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HOMOTOPY ANALYSIS OF MAGNETIC FIELD EFFECT ON FREE CONVECTION FLOW PAST A SEMI-INFINITE FLAT PLATE

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Abstract: In this paper, Homotopy Analysis Method (HAM) is implemented to obtain an approximate solution (a non-zero auxiliary function) to the coupled nonlinear ordinary differential equations emerging from the magnetic field effect on free convection flow of an incompressible, viscous laminar fluid in the presence of buoyancy force past a semi-infinite flat plate. The result obtained by this method provides a discussion on the effects of these parameters on velocity, temperature and concentration profiles, the skin-friction coefficient, rate of heat transfer and Sherwood and validates the potential applicability of HAM in solving partial differential equations.
Keywords: Semi-infinite flat plate, HAM, non-zero auxiliary function, Sherwood number, magnetic field, approximate solution

1. INTRODUCTION

Free convective heat and mass transfer have been studied due to their application in metrology, astrophysics, geophysics, solar physics, aeronautics and electronics. This has prompted many researchers into this grey area. Similarly, the use of approximate solution methods as an alternative to validate the great potential of the methods in solving partial differential equations has attracted many researchers. Liao (1992) employed the basic ideas of the Homotopy in topology to propose a general analytical approach for nonlinear problems. This came to light when he researched into Homotopy analysis method (Liao, 2003). Comparison between the Homotopy analysis method and Homotopy perturbation method was looked into by Liao (2005).Majid et al. (2010) studied application of Laplace decomposition method to solve nonlinear coupled partial differential equations. Approximate solution for the coupled nonlinear equations using Homotopy analysis method was studied by Suping and Li (2011). Shit and Haldar (2012) studied the effect of thermal radiation and temperature dependent viscosity on free convective flow and mass transfer of an electrically conducting fluid over an isothermal sheet.

Farooq et al. (2012) looked into application of He's method in solving a system of nonlinear coupled equations arising in non-newtonian fluid mechanics. Thermal conductivity and its effects on compressible boundary layer flow over a circular cylinder were studied by Anselm and Koriko (2013). Animasaun and Oyem (2014) investigated the effect of variable viscosity, Dufour, Soret and thermal conductivity on free convective heat and mass transfer of non-Darcian flow past flat surface. Combined effects of viscous dissipation and magnetic field on MHD free convection flow with thermal conductivity over a vertical plate was studied by Oyem et al. (2015). Similarly, Oyem et al. (2015) studied MHD free convective heat transfer on reacting flow over a vertical plate with constant thermal conductivity. However in this paper, the Homotopy Analysis Method is used to obtain an approximate solution to a system of coupled nonlinear equations and the various effects some thermo-physical properties in free convection fluid flow past a semi-infinite flat plate.

2. PROBLEM FORMULATION

A steady two-dimensional, incompressible free convective fluid flow past a semi-infinite flat plate is considered. The flow is assumed to be taken along the plate in the x −axis and the y −axis is normal to the plate. A uniform magnetic field is applied in the direction perpendicular to the plate and all the physical properties of the fluid are assumed to be constant. When the Boussinesq's approximation is entreated under these assumptions, the flow is governed by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma\beta_0^2 u}{\rho} \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \quad (4)$$

subject to the boundary conditions

$$u=0 \quad v=0 \quad T=T_w C=C_w at y=0 \quad (5)$$

$$u\to 0 \quad T\to T_\infty C\to C_\infty as y\to\infty$$

where u, v are the velocity components in x, y directions respectively, ρ is density of the fluid, c_p is the specific heat capacity at constant pressure, υ is the kinematic viscosity, g is the acceleration due to gravity, σ is electrical conductivity, T is temperature of the fluid, β and β_0 are coefficient of volumetric expansion and magnetic field intensity, $\kappa(T)$ is variable thermal conductivity of the fluid, D is the coefficient of mass diffusivity, Q_0 is heat absorption coefficient and C is the fluid species concentration. The radiative heat flux q_r is describes by the Rosseland approximation given by:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (6)$$

where σ^* is the Stefan-Boltzman constant and k^* is the mean absorption coefficient. The temperature difference within the flow is assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature T using a truncated Taylor's series about the free stream temperature T_∞ and neglecting higher order terms, one obtains:

$$T^4=4T_\infty{}^3T-3T_\infty{}^4 \quad (7)$$

Introducing the stream functions $\psi(x,y)$ in equation (8) into equations (1) – (4),

$$u=\frac{\partial\psi}{\partial y} \quad \text{and} \quad v=-\frac{\partial\psi}{\partial x} \quad (8)$$

equation (1) is satisfied. In view of equations (6) and (7), equations(2) - (4) subject to the boundary conditions (5) is transformed using the following dimensionless quantities:

$$\eta=y\sqrt{\frac{U_0}{\upsilon x}};\ \psi=\sqrt{x\upsilon U_0}f(\eta);\ \theta=\frac{T-T_\infty}{T_w-T_\infty};\ \phi=\frac{C-C_\infty}{C_w-C_\infty};$$

$$M=\frac{\sigma\beta_0^2 x}{\rho U_0};\ P_r=\frac{\upsilon}{\alpha};\ G_r=\frac{g\beta x(T_w-T_\infty)}{U_0^2} \quad (9)$$

$$E_c=\frac{U_0^2}{c_p(T_w-T_\infty)};\ G_c=\frac{g\beta^* x(C_w-C_\infty)}{U_0^2};R_a=\frac{\kappa k^*}{4\sigma^* T_\infty^3};\ S_c=\frac{v}{D}$$

whereθ is the dimensionless temperature, $G_r$thermal Grashof number,M is the magnetic field parameter, $G_c$is Solutal Grashof number, $P_r$is Prandtl number, E_cEckert number,R_a is thethermal radiation parameter and S_c is Schmidt number. The resulting coupled nonlinear ordinary differential equation becomes

$$\frac{d^3f}{d\eta^3}+\frac{1}{2}f(\eta)\frac{d^2f}{d\eta^2}+G_r\theta(\eta)+G_c\phi(\eta)-M\frac{df}{d\eta}=0 \quad (10)$$

$$\left(1+\frac{4}{3R_a}\right)\frac{d^2\theta}{d\eta^2}+\frac{1}{2}P_r f(\eta)\frac{d\theta}{d\eta}+\left(\frac{d\theta}{d\eta}\right)^2+P_rE_c\left(\frac{d^2f}{d\eta^2}\right)^2=0 \quad (11)$$

$$\frac{d^2\phi}{d\eta^2}+\frac{1}{2}S_c f\frac{d\phi}{d\eta}=0 \quad (12)$$

with boundary and initial conditions:

$$f = 0\,,\ \ f' = 1\,,\ \ \theta = 1\,,\ \ \phi = 1 \qquad at\ \eta = 0$$
$$f' = 0\,,\ \ \theta = 0\,,\ \ \phi = 0 \qquad as\ \eta \to \infty \tag{13}$$

3. SOLUTION BY HOMOTOPY ANALYSIS METHOD (HAM)

In order to get a physical insight into the coupled nonlinear ordinary differential equations (10) – (12) subject to boundary conditions (13), a new kind of Homotopy Analysis Method (HAM) with non-zero auxiliary parameter $\hbar$ and a non-zero auxiliary function $H(\eta)$ is constructed.

Consider the differential equation:

$$N[f(\eta)] = 0 \tag{14}$$

where N is a nonlinear operator and $f(\eta)$ (the unknown function) is an approximate solution of equation (14). Let $f_o(\eta)$ be an initial approximation of $f(\eta)$, $H(\eta)$ the auxiliary function and L an auxiliary operator with the property such that:

$$L[f(\eta)] = 0 \text{ when } \ f(\eta) = 0. \tag{15}$$

Instead of using the traditional Homotopy

$$H[f(\eta; q); q] = (1-q)L[f(\eta; q) - f_o(\eta)] + qN[f(\eta; q)] \tag{16}$$

we consider a non-zero auxiliary parameter $\hbar$ and a non-zero auxiliary function $H(\eta)$ to construct a new kind of Homotopy of the form:

$$\left.\begin{aligned} H[f(\eta); q, \hbar, H(\eta)] = (1-q)L\big[f\big(\eta; q, \hbar, H(\eta)\big) - f_o(\eta)\big] \\ +q\hbar H(\eta)N\big[f\big(\eta; q, \hbar, H(\eta)\big)\big]\end{aligned}\right\} \tag{17}$$

where $q \in [0,1]$ is an embedding parameter and $f(\eta, q)$ is a function of η and q. When, $q = 0$, equation (17) becomes

$$H[f(\eta); 0, \hbar, H(\eta)\,] = L[f(\eta; 0, \hbar, H(\eta)) - f_o(\eta)] \tag{18}$$

Thus, in finding a solution of $H[f(\eta); 0, \hbar, H(\eta)] = 0$ using equation (15), the R.H.S. of equation (18) becomes

$$f(\eta; 0, \hbar, H(\eta)\,) = f_o(\eta) \tag{19}$$

which is the solution of $H[f(\eta); 0, \hbar, H(\eta)\,] = 0$. Similarly, when $q = 1$, equation (17) becomes

$$H[f(\eta); 1, \hbar, H(\eta)\,] = -\hbar H(\eta)N\big[f\big(\eta; 1, \hbar, H(\eta)\big)\big]. \tag{20}$$

Consideringequation (14) and the solutions

$$H[f(\eta); 1, \hbar, H(\eta)\,] = 0$$
$$-\hbar H(\eta)N\big[f\big(\eta; 1, \hbar, H(\eta)\big)\big] = 0$$
$$H[f(\eta; 1, \hbar, H(\eta))] = 0$$

such that $\hbar H(\eta) \neq 0$, we have

$$N\big[f\big(\eta; 1, \hbar, H(\eta)\big)\big] = N[f(\eta)] \tag{21}$$

and in algebraic form as

$$f\big(\eta; 1, \hbar, H(\eta)\big) = f(\eta). \tag{22}$$

Analysing the physical background and the initial boundary conditions of the non-linear differential problems, it is possible to know the base functions which are proper to represent the solutions, even without solving the given nonlinear problems. In view of the boundary conditions (4) and (5), $f(\eta)$ can be expressed by the base function

$$\langle \eta^j \exp(-nj)\ |j \geq 0, n \geq 0\rangle \tag{23}$$

where the solutions of $f(\eta), \theta(\eta)$ and $\phi(\eta)$ are represented in series form as:

$$f(\eta) = a_{0,0}^0 + \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{n,k}^k \eta^k \exp(-nj) \tag{24}$$

$$\theta(\eta) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} b_{n,k}^k \eta^k \exp(-nj) \tag{25}$$

$$\phi(\eta) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} c_{n,k}^k \eta^k \exp(-nj) \tag{26}$$

where, $a_{n,k}^k$, $b_{n,k}^k$, $c_{n,k}^k$ are coefficients. As long as the set of base functions are determined, the auxiliary function $H(\eta)$, initial approximations $f_o(\eta), \theta_o(\eta)$ and $\phi_o(\eta)$ and auxiliary Linear Operators L_f, L_θ, L_ϕ must be chosen in such a way that all solutions exist and can be expressed by this set of base functions. Therefore, in framing the Homotopy Analysis Method (HAM), we apply the rule of solution expressions in choosing the auxiliary function $H(\eta)$, initial approximations $f_o(\eta)$, $\theta_o(\eta)$and$\phi_o(\eta)$ and auxiliary linear operators L_f , L_θ , L_ϕ.

Invoking the rule of solution expressions for $f(\eta)$, $\theta(\eta)$and $\phi(\eta)$ for equations (1) – (3) together with the boundary conditions (4) and (5), the initial guesses for $f_o(\eta)$, $\theta_o(\eta)$and$\phi_o(\eta)$ which satisfies both the initial and boundary conditions (4) and (5) are given as:

$$\left.\begin{aligned} f_o(\eta) &= 1 - \exp(-\eta) \\ \theta_o(\eta) &= \exp(-\eta) \\ \phi_o(\eta) &= \exp(-\eta) \end{aligned}\right\} \quad (27)$$

with linear operators L_f , L_θ , L_ϕ as

$$L_f[f(\eta, q)] = \frac{\partial^3 f(\eta; q)}{\partial \eta^3} - \frac{\partial f(\eta; q)}{\partial \eta} \quad (28)$$

$$L_\theta[\theta(\eta, q)] = \frac{\partial^2 \theta(\eta; q)}{\partial \eta^2} - \theta(\eta; q) \quad (29)$$

$$L_\phi[\phi(\eta, q)] = \frac{\partial^2 \phi(\eta; q)}{\partial \eta^2} - \phi(\eta; q). \quad (30)$$

The linear operators L_f , L_θ , $L_\phi$therefore, have the following properties

$$\left.\begin{aligned} &L_f[C_1 + C_2 \exp(-\eta) + C_3 \exp(\eta)] = 0 \\ &L_\theta[C_4 \exp(-\eta) + C_5 \exp(\eta)] = 0 \\ &L_\phi[C_6 \exp(-\eta) + C_7 \exp(\eta)] = 0 \end{aligned}\right\} \quad (31)$$

where C_1 ,C_2, C_3, C_4, C_5, C_6 and C_7 are constants. The linear operators were solved using Wolfram Mathematica to obtain the values of skin friction coefficient$f''(0)$, Nusselt number$-\theta'(0)$ and Sherwood number$-\phi'(0)$ for various values of magnetic field parameterM as shown in table 1. The effect on the flow field in terms of velocity, temperature and concentration profiles are displayed in figures 1 – 15.

Table 1: Values of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$at $G_r = 1.0$, $G_m = 1.0$, $E_c = 0.5$, $S_c = 0.22$, $R_a = 0.7$, $P_r = 0.71$for various values of M

M	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	−1.307142349845885	−0.42369812683995906	−1.5668679828480405
1.0	−1.2929399313836567	−0.4358215769389489	−1.6241399768826683
1.5	−1.1561828728188863	−0.44675272644788966	−1.5964497396420863
2.0	−0.8323353371052551	−0.4502471478922309	−1.5999144272653498

4. RESULTS AND DISCUSSION

In this paper, the effect of magnetic field parameter on free convection past a semi-infinite plate had been considered. The effect of the emerging parameters; magnetic field parameter M, Prandtl number P_r, Schmidt number S_c, thermal Grashof number G_r, Solutal Grashof number G_c and Eckert number E_c on the flow characteristics are displayed in figures 1–15 and the results obtained in terms of skin friction coefficient, Nusselt number and Sherwood number for various values of magnetic field parameter are shown in table 1. It was observed from table 1 that the magnitude of the skin friction coefficient increases with increasing values of magnetic field parameter M while, the Nusselt number and Sherwood number decreases with increasing values of M.

Figures 1 − 3 depicts the effect of various values of magnetic field parameter M on velocity, temperature and concentration profiles at$G_r = 1.0$, $G_c = 1.0$, $E_c = 0.5$, $S_c = 0.22$, $R_a = 0.7$, $P_r = 0.71$. In figure 1, velocity profilesare considerably reduced with the increase in the value of M. This is because, increasing the magnetic field strength normal to the flow in an electrically conducting fluid produces a drag force known as the Lorentz force, which acts against the flow. Also, application of a moderate magnetic field normal to the flow can be used as a stabilizing mechanism, delaying the transition from laminar to turbulent flow. Figure 2 and 3 shows that increase in M, increases both in temperature and concentration profile. It shows that application of the magnetic field produces heat in the fluid and thereby reduces heat and mass transfers from the wall and making the fluid temperature and concentration distributions to increase.The effect of the Prandtl number P_r on velocity, temperature and concentration profiles are shown in Figures 4 − 5. From figures 4 and 6, it was observed that as P_r increases, velocity adn temperature profiles decreases away from the plate towards the free stream indicating that the fluid is increasingly viscous as P_r rises and magnitude of P_r substantially reduces the temperature of the fluid. But in figure 6, concentration profiles increases with increasing values of P_r.

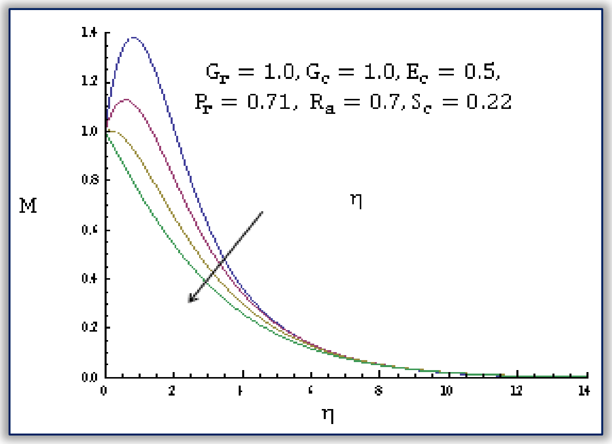

Figure 1: Effect of various values of M on velocity profiles

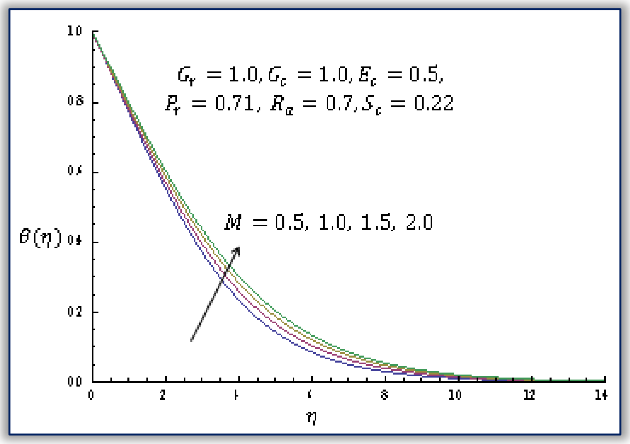

Figure 2: Temperature profiles for various values ofM

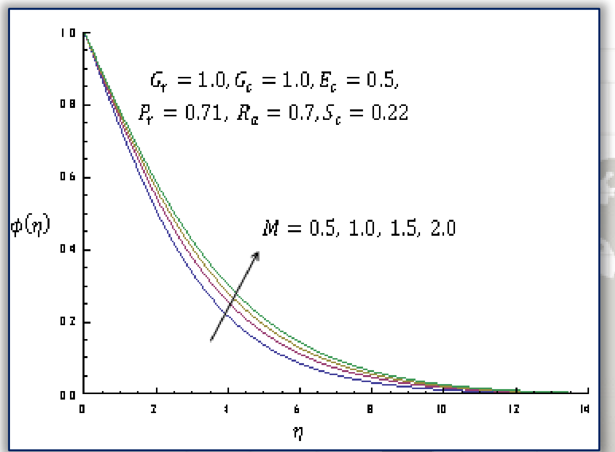

Figure 3: Effect of various values ofM on concentration profiles

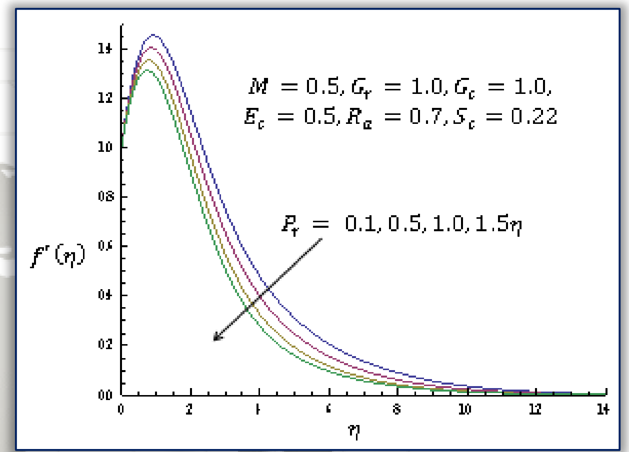

Figure 4: Effect of different values ofP_r on velocity profiles

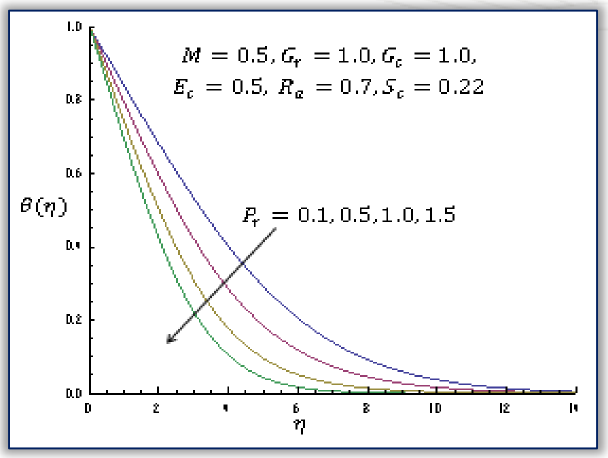

Figure 5: Effect of different values of P_r on temperature profiles

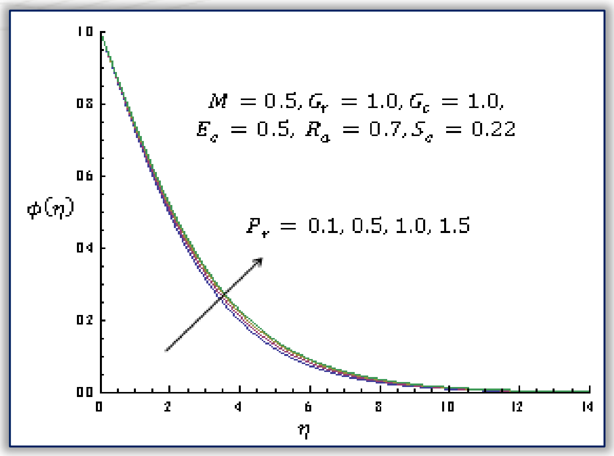

Figure 6: Effect of P_r on concentration profiles

For the precribed values of the physical parameters, the influence of the Schmidt number S_c on dimensionless velocity and concentration profiles are displayed in figures 7 and 8. Schmidt number S_c represents the ratio of momentum to the mass diffusivitythus, measuring the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers(Gebhart et al. (1988)). From figures 7 and 8, it was revealed that as S_c increases, velocity and concentraton profiles decreases along the plate towards the free stream. These reductions in the velocity and concentration profiles clearly observes that the dimensionless concentration profile decay from a maximum concentration of 1.0 at $\eta = 0$ (the wall boundary condition) to zero at the freestream. Thus, if we critically look at Schmidt number which is dependent on mass diffusion D, an increase in S_c corresponds to a decrease in mass diffusion hence, a reduction in concentration profiles.The effect of the Eckert number E_c on temperature profiles is illustrated in Figure9. Eckert number E_c expresses the ratio of a flow's kinetic energy to the boundary layer enthalpy difference and is used to characterize viscous thermal dissipation of convection (Eckert and Darake (1972), Cengel and Cimbala (2006)). The influence of viscous dissipation on the flow field is to increase the energy, producing greater fluid temperature and buoyancy force. Figure 9 therefore depicts that temperature profiles increases with increasing values in $E_c$thereby, boosting temperatureas internal energy is increased.

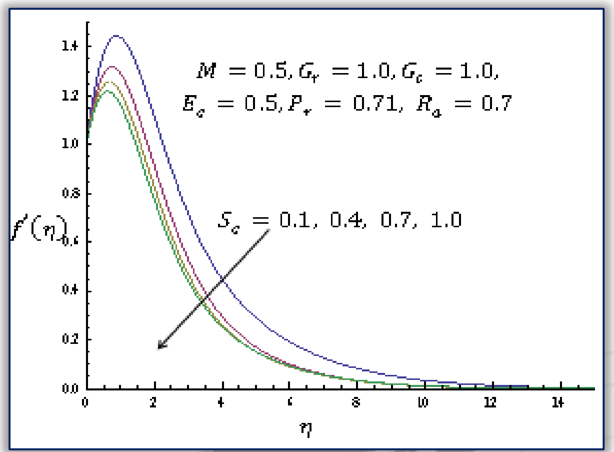

Figure 7: Effect of S_c on velocity profiles

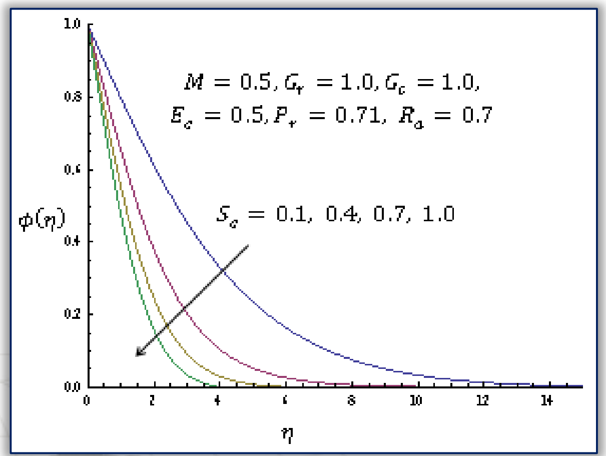

Figure 8: Effect of S_c on concentration profiles

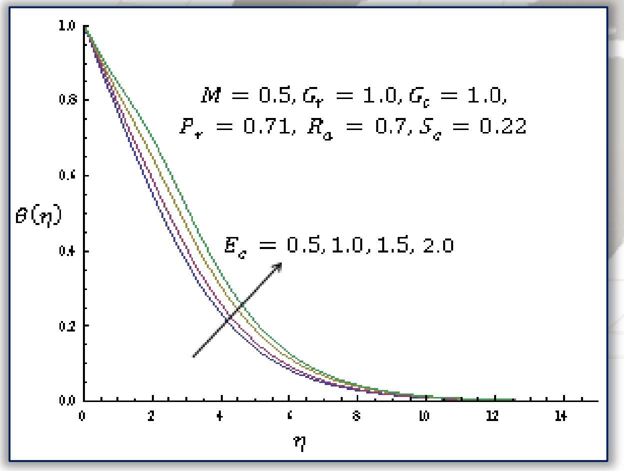

Figure 9: Effect of E_c on temperature profiles

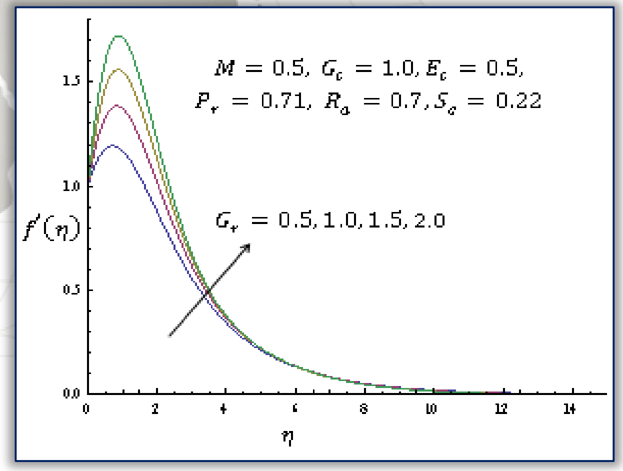

Figure 10: Effect of G_r on velocity profiles

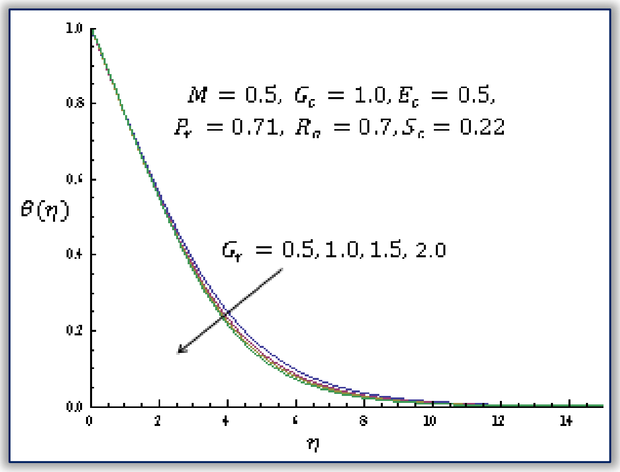

Figure 11: Effect of G_r on temperature profiles

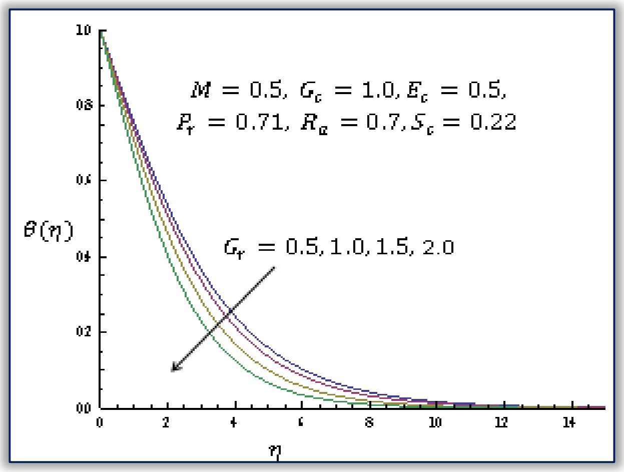

Figure 12: Effect of G_r on concentration profiles

The effects of thermal and solutal Grashof number on velocity, temeprature and concentration profiles are shown in Figures 10 – 15. Figure 10 revealed that increase in G_r results in the increase in velocity profiles over the region. It is apparent from the figure 10 that there is rapid rise in the dimensionless velocity along the wall to a peak and gradually away from the plate. From figures11 and 12, temperature and concentration profiles decrease with increasing values in G_r and this result is in agreement with Ibrahim (2014). Similarly, effects of the solutal Grashof number G_c on velocity, temperature and concentration profiles are presented in figures 13 – 15. It was observed that velocity profiles increases with increase in G_c(fluid velocity is boostedindicating that buoyancy has an acccelerating effect on the flow field). Likewise, both temperature and concentration profiles decrease in value with increase in G_c as shown in figures 14 and 15.

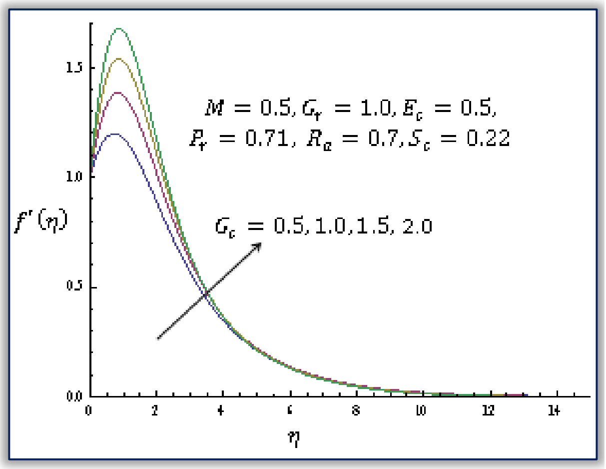

Figure 13: Effect of G_c on velocity profiles

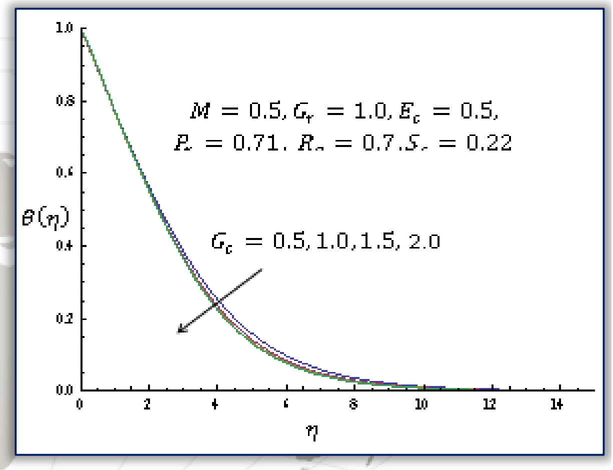

Figure 14: Effect of G_c on temperature profiles

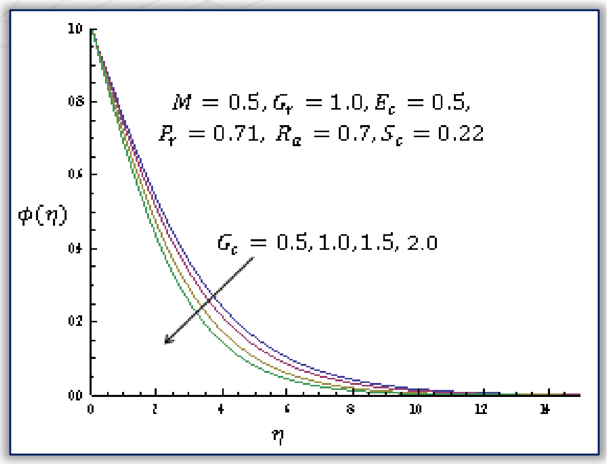

Figure 15: Effect of G_c on concentration profiles

5. CONCLUSION

In this paper, we have been able to employ HAM to solve a free convection fluid flow problem past a semi-infinite flat plate. Analytical expressions for velocity, temperature and concentration profiles have been obtained (table 1). Consequently, it is agreed that HAM can be used alternatively to solve nonlinear coupled ordinary differential equations which do not have exact analytical solutions. It was also observed that the effect of the physical parameters on velocity, temperature and concentration profiles revealed that the flow field characteristics can be controlled using magnetic field.

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