} H}{\partial R \partial Q} \right)^{2} + \phi_{3} p^{4} \left(\frac{\partial^{2} H}{\partial Q^{2}} \right)^{2} \right] \partial R \partial Q}{\int_{0}^{1} \int_{0}^{1} H^{2} \partial R \partial Q}$$
(24)

Similarly, for aspect ratio p = b/a, the λ^2 can be expressed in terms of p and a as:

$$\lambda^{2} = \frac{\frac{D_{x}}{a^{4}\rho h} \int_{0}^{1} \int_{0}^{1} \left[\phi_{1} \left(\frac{\partial^{2} H}{\partial R^{2}} \right)^{2} + \frac{2\phi_{2}}{p^{2}} \left(\frac{\partial^{2} H}{\partial R \partial Q} \right)^{2} + \frac{\phi_{3}}{p^{4}} \left(\frac{\partial^{2} H}{\partial Q^{2}} \right)^{2} \right] \partial R \, \partial Q}{\int_{0}^{1} \int_{0}^{1} H^{2} \, \partial R \, \partial Q}$$
(25)

3.2 Application of Rayleigh-Ritz Method

Let the partial differentials of the deflection function, w, in terms of the dimensionless parameters R and Q, be expressed as:

$$w'^{R} = \frac{\partial w(R,Q)}{\partial R}$$
(26)

$$w''^{R} = \frac{\partial^{2} w(R,Q)}{\partial R^{2}}$$
(27)



165 | Fascicule 1



$$w'^{Q} = \frac{\partial w(R,Q)}{\partial Q}$$
(28)

$$w^{\prime\prime Q} = \frac{\partial^2 w(R,Q)}{\partial Q^2}$$
(29)

$$w^{\prime\prime RQ} = \frac{\partial^2 w(R,Q)}{\partial R \partial Q}$$
(30)

The double integrals of the squares of w''^R , w''^Q , and w''^{RQ} are calculated and given by the Eqs. (31) – (34).

$$\int_0^1 \int_0^1 (w''^R)^2 \,\partial R \,\partial Q = A^2(1.8)(0.049206) = 0.088571A^2 \tag{31}$$

$$\int_{0}^{1} \int_{0}^{1} (w''^Q)^2 \,\partial R \,\partial Q = A^2(4.8)(0.007539683) = 0.0361905A^2 \tag{32}$$

$$\int_{0}^{1} \int_{0}^{1} (w''^{RQ})^2 \,\partial R \,\partial Q = A^2(0.48571)(0.085714) = 0.0416321A^2$$
(33)

$$\int_{0}^{1} \int_{0}^{1} w^{2} \partial R \partial Q = A^{2}(0.04920635)(0.007539) = 0.00037096667A^{2}$$
(34)

Substituting these values into the fundamental frequency equation fundamental frequency for p = a/b, as:

$$\lambda^{2} = \frac{\frac{D_{x}}{b^{4}\rho h} \left[0.088571 \frac{\phi_{1}}{p^{4}} + 0.0416321 * \frac{2\phi_{2}}{p^{2}} + 0.0361905\phi_{3} \right]}{0.00037096667}$$
(35)

The Eq. (35) further reduces to:

$$\lambda^{2} = \frac{D_{x}}{b^{4}\rho h} \left[238.757 \frac{\phi_{1}}{p^{4}} + 224.452 \frac{\phi_{2}}{p^{2}} + 97.557 \phi_{3} \right]$$
(36)

Re-arranging Eq. (36) in terms of plate dimension 'a' and aspect ratio, p, gives: $\lambda^2 = \frac{D_x}{4} [238.757\phi_1 + 224.452\phi_2 p^2 + 97.557\phi_3 p^4]$ (37)

$$e^{2} = \frac{\frac{D_{X}}{a^{4}\rho h} \left[0.088571 \phi_{1} + 0.0416321 * \frac{2\phi_{2}a^{2}}{b^{2}} + 0.0361905 * \frac{\phi_{3}a^{4}}{b^{4}} \right]}{0.00037006667}$$

The Eq. (38) reduces further to:

$$= \frac{D_x}{a^4 \rho h} \left[238.757 \phi_1 + 224.452 \frac{\phi_2 a^2}{b^2} + 97.557 \frac{\phi_3 a^4}{b^4} \right]$$
(39)

Then, for the reciprocal of the aspect ratio, that is, p = b/a:

$$\lambda^{2} = \frac{D_{x}}{a^{4}\rho h} \left[238.757 \phi_{1} + 224.452 \frac{\phi_{2}}{p^{2}} + 97.557 \frac{\phi_{3}}{p^{4}} \right]$$
(40)

where ϕ_1 , ϕ_2 , and ϕ_3 are as defined in Eqs. (18a) – (18c).

 λ^2

The square roots of Eqs. (36), (39), and (40) give the required expressions for computing the fundamental frequencies, λ , of an SSSC rectangular thin orthotropic plate experiencing free vibration. **RESULTS AND DISCUSSION**

The square root of Eq. (40) was used to study the variation of fundamental frequency, λ , with various aspect ratios, p, and combinations of flexural rigidities, ϕ_1 , ϕ_2 , and ϕ_3 . The results were validated by comparing them with solutions obtained by Pilkey [8]. The equations of square of fundamental frequency derived by Pilkey [8] for a vibrating SSSC thin rectangular orthotropic plate for p = a/b and α = b/a, respectively, are:

$$\lambda^{2} = \frac{1}{b^{4}\bar{m}} \left[\frac{237.815D_{x}}{p^{4}} + \frac{283.61B}{p^{2}} + 97.409D_{y} \right]$$
(41)

$$\lambda^{2} = \frac{1}{a^{4}\bar{m}} \left[237.815 D_{x} + \frac{283.61B}{\alpha^{2}} + \frac{97.409 D_{y}}{\alpha^{2}} \right]$$
(42)



Figure 2: Graph of fundamental frequency against aspect ratio for various combinations of flexural rigidity

where p and α represent the aspect ratios of the plate in terms of a/b and b/a, respectively.

The results obtained in this work and those of Pilkey [8], are presented in Tables 1 – 3. A cursory look at Tables 1 – 3, shows that the fundamental frequencies, λ , obtained by Pilkey [8] agree very closely with the solution obtained in this work for various aspect ratios and combinations of flexural rigidities, ϕ_i (where i = 1, 2, and 3). As a matter of fact, the maximum percentage difference of –5.087% occurred when the aspect ratio, p, is equal to 0.8 and the flexural rigidities, ϕ_1 , ϕ_2 , and ϕ_3 are all equal to unity. The average percentage



(38)



differences of the fundamental frequency for flexural rigidity ratios, φ_1 , φ_2 , and φ_3 in Tables 1, 2, and 3, are -3.404%, -2.029%, and -2.456%, respectively. The closeness of the two results (i.e. results from this paper and Pilkey's solution) increases with increasing aspect ratio, p.

The graph of fundamental frequency, λ , obtained in this work versus aspect ratio, p = b/a (ranging from values 0.3 to 1), was plotted for various combinations of flexural rigidities, ϕ_i , as shown in Figure 2. From Figure 2, it was observed that the frequency, λ , decreased with increasing aspect ratio, p, and decreasing flexural rigidities, ϕ_i . The curves tend to converge as the aspect ratio, p, approaches unity, and becomes parallel to the aspect ratio, p, axis.

Aspect	Fundamental frequency	Fundamental frequency, λ		Difformer	Percentage difference
ratio,	squared, λ^2 from the	Present work,	Pilkey's	(2 - 2)	$\left[\frac{(\lambda_1 - \lambda_2)}{0}\right]_{0/6}$
p = b/a	present work	λ ₁	solution, λ_2	$(\lambda_1 - \lambda_2)$	$\begin{bmatrix} \lambda_2 \end{bmatrix}$ 70
0.1	998254	999.127	1001.344	-2.217	-0.221
0.2	66823.18	258.502	261.168	-2.666	-1.021
0.3	14776.74	121.560	124.157	-2.597	-2.092
0.4	5454.402	73.840	76.259	-2.419	-3.172
0.5	2697.477	51.937	54.137	-2.200	-4.064
0.6	1614.989	40.187	42.157	-1.970	-4.673
0.7	1103.141	33.214	34.962	-1.748	-5.000
0.8	827.6395	28.769	30.311	-1.542	-5.087
0.9	664.5505	25.779	27.137	-1.358	-5.004
1.0	560.7760	23.680	24.876	-1.196	-4.808
1.1	490.8873	22.156	23.211	-1.055	-4.545
1.2	441.6736	21.016	21.949	-0.933	-4.251
1.3	405.7263	20.143	20.970	-0.827	-3.944
1.4	378.6682	19.459	20.196	-0.737	-3.649
1.5	357.7840	18.915	19.573	-0.658	-3.362
1.6	341.3196	18.475	19.065	-0.590	-3.095
1.7	328.1026	18.114	18.644	-0.530	-2.843
1.8	317.3256	17.814	18.293	-0.479	-2.618
1.9	308.4180	17.562	17.996	-0.434	-2.411
2.0	300.9673	17.348	17.743	-0.395	-2.226

Table 1: Fundamental frequencies of SSSC plate under free vibration for flexural rigidity, $\varphi_1 = \varphi_2 = \varphi_3 = 1$, and aspect ratio, p = b/a

Table 2: Fundamental frequencies of SSSC plate under free vibration for flexural rigidities, $\phi_1 = 1, \phi_2 = 0.5, \phi_3 = 1$, and aspect ratio, p = b/a

Aspect	Fundamental frequency	Fundamental frequency, λ		Difforma	Percentage difference
ratio,	squared, λ^2 from the	Present work,	Pilkey's	$(\lambda - \lambda)$	$\left[\frac{(\lambda_1 - \lambda_2)}{0}\right] 0/6$
p = b/a	present work	λ ₁	solution, λ_2	$(\lambda_1 - \lambda_2)$	$\begin{bmatrix} \lambda_2 \end{bmatrix}^{70}$
0.1	987031.4	993.495	994.238	-0.743	-0.075
0.2	64017.53	253.017	254.290	-1.273	-0.501
0.3	13529.79	116.318	117.640	-1.322	-1.124
0.4	4750.990	68.927	70.208	-1.281	-1.825
0.5	2248.573	47.419	48.617	-1.198	-2.464
0.6	1303.251	36.101	37.193	-1.092	-2.936
0.7	874.1079	29.565	30.544	-0.979	-3.205
0.8	652.2864	25.540	26.405	-0.865	-3.276
0.9	525.9999	22.935	23.693	-0.758	-3.199
1.0	448.5400	21.179	21.841	-0.662	-3.031
1.1	398.1385	19.953	20.531	-0.578	-2.815
1.2	363.7389	19.072	19.577	-0.505	-2.580
1.3	339.3203	18.421	18.863	-0.442	-2.343
1.4	321.4101	17.928	18.317	-0.389	-2.124
1.5	307.9057	17.547	17.891	-0.344	-1.923
1.6	297.4813	17.248	17.552	-0.304	-1.732
1.7	289.2701	17.008	17.278	-0.270	-1.563
1.8	282.6879	16.813	17.055	-0.242	-1.419
1.9	277.3304	16.653	16.869	-0.216	-1.280
2.0	272.9108	16.520	16.714	-0.194	-1.161





Aspect	Fundamental frequency	Fundamental frequency, λ		Difforence	Percentage difference
ratio,	squared, λ^2 from the	Present work,	Pilkey's	$(\lambda - \lambda)$	$\begin{bmatrix} (\lambda_1 - \lambda_2) \end{bmatrix} 0/6$
p = b/a	present work	λ_1	solution, λ_2	$(n_1 - n_2)$	$\begin{bmatrix} \lambda_2 \end{bmatrix}^{\prime 0}$
0.1	499246.4	706.574	708.109	-1.535	-0.217
0.2	33530.97	183.115	184.988	-1.873	-1.012
0.3	7507.750	86.647	88.463	-1.816	-2.053
0.4	2845.580	53.344	55.013	-1.669	-3.034
0.5	1468.117	38.316	39.802	-1.486	-3.733
0.6	926.8732	30.445	31.741	-1.296	-4.083
0.7	670.9488	25.903	27.019	-1.116	-4.130
0.8	533.1983	23.091	24.047	-0.956	-3.976
0.9	451.6538	21.252	22.070	-0.818	-3.706
1.0	399.7615	19.994	20.696	-0.702	-3.392
1.1	364.8221	19.100	19.704	-0.604	-3.065
1.2	340.2153	18.445	18.968	-0.523	-2.757
1.3	322.2416	17.951	18.406	-0.455	-2.472
1.4	308.7126	17.570	17.968	-0.398	-2.215
1.5	298.2705	17.271	17.620	-0.349	-1.981
1.6	290.0383	17.031	17.339	-0.308	-1.776
1.7	283.4298	16.835	17.109	-0.274	-1.601
1.8	278.0413	16.675	16.918	-0.243	-1.436
1.9	273.5875	16.540	16.758	-0.218	-1.301
2.0	269.8622	16.427	16.622	-0.195	-1.173

Table 3: Fundamental frequencies of SSSC plate under free vibration for flexural rigidities,
$\omega_{r} = 1, \omega_{r} = 0.5, \omega_{r} = 0.5$ and aspect ratio, $p = b/a$

CONCLUSION

An approximate method for analyzing the vibration characteristics of a freely vibrating rectangular thin orthotropic plate using Taylor-Maclaurin series formulated shape function was presented in this work. The closeness of the results of this work with Pilkey's solution, indicates that the newly derived equations given in Eqs. (39) and (40) can be used to compute fundamental frequencies of a free vibrating SSSC rectangular thin orthotropic plate. The use of Taylor-Maclaurin series in Rayleigh-Ritz method to formulate new equations for the SSSC rectangular thin orthotropic plate is a simple and versatile approach that can be applied to various rectangular plate cases.

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