[1.]Jelena PETROVIĆ, [2.]Živojin STAMENKOVIĆ, [3.]Miloš KOCIĆ, [4.]Milica NIKODIJEVIĆ, [5.]Jasmina BOGDANOVIĆ-JOVANOVIĆ

MHD FLOW AND HEAT TRANSFER IN POROUS MEDIUM WITH INDUCED MAGNETIC FIELD EFFECTS

[1-3,5.]Faculty of Mechanical Engineering Niš, SERBIA
[4.] Faculty of Occupational Safety Niš, SERBIA

Abstract: The magneto-hydrodynamic (MHD) flow and heat transfer of viscous incompressible fluid through porous medium has been considered in the paper. Fluid flows through porous medium between two parallel fixed isothermal plates which have been kept at the two constant different temperatures. External applied magnetic field is homogenous and perpendicular to the plate. Due to the fluid flow magnetic field is induced along the fluid flow direction. The general equations that describe the discussed problem (momentum, magnetic induction and energy equation) under the adopted assumptions are reduced to ordinary differential equations and closed-form solutions are obtained. Solutions with appropriate boundary conditions for velocity, induced magnetic field and temperature have been obtained. The influences of Hartmann number, Reynolds magnetic number, suction parameter and porosity parameter have been presented graphically to show their effects on the flow and heat transfer characteristics.
Keywords: MHD flow, heat transfer, porous medium

1. INTRODUCTION

The flow and heat transfer of electrically conducting fluids in channels and circular pipes under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps, accelerators and flowmeters and have applications in nuclear reactors, filtration, geothermal systems and others. The requirements of modern technology have stimulated the interest in fluid flow studies, which involve the interaction of several phenomena. One of these phenomena is certainly viscous flow of electrically conducting fluid through porous medium in the present of magnetic field. The mathematical theory of the flow of fluid through a porous medium was initiated by Darcy [1]. For the steady flow, he assumed that viscous forces were in equilibrium with external forces due to pressure difference and body forces.

The flow and temperature distribution through porous channels is of great importance in range of scientific and engineering domains, including earth science, nuclear engineering and metallurgy. Cunningham and Williams [2] reported several geophysical applications of flow in porous medium. McWhirter et al. [3] reported the experimental results of the MHD flow in a porous medium required for the design of a blanket of liquid metal around a thermonuclear fusion-fission hybrid reactor. Research results presented by Prescott and Incropera [4] and Lehmann et al. [5] show that applied permanent magnetic field during the solidification process modify the intensity of the interdendritic flow of the metallic liquid in the mushy zone, i.e. a porous medium.

The MHD heat transfer can be divided in two parts. One contains problems in which the heating is an incidental by product of electromagnetic fields as in MHD generators and pumps etc, and the second consists of problems in which the primary use of electromagnetic fields is to control the heat transfer (Toshivo Tagawa et al. [6]).

Several analytical and numerical works in the literature are devoted to the study of the MHD flow of a conducting fluid through a porous medium between two parallel fixed plates[7-9]. Vidhya and Kesavan [10] considered an incompressible viscous fluid flow and temperature distribution in a porous medium between two vertical parallel plates and the problem is analyzed analytically. Recently Singh and Kumar [11] have investigated the heat and mass transfer MHD flow through porous medium. Steady flow of a viscous fluid through a saturated porous medium of finite thickness, impermeable and thermally insulated bottom and the other side is stress free, at a constant temperature was studied by Mounuddin and Pattabhiramacharyulu [12]. Rahman et al. [13] considered magneto hydrodynamic mixed convection in a horizontal channel with an open cavity.

In spite of all the previous studies, EMHD Poiseuille viscous flow through porous medium in a horizontal channel with insulated and impermeable walls in the presence of viscous dissipation and Joule heating[14] has received little attention. Motivated by the above referenced works and the numerous possible industrial applications of the problem, we have extended research in this field. Hence we have considered, the steady flow of electrically conductive viscous fluid through the porous medium in the presence of externally applied magnetic field and induced magnetic field due to fluid motion.

2. MATHEMATICAL MODEL

The problem of the MHD flow and heat transfer of viscous incompressible fluid through porous medium has been considered in this paper. Fully developed flow takes place between parallel plates that are at a distance h, as shown in Figure 1. The upper and lower plate have been kept at the two constant temperatures T_{w1} and T_{w2} . In order to form mathematical model of described flow and heat transfer problems continuity, extended Navier-Stokes, energy and magnetic inductionequations are used. From the fact that flow is fully developed and two-dimensionalvelocity component in y direction is constant and different from zero because in this case there are sources (sinks) on plates.

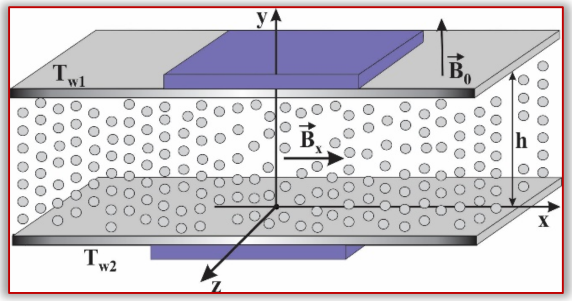


Figure 1. Physical model and coordinate system

Then fluid velocity $\vec{W}$, externally applied magnetic field of induction $\vec{B}$ and current density vector $\vec{j}^*$ are:

$$\vec{W} = u(y)\vec{i} + v\vec{j},\ \vec{B} = B_x(y)\vec{i} + B_0\vec{j},\ \vec{j}^* = \frac{1}{\mu_0}\nabla \times \vec{B}. \tag{1}$$

Using previously defined vectors in Navier-Stokes equation and projecting them on axes x and y, following equations are obtained:

$$\rho v\frac{du}{dy} = -\frac{\partial p}{\partial x} + \mu\frac{d^2u}{dy^2} - \frac{\mu}{K^*}u + \frac{B_0}{\mu_0}\frac{dB_x}{dy}, \tag{2}$$

$$\frac{\partial p}{\partial y} + \frac{\mu}{K^*}v + \frac{B_x}{\mu_0}\frac{dB_x}{dy} = 0. \tag{3}$$

The momentum and energy equation for described flow and heat transfer problem takes the following form:

$$\frac{1}{\sigma\mu_0}\frac{d^2B_x}{dy^2} - v\frac{dB_x}{dy} + B_0\frac{du}{dy} = 0, \tag{4}$$

$$k\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 + \frac{\mu}{K^*}u^2 + \frac{1}{\sigma\mu_0^2}\left(\frac{dB_x}{dy}\right)^2 = 0. \tag{5}$$

Boundary conditions are represented with following equations:

$$u(0) = 0,\ u(h) = 0,\ B_x(0) = 0,\ B_x(h) = 0,\ T(0) = T_{w2},\ T(h) = T_{w1}. \tag{6}$$

Equations (2)-(5) with boundary conditions (6) represent the mathematical model of described flow and heat transfer problems in porous medium. It is convenient to transform the equations and boundary conditions to a nondimensional form. Using following transformations:

$$y^* = \frac{y}{h},\ u^* = \frac{u}{U},\ \Theta = \frac{T - T_{w2}}{T_{w1} - T_{w2}},\ b = \frac{B_x}{B_0},\ U = \frac{h^2P}{\mu},\ P = -\frac{\partial p}{\partial x} = \text{const},\ \beta = \frac{v}{U},\ \Lambda = \frac{h^2}{K^*},$$

$$Rm = \sigma\mu_0 hU,\ Pr = \frac{\mu c_p}{k},\ Re = \frac{hU}{\nu},\ Ec = \frac{U^2}{c_p\left(T_{w1} - T_{w2}\right)},\ Ha = Bh\sqrt{\frac{\sigma}{\mu}}. \tag{7}$$

equations (2)-(5) and boundry conditions (6) are transformed into form which represents nondimensional mathematical model of this problem:

$$\frac{d^2u^*}{dy^{*2}} - \beta Re\frac{du^*}{dy^*} - \Lambda u^* + \frac{Ha^2}{Rm}\frac{db}{dy^*} + 1 = 0, \tag{8}$$

$$\frac{du^*}{dy^*} - \beta\frac{db}{dy^*} + \frac{1}{Rm}\frac{d^2b}{dy^{*2}} = 0, \tag{9}$$

$$\frac{d^2\Theta}{dy^{*2}} + Pr\,Ec\left[\left(\frac{du^*}{dy^*}\right)^2 + \Lambda u^{*2} + \frac{Ha^2}{Rm^2}\left(\frac{db}{dy^*}\right)^2\right] = 0, \tag{10}$$

$$\frac{\partial}{\partial y^*}\left(\frac{\mu_0 p}{B_0^2}\right) + b\frac{db}{dy^*} + \beta\Lambda\frac{Rm}{Ha^2} = 0, \tag{11}$$

$$u^*(0) = 0,\ u^*(1) = 0,\ b(0) = 0,\ b(1) = 0,\ \Theta(0) = 0,\ \Theta(1) = 1. \tag{12}$$

The solutions of equations (8) with boundary conditions have the following forms:

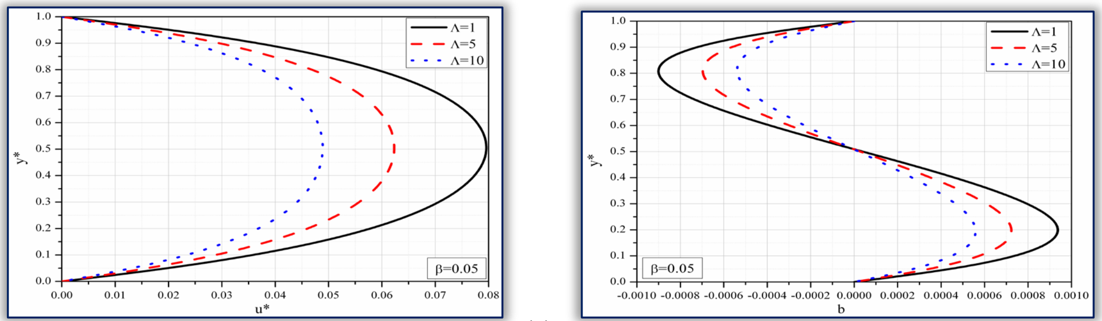
$$u^*(y^*) = C_1 \exp(n_1 y^*) + C_2 \exp(n_2 y^*) + C_3 \exp(n_3 y^*) + \frac{1}{\Lambda}, \tag{13}$$

$$b(y^*) = \frac{Rm}{Ha^2}\left[L_{33} C_1 \exp(n_1 y^*) + L_{34} C_2 \exp(n_2 y^*) + L_{35} C_3 \exp(n_3 y^*) + \frac{\beta Re}{\Lambda} + C_4\right], \tag{14}$$

$$\begin{aligned} \theta(y^*) = -Pr\,Ec[&S_{31}\exp(2n_1 y^*) + S_{32}\exp(2n_2 y^*) + S_{33}\exp(2n_3 y^*) + \\ &S_{34}\exp((n_1+n_2)y^*) + S_{35}\exp((n_1+n_3)y^*) + S_{36}\exp((n_2+n_3)y^*) + \\ &+S_{37}\exp(n_1 y^*) + S_{38}\exp(n_2 y^*) + S_{39}\exp(n_3 y^*) + \frac{1}{2\Lambda} y^{*2} + G_1 y^* + G_2] \end{aligned} \tag{15}$$

3. RESULTS AND DISCUSSION

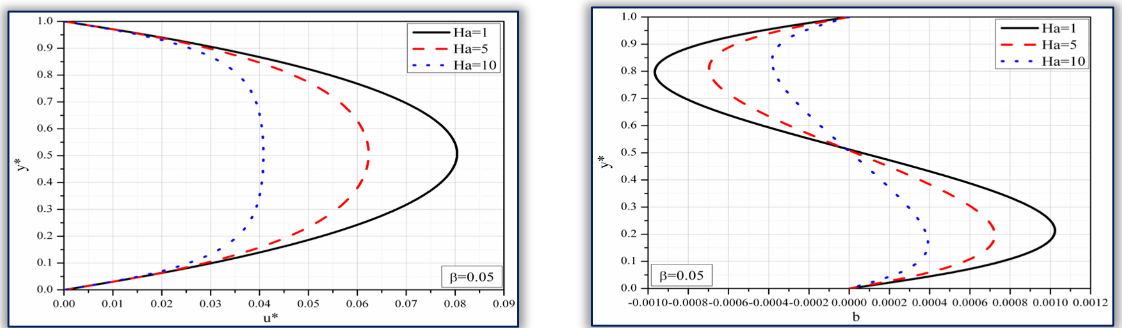
For the sake of transparency, to draw conclusions, part of the results for dimensionless longitudinal velocity,induced magnetic field and temperature has been presented graphically in Figures 2a to 5b.



(a) (b)

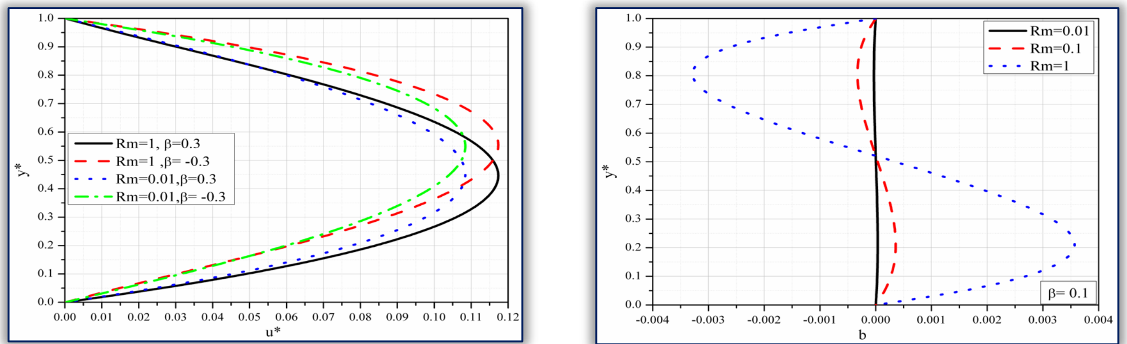
Figure 2. Effect of porosity parameter on velocity and induced magnetic field

It can be clearly seen from Figures 2a and 2b that increase of the porosity parameter reduces the velocity and induced magnetic field. Increase of Hartmann number i.e. increase of externally applied magnetic field also results in the reduction of velocity and induced magnetic field (Figures 3a and 3b).



(a) (b)

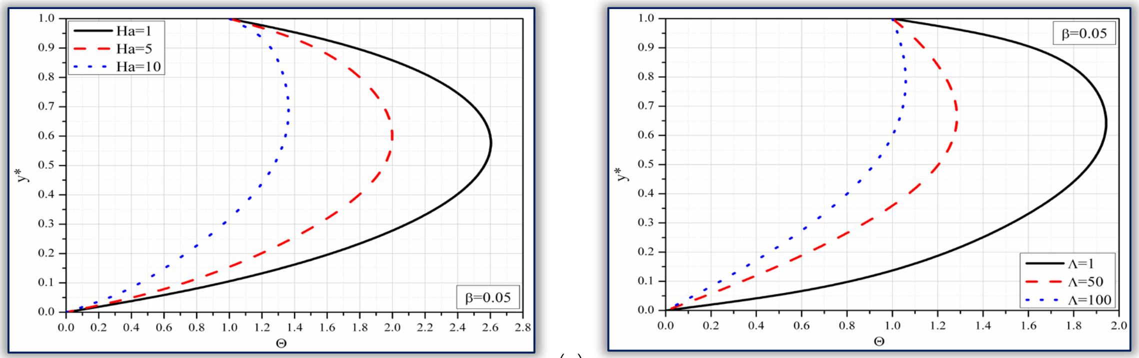
Figure 3. Effect of Hartmann number on velocity and induced magnetic field



(a) (b)

Figure 4. Effect of Reynolds magnetic number on velocity and induced magnetic field

Increase of Reynolds magnetic number increases velocity and induced magnetic field as shown in figures 4. It can be clearly seen from Figures 5 that decrease of Hartmann number and porosity parameter increase the temperature field.



(a) (b)

Figure 5. Effect of Hartmann number and porosity parameter on temperature

5. CONCLUSIONS

The magnetohydordynamic (MHD) flow and heat transfer of viscous incompressible fluid through porous medium has been investigated in the paper. Fluid flow through homogeneous and isotropic porous medium between two parallel fixed isothermal plates. Effects of Hartmann number, porosity parameter and Reynolds magnetic number on the heat and mass transfer have been analyzed.

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