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MECHANICAL BUCKLING OF DOUBLE-LAYERED GRAPHENE SHEETS EMBEDDED IN AN ELASTIC MEDIUM USING A NONLOCAL NEW FIRST-ORDER DEFORMATION THEORY

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Abstract: This paper investigates the buckling behavior of double-layered graphene sheets (GSs) with vaious boundary conditions. It is assumed that two graphene sheets are bonded by an internal elastic medium and surrounded by external elastic fundation. Governing equations are derived based on new first-order sher deformation theory (NFSDT), while the expressions for the buckling load are given in explicit form for a graphene sheets with all edges simply supported (SSSS) and all edges clamped (CCCC). The equations of equilibrium of the nonlocal model have been derived by using the vertical displacement method. Two characteristic types of buckling are considered (in-phase buckling and out-of-phase buckling). The influence of small scale coefficient, aspect ratio, stiffness of internal elastic medium and external elastic fundation on the nonlocal buckling load is explained.
Keywords: Analytical modelling, Buckling, Nano-structures

1. INTRODUCTION

The double- nanoplate systems (DNPS) can be found in nanocomposites structures such as multiple GSs dispersed in a polymer matrix. DNPS consist of two GSs bonded by an elastic medium, e.g. polymer resin. The internal elastic medium between the two layers of GSs is modelled in the Winkler model. The surrounding elastic medium is modelled by the two-parameter Pasternak-type foundation.The first parameter presents normal pressure, while the other presents the influence of shear stress. Following the production of carbon nanotubes (CNTs) [1] and GSs [2] these two nanostructural elements, due to their extraordinary mechanical, chemical, electronic and thermal conductivity properties [3-5] have very often been used as components in microelectro-mechanical systems (MEMS) and nano electro-mechanical systems (NEMS). Composite materials are one of the most significant applications of multilayer nanoplate structures.

Every potential application of GSs requires very good knowledge of their mechanical behaviour. Because of the very small dimensions of nanostructure elements, it is very difficult to perform experimental research. Also, molecular dynamic (MD) simulation is highly computationally expensive in the analysis of GSs with large numbers of atoms. Behfar and Naghdabadi [6] used classical elasticity theory to investigate the vibration behaviour of orthotropic multi-layered graphene sheets embedded in an elastic medium. Liew et al. [7] investigated the vibrations of isotropic multi-layered GSs embedded in an elastic matrix using a classical continuum model. Classical elasticity theory is a scale free theory and cannot handle such small effects. As a result, the mechanical behaviour of the nanostructures cannot be described by classical elasticity theory successfully. This made Eringen formulates nonlocal elasticity theory [8] as a modification of the classical elasticity theory. In Eringen's nonlocal elasticity theory the small scale effects have been taken into account assuming that the stress tensor in the observed point depends on the deformation tensor in all other points of the entire domain occupied by the material. In the classical elasticity theory, the state of stress in the observed point depends on the state of deformation in that point only. Peddiesson et al. [9] were the first to apply the theory of nonlocal elasticity to the analysis of static deformations of Euler-Bernoulli nanobeams. Sudak [10] applied nonlocal continuum mechanics to the buckling analysis of multiwalled CNTs. Duan and Wang [11] were

the first to formulate the nonlocal continuum plate model while researching the influence of small scale parameter on the bending of circular nanoplates. Sakhaee-Pouret al. [12] applied the nonlocal continuum plate model for the vibration analysis of single-layered GSs.

Recently, by applying the nonlocal continuum plate model, a number of papers concerned with the analysis of the mechanical behaviour of double-nanoplates systems have been published. Murmu and Adhikari [13] studied the nonlocal vibration of bonded double-nanoplate systems. Murmu et al. [14] investigated nonlocal buckling of double-nanoplate-systems under biaxial compression. Pouresmaeeli et al. [15] presented the vibration of double-orthotropic nanoplates embedded in an elastic medium. The external medium was modelled as Winkler type foundation. Recently, Radić and Jeremić [16] studied the thermal buckling of double-layered grapheme sheets embedded in Pasternak elastic medium using nonlocal new first-order shear deformation theory. Their work drew from Kirchhoff's nonlocal plate theory.

Pradhan and Phadikar [17] reformulated the classical plate theory (CLPT) and first-order shear deformation theory (FSDT) of plates by using Eringen's nonlocal differential constitutive relation. The derived equations were solved for the case of simply supported boundary conditions by applying Navier's approach. Pradhan [18] analyzedthe buckling behaviour of grapheme by using the reformulated higher- order shear deformation theory (HSDT). Recently, Hashemi and Samaei [19] have analysed the buckling behaviour of micro/nanoscale plates based on nonlocal first-order shear deformation theory (FSDT). Samaei et al. [20] also used nonlocal FSDT to analyse the buckling behaviour of a single-layered graphene sheet embedded in an elastic medium. Thai et al. [21] investigated the bending, buckling and free vibration of functionally graded sandwich plates under various boundary conditions using a new first-order shear deformation theory (NFSDT). A verification study demonstrated that the NFSDT was more accurate in relation to FSDT and comparable to higher-order shear deformation theory (HSDT). Besides that, NFSDT is significantly simpler compared to FSDT and HSDT because of its lower number of unknown parameters.

In this paper, using Eringen's nonlocal differential constitutive relation local NFSDT has been reformulated and used for the analysis of nonlocal buckling load of double-layered GSs under various boundary conditions. Two different boundary conditions have been discussed in the analysis of the nonlocal buckling load of double-layered GSs.

2. THEORETICAL FORMULATION

Double-layered GSs are surrounded by an external elastic medium. For mathematical modelling, the connection between the two layers of GSs is modelled with identical vertical springs whose stiffness is C. The external medium is modelled as a Pasternak type foundation, which includes Winkler modulus parameter K_W and shear modulus parameter K_G of the external elastic medium. Both GSs layers have the length L_X, width L_y, and thickness *h.* We will associate the nanoplates with the coordinate system in which the x- and y-axes are located in the mid plane, (z=0), while the coordinate beginning is placed in the corner of the nanoplate, as shown in Figure 1.

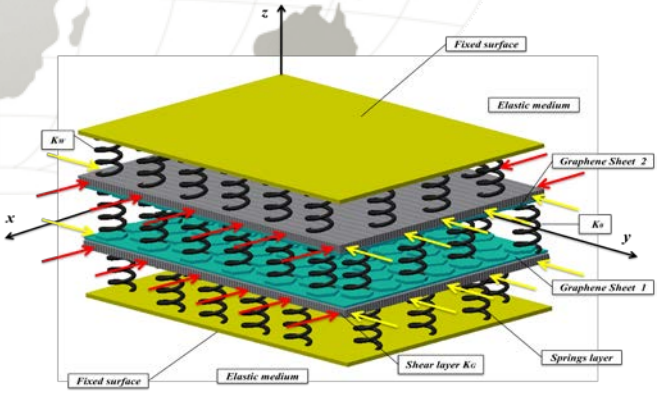

Figure 1. Double-layered graphene sheet embedded in an elastic medium

2.1. Nonlocal elasticity theory for isotropic nanoplate

Nonlocal elastic theory assumes that the stress state at a reference point *x* in an elastic continuum depends not only on strain at that point but also on the strain states at all other points *x'*on the domain. The basic equations for a linear, homogenous, isotropic, nonlocal elastic body are given by Ref. [20]. The components of nonlocal stress tensor $\sigma_{ij}(x)$ are given in the form of the following expression:

$$\sigma_{ij}(x) = \int_V \lambda(|x'-x|,\xi)\sigma_{ij}^L(x')dV(x') \quad i,j = x,y,z \tag{1}$$

where $\sigma_{ij}^L(x')$ is the classical or local stress tensor at any point x' in the body, kernel function $\lambda(|x'-x|,\xi)$ represents the nonlocal modulus and it defines the influence of deformation in the point x' to the stress in the observed point x, $\xi = e_0 a/\ell$ is a material constant that depends on the internal

characteristic length a (e.g. lattice parameter, granular size, distance between C-C bonds) and external characteristic length ℓ (e.g. crack length, wave length); and e_0 is the constant which depends on the type of material and it is determined independently for each material, on the basis of experimental results or some other valid method. A classical or local stress tensor at any point x' can be expressed via the deformation tensor by the generalized Hook's law:

$$\sigma_{ij}^{L}(x') = C_{ijkl}\varepsilon_{kl}(x') \tag{2}$$

where C_{ijkl} is the fourth-order elasticity tensor and $\varepsilon_{kl}(x')$ is the strain tensor.

The linear diferential operator is an aproximate model of the kernel obtained by matching the Fourier transforms of the kernel in the wave number space with the dispersion curves of lattice dynamics. It can be shown that the linear operator operator for the two-dimensional case has the following form:

$$¢ = 1-(e_0 a)^2 \nabla^2 \tag{3}$$

where ∇^2 is the Laplacian operator which is defined by $\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$, $e_0 a$ is the nonlocal parameter.

2.2. The generalized displacement field

The displacement field for the new FSDT is given by

$$u_x(x,y,z) = u(x,y) - z\frac{\partial \varphi}{\partial x} \tag{4}$$

$$u_y(x,y,z) = v(x,y) - z\frac{\partial \varphi}{\partial y} \tag{5}$$

$$u_z(x,y,z) = w(x,y) \tag{6}$$

where u, v, w, $\partial\varphi/\partial x$ and $\partial\varphi/\partial y$ are unknown displacement functions. Nonlocal constitutive equations for one nanoplate have the following form:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{Bmatrix} - (e_0 a)^2\nabla^2\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{Bmatrix} = \begin{Bmatrix} C_{11} & C_{12} & 0 & 0 & 0\\ C_{21} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{66} & 0 & 0\\ 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & C_{44}\end{Bmatrix} \tag{7}$$

where elastic constants C_{ij} depend on Young's modulus E and Poisson's ratio ν

$$C_{11} = C_{22} = \frac{E}{1-\nu^2},\quad C_{12} = C_{21} = \frac{\nu E}{1-\nu^2},\quad C_{44} = C_{55} = C_{66} = \frac{E}{2(1+\nu)} \tag{8}$$

3. EQUATIONS OF EQUILIBRIUM

Equations of equilibrium have been derived by using the virtual displacement method, which presents a special case of Hamilton's principle for static problems. According to the principle of virtual displacement, the first variation of potential energy of the system must be equal to zero

$$\delta U + \delta V = 0 \tag{9}$$

where δU is virtual strain energy and δV is the virtual work done by the applied forces of the external elastic foundation, internal elastic medium, and in-plane forces.

In order to solve the problem of the buckling of one nanoplate by applying NFSDT we need to solve the system of two partial differential equations (equations that correspond to the displacements due to bending $\delta w_1, \delta\varphi_1$):

$$\delta w_1:\ K_s G h\nabla^2(w_1-\varphi_1) + \left[1-(e_0 a)^2\nabla^2\left(-K_W w_1 + K_G\nabla^2 w_1 + C(w_2-w_1) + N_{cr}\nabla^2 w_1\right)\right] = 0 \tag{10}$$

$$\delta\varphi_1:\ D\nabla^4\varphi_1 + K_s G h\nabla^2(w_1-\varphi_1) = 0 \tag{11}$$

Eqs. (10) and (11) are related to graphene sheet 1. In the same way the equations of equilibrium for graphene sheet 2 are obtained:

$$\delta w_2:\ K_s G h\nabla^2(w_2-\varphi_2) + \left[1-(e_0 a)^2\nabla^2\left(-K_W w_2 + K_G\nabla^2 w_2 + C(w_1-w_2) + N_{cr}\nabla^2 w_2\right)\right] = 0 \tag{12}$$

$$\delta\varphi_2:\ D\nabla^4\varphi_2 + K_s G h\nabla^2(w_2-\varphi_2) = 0 \tag{13}$$

In Ref. [21], the use of shear correction factor has been avoided, so that transverse shear stiffness exists in equations of equilibrium, which is calculated directly from the equations of equilibrium via the transverse shear stress. In the case of nanoplates, it can be proved easily that the expression for the

transverse shear stiffness is reduced to the value $K_s Gh$ which occurs in the FSDT equations and it has been used in Eqs. (10)-(13).

4. ANALYTICAL BUCKLING SOLUTIONS

Two different ways of nanoplate support will be explored in this paper. The edge of a nanoplate in our cases can be clamped (C) or simply supported (S). For NFSDT these two boundary conditions are mathematically expressed in the following ways:

clamped edge (C)

$$w_i = \frac{\partial \varphi_i}{\partial x} = \frac{\partial \varphi_i}{\partial y} = 0, \text{ at } x=0,L_x$$

$$w_i = \frac{\partial \varphi_i}{\partial x} = \frac{\partial \varphi_i}{\partial y} = 0, \text{ at } y=0,L_y \qquad i=1,2; \tag{14}$$

and simply supported edge (S)

$$M_{xxi} = w_i = \frac{\partial \varphi_i}{\partial y} = 0, \text{ at } x=0,L_x$$

$$M_{yyi} = w_i = \frac{\partial \varphi_i}{\partial x} = 0, \text{ at } y=0,L_y \quad i=1,2; \tag{15}$$

The solution of the system of partial differential Eqs. (10-13) under boundary conditions which are defined in Eqs. (14-15), can be obtained if we assume the displacement functions in the following form

$$\begin{aligned} w_1(x,y) &= W_{1mn} X(x) Y(y) \\ \varphi_1(x,y) &= \varphi_{1mn} X(x) Y(y) \\ w_2(x,y) &= W_{2mn} X(x) Y(y) \\ \varphi_2(x,y) &= \varphi_{2mn} X(x) Y(y) \end{aligned} \tag{16}$$

where $(W_{1mn}, \varphi_{1mn}, W_{2mn}, \varphi_{2mn})$ are the unknown coefficients.

The functions $X(x)$ and $Y(y)$ are given in Table 1. and suggested in Ref. [21] and [22] so that they satisfy different boundary conditions in Eqs. (14)-(15).

Table 1. The admissible functions X(x) and Y(y).

Boundary conditions					The functions X(x) and Y(y)	
Notation	x=0	y=0	X=Lx	Y=Ly	X(x)	Y(y)
SSSS	S	S	S	S	$\sin(\alpha x)$	$\sin(\beta y)$
CCCC	C	C	C	C	$\sin^2(\alpha x)$	$\sin^2(\beta y)$

In that, $\alpha = m\pi/L_X$ and $\beta = n\pi/L_y$, m and n are the half wave numbers. Substituting Eq. (16) into Eqs. (10-13) the analytical solution can be obtained from

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 \\ A_{21} & A_{22} & 0 & 0 \\ A_{31} & 0 & A_{33} & A_{34} \\ 0 & 0 & A_{43} & A_{44} \end{bmatrix} \begin{Bmatrix} W_{1mn} \\ \varphi_{1mn} \\ W_{2mn} \\ \varphi_{2mn} \end{Bmatrix} = 0 \tag{17}$$

In this section by solving Eq.(17), the values of the critical buckling load will be determined for two different support conditions.

By solving Eq. (17) the values of critical buckling load obtained for two characteristic buckling cases.

$$N_{cr1} = \frac{K_s GhDV_1V_2 + \left[K_W(\mu V_1 - V_3) + K_G(V_1 - \mu V_2)\right](DV_2 - K_s GhV_1)}{(V_1 - \mu V_2)(DV_2 - K_s GhV_1)} \tag{18}$$

$$N_{cr2} = \frac{K_s GhDV_1V_2 + \left[(K_W + 2C)(\mu V_1 - V_3) + K_G(V_1 - \mu V_2)\right](DV_2 - K_s GhV_1)}{(V_1 - \mu V_2)(DV_2 - K_s GhV_1)} \tag{19}$$

The equations (18) and (19) are valid in the case of SSSS and CCCC and the mark $\mu = (e_0 a)^2$ has been used.

The values for V_1, V_2, and V_3 depend on the type of nanoplates support:

a) SSSS nanoplates: $V_1 = -\frac{L_x L_y}{4}(\alpha^2 + \beta^2)$, $\quad V_2 = \frac{L_x L_y}{4}(\alpha^2 + \beta^2)^2$, $\quad V_3 = \frac{L_x L_y}{4}$

b) CCCC nanoplates: $V_1 = -\frac{3L_x L_y}{16}(\alpha^2 + \beta^2)$, $\quad V_2 = \frac{L_x L_y}{2}\left(\frac{3}{2}\alpha^4 + \alpha^2\beta^2 + \frac{3}{2}\beta^4\right)$, $\quad V_3 = \frac{9L_x L_y}{64}$

In the case of in-phase buckling, the two nanoplates buckled synchronously ($w_1 = w_2$). In the case of out-of-phase buckling, the two nanoplates buckled asynchronously ($w_1 \neq w_2, w_1 = -w_2$). It is known that in the case of in-phase buckling, the critical buckling load does not depend on the stiffness modulus C , which is different from out-of-phase buckling where the critical buckling load is proportional to the double value of the stiffness parameter C .

5. NUMERICAL RESULTS

This section shows a few numerical examples of the buckling behaviour of double-layered GSs that have been embedded in a Pasternak elastic medium. The mechanical properties of GSs are the same as in Ref. [25]. Although the value of the nonlocal parameter $e_0 a$ is not accurately known for the GSs, it can be seen from the available literature that its most often value for GSs and CNTsis $e_0 a = 0 \div 2\,nm$. The nonlocal parameter has been taken in the same interval in Ref. [13,25,26].

The various non-dimensional parameters used are

$$K_{WN} = \frac{K_W L_x^4}{D}, \quad K_{GN} = \frac{K_G L_x^2}{D}, \quad C_N = \frac{C L_x^4}{D}$$

Table 2. Validation of the results for critical buckling load for single-layered graphene sheets for all edges simply supported obtained from molecular dynamic simulation.

Lx = Ly(nm)	MD simulation (nN/nm)	Present Study (nN/nm)
4.99	1.0837	1.0662
10.77	0.4331	0.4320
18.51	0.1714	0.1740
26.22	0.0889	0.0911
33.85	0.0554	0.0558
41.78	0.0372	0.0370

In Table 2, the comparison of results is given for critical buckling loads on square monolayer simple-supported GSs with the results obtained by the application of molecular dynamic simulation in Ref. [23]. Young's modulus of the GSs is assumed as E = 1 TPa, Poisson's ratio ν =0.16, h =0.34 nm, $K_w = K_G = 0$, and $C = 0$. From Table 2, a very high concordance of the results in the present study with the results in Ref. [11] can be noticed. In Ref. [23], it has been demonstrated that the optimal value of the nonlocal parameter for First-Order Shear Deformation Theory is $(e_0 a)^2 = 1.81 nm^2$. The value of the Winkler modulus parameter K_{WN} , for the surrounding polymer matrix is varied from 0 to 500, what for $L_x = 10nm$ corresponds to the change $K_W = 0\,GPa/nm \div 0.185\,GPa/nm$, while the shear modulus parameter K_{GN} is varied from 0 to 20,what for $L_x = 10nm$ corresponds to the change $K_G = 0\,nN/nm \div 0.771\,nN/nm$.

In Ref. [27] the value for C is varied from 0.005 GPa/nm to 0.1 GPa/nm . In a dimensionless form it corresponds to the change C_N from 5 to 270. For the present study, the transverse shear correction factor is taken as 0.8667. The same value for the transverse shear correction factor has been taken in Ref. [19]. In the following figures on the coordinate axis, the values 10^{-4} N_{cr} are used.

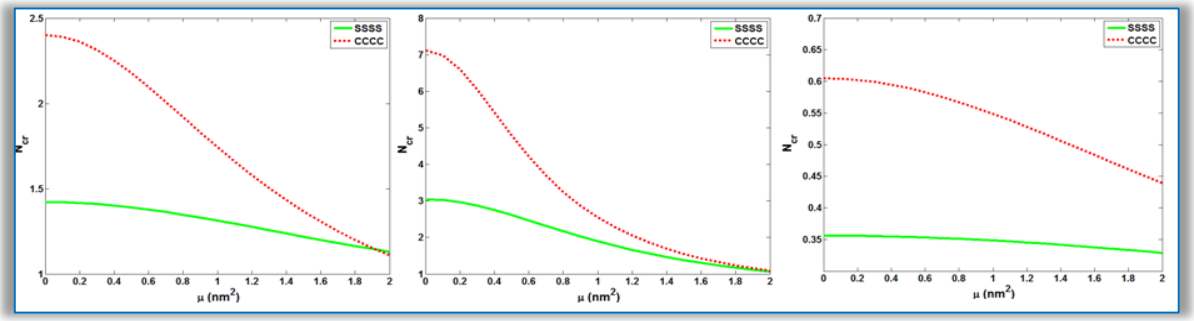

Figure 2. Small scale effect on critical buckling load of double-layered graphene sheets at various nonlocal parameters and boundary conditions for in-phase buckling $(K_{WN} = 250, K_{GN} = 10, C_N = 0)$ for (a) $m = n = 1, L_x = L_y = 10\,nm$, (b) $m = n = 2, L_x = L_y = 10\,nm$ and (c) $m = n = 1, L_x = L_y = 20\,nm$.

In all three figures, the highest value of the critical buckling load is for CCCC nanoplates, while the lowest value is for SSSS nanoplates. In all cases, with the increase of the nonlocal parameter value, the value of critical buckling load is decreased nonlinearly. The highest nonlinearity is present at CCCC nanoplates, and the lowest at SSSS nanoplates.

The nonlinearity increases with the increase of the half wave number, so that, in cases where $L_x = L_y = 10\,nm$ and $m = n = 2$ are at higher values for nonlocal parameter the value of critical buckling load becomes very close for two observed cases. It is known that with the increase of the number of half-waves the intensity of nonlocal effect increases. It is obvious that the influence of nonlocal effect on the decrease of the value of critical buckling load depends very intensively on the way in which the nanoplates are supported. At higher values of the nonlocal parameter, the difference in the value of critical buckling load is significant for all two boundary conditions.

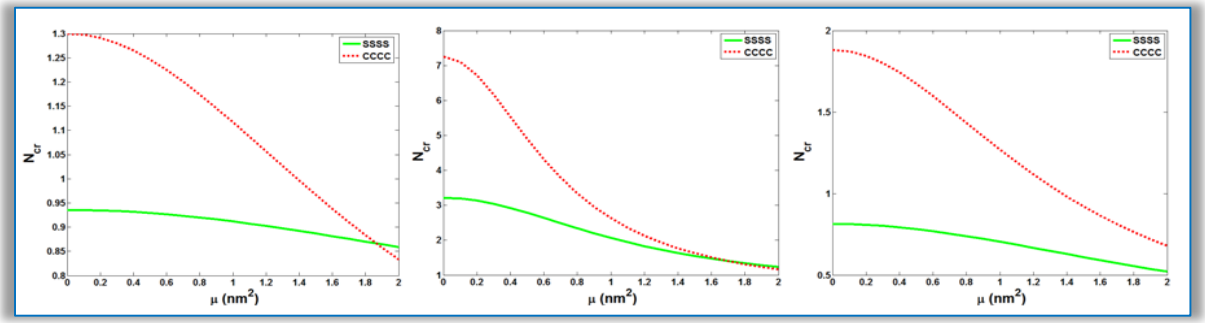

Figure 3. Small scale effect on critical buckling load of double-layered graphene sheets at various nonlocal parameters and boundary conditions for out-of-phase buckling
($K_{WN} = 250, K_{GN} = 10, C_N = 200$) for (a) $m = n = 1, L_x = L_y = 15\,nm$, (b) $m = n = 2, L_x = L_y = 10\,nm$ and (c) $m = n = 2, L_x = L_y = 20\,nm$

In Figure 3b, it can be noticed that in the case of out-of- phase buckling at $L_x = L_y = 10\,nm$ and $m = n = 2$ at high values of nonlocal parameter ($\mu \approx 2\,nm^2$) the value of critical buckling load is very close for the case of CCCC and SSSS nanoplates. In the case of Figure 3a, where $L_x = L_y = 15nm$ and $m = n = 1$ the buckling behaviour is the same as in the case of the in-phase buckling in Figure 2a. In the case of Figure 3c when $L_x = L_y = 20\,nm$ and $m = n = 2$, it can be seen that the difference in the values of critical buckling load at higher values of nonlocal parameters is important for all two ways of support.

6. CONCLUSIONS

In this paper is applied a new nonlocal first-order shear deformation theory to illustrate the nonlocal buckling behaviour of double-layered GSs embedded in an elastic medium. The equations of equilibrium are derived by using the virtual displacement method. The critical buckling load was obtained analytically for two characteristic cases of buckling. Numerical results discuss the effects of the nonlocal parameter, plate aspect ratio, elastic foundation stiffnesses and boundary conditions on the critical buckling load for two characteristic cases of buckling of the double-layered GSs. The following conclusions may be drawn from the present paper:

1. For both characteristic cases of buckling, the value of the critical buckling load decreases with the increase of the value of the nonlocal parameter. The way in which the plates are supported significantly influences the intensity of the decrease of the critical buckling load.
2. The nonlocal effect is the most prominent at CCCC nanoplates, and the lowest at SSSS nanoplates for both characteristic cases.
3. In both characteristic cases of buckling, the value of the critical buckling load decreases with the increase of the nonlocal parameter value. That decrease of the value of the critical buckling load is the most intensive at CCCC nanoplates (Fig 2.b and Fig. 3.b).

Note

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