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STUDY OF AXI-SYMMETRIC FREEZING AND MELTING OF AN AGITATED BATH MATERIAL AROUND A LOW MELTING TEMPERATURE CYLINDRICAL SOLID ADDITIVE

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Abstract: To reduce the time for unavoidable freezing and melting of agitated bath material onto a low melting temperature cylindrical solid additive with the growth of the frozen layer of negligible thermal resistance during melt preparation of required composition for production of steel and cast iron and consequently decreasing the production time, cost, requirement of energy and impact on environment makes the product globally competitive. With this objective, a non-dimensional lump-integral model is evolved for this event. It exhibits its control by non-dimensional independent parameters: the property-ratio, γ, the melting temperature-ratio, θ_{ab}, the heat capacity-ratio, C_r of the additive-bath system, the bath material phase-change parameter, the Stefan number S_{tb}, and that of the additive, S_{ta}, and the bath condition the conduction factor, C_{of}. Moreover, the model gives close-form solutions for the freezing and melting and associated heating and melting of additive in terms of these parameters. When the frozen layer, R^*_{bm}, is represented by the frozen layer per unit Stefan number, S_{tb}, of the bath material and time τ^* as per unit property-ratio, γ, the solution for R^*_{bm} remains only in terms of S_{ta}, θ_{ab}, and C_{of}. The plots of these solutions indicate that decreasing C_{of} for certain θ_{ab} and S_{ta}, or reduction in θ_{ab} for prescribed C_{of} and S_{ta} or Increasing S_{ta} for given C_{of} and θ_{ab} diminishes the total time of freezing and melting. The associated heating and melting of the additive are also found. When the bath is at the freezing temperature of the bath material, only freezing occurs. Its closed form solution is derived. This model is validated with that of the literature by transforming it to only melting of the cylindrical additive initially at its melting temperature subjected to constant temperature heat injection. A close agreement is found.
Keywords: Alloyants addition, Alloyants-melt bath system, Mathematical modeling, Freezing and Melting

1. INTRODUCTION

To compete in aggressive global market, steel and cast iron of different compositions are required to be produced with high productivity, reduced cost, and with no adjustment in their quality. Prior to their casting, their melts are prepared by immersing and assimilating alloyants, called additives, in the melt bath, and then are treated by several metallurgical processes. Here, an unavoidable event of freezing and melting of the bath material around the additive occurs soon after their immersing in the bath. It sets in high temperature gradient towards the additive side in the initial period resulting in the requirement of the conductive heat by the additive far greater than the convective heat supplied from the bath. Since the conductive heat needed by the additive is met by the sum of the convective heat of the bath, and the latent heat of fusion generated owing to the freezing of the bath material onto the additive, the excess conductive heat is balanced by the latent heat of fusion of the bath material evolved due to its freezing onto the additive. With passing of the time, the temperature gradient onto the additive side and, consequently, the conductive heat requirement by the additive decrease until the conductive heat becomes the same as that of the bath convective heat. After this time, the conductive heat is lower than the available bath convective heat due to which the extra convective heat melts the frozen layer. During the entire period of the freezing and melting of the bath material, the additive of low melting temperature gets heated and melts with increase in the melt depth, and the heat penetration depth. Such an event is performed in a certain time, and dependent upon the thermo-physical properties of the additive and the bath material, their temperatures at the initiation of this event, the bath condition represented by the bath convection, and the shape and size of the additive. In order to reduce the time of this event, for a given additive-bath system, only the convective heat of the bath that decides the extent of the frozen layer formed needs to be controlled. To increase this heat that permits the formation of a smaller thickness of the frozen layer, which melts in a lesser time, the heat transfer co-efficient of the bath is required to be increased. It is obtained by making the bath highly agitated. The thermal resistance of such a thickness including that of the agitated bath is much less than that of the additive. Moreover, less time taken in the freezing and melting results in diminishing the production time and increasing the productivity.

Investigation of this event with the additive having the melting temperature lower than that of the freezing temperature of the bath material seldom appears in the literature. However, for the thermal resistance of the additive comparable with that of the frozen layer of the bath onto the additive, the freezing and melting was studied in case of plate [1], cylindrical [2-6], and spherical [7-12] shaped additives. Their melting temperature was higher than the freezing temperature of the bath material and their thermo-physical properties were taken to be uniform but different. It was found that increasing the bath convective heat by raising the value of the heat transfer co-efficient or decreasing the freezing temperature of the bath material reduces the development of the frozen layer thickness and the time of the freezing and melting for the plate additive [1] and the cylindrical additive [5] made of titanium. For the spherical additive [2] , this prediction was implicit, When the thermal resistance of the frozen layer is negligible with respect to that of the high melting temperature plate [13], cylindrical [14], and spherical [15] additives, the close-form solutions for the freezing and melting were reported. The closed-form solution was also obtained for the plate with temperature-dependent heat capacity [16]. Recently, the literature exhibited the closed-form expressions for the instant equilibrium temperature at the interface between the high melting temperature [17] and the low melting temperature [18] cylindrical additives once the freezing of the bath material onto these got initiated after plunging them in the bath. It was also found for the high melting temperature plate additive [19].

The current problem relates to the axi-symmetric freezing and melting of the bath material onto a cylindrical additive, when its melting temperature is lower than the freezing temperature of the bath material. The bath is highly agitated, and the thermal resistance of the frozen layer developed is assumed to be negligible in comparison with that of the additive. Its non-dimensional mathematical model in lump- integral format is evolved. It makes the event dependence upon the non-dimensional independent parameters: the property-ratio, γ, the melting temperature-ratio, θ_{ab}, and the heat capacity-ratio, C_r, of the additive-bath system, the Stefan number of the additive, S_{ta} and that of the bath material, S_{tb} and the condition of the bath denoted by the conduction factor, C_{of} and leads to close- form solutions for the freezing and melting of the bath material onto the additive and heating and melting of the additive, but these solutions are only in terms of θ_{ab}, S_{ta} and the conduction factor, C_{of} of the bath once the frozen layer, R^*_{bm} is represented by the frozen layer per unit Stefan number, S_{tb} of the bath material and the time, τ^*, the time, τ per unit property-ratio. The solutions for the freezing with no melting, along with the heating and the melting of the additive are also obtained. To validate the present problem with that of the literature, it is transformed to the melting of the cylindrical additive initially at its melting temperature by a constant temperature heat injection. A close agreement is found.

2. FORMULATION OF THE PROBLEM

To model mathematically the freezing and melting of an agitated bath material around a low melting temperature solid cylindrical shaped additive, the bath is considered to be at a uniform temperature, T_b greater than its freezing temperature, T_{bf}. In this bath, a cylindrical additive at an initial temperature, T_{ai} lower than its melting temperature, T_{af} is immersed. This temperature, T_{af} is also less than the freezing temperature, Tbf of the bath material. These temperature set up a temperature field Tai<Tam<Tbm<Tb in the additive-melt bath system, Fig.1. Moreover, the bath material immediately begins to freeze around the surface of the immersed additive, the interface formed between the freezing layer and the additive acquires an equilibrium temperature, Te, that resides between T_{ai} and T_{bf} ($T_{bf}>T_e>T_{af}>T_{ai}$) and melting along with heating of the additive initiates. As the time progresses, the interface temperature, T_e, and the thickness of the frozen layer of the bath material onto the additive increase. The heat penetration depth,

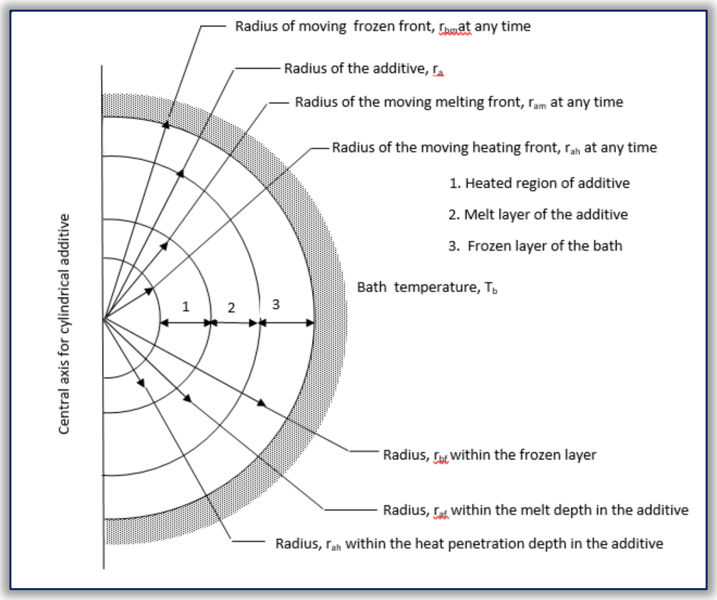

Figure 1: Diagram related to freezing and melting of the agitated bath material onto a low melting temperature cylindrical additive with its melting and heating

and the melt depth in the additive also increase. These occurrences continue until the heat conducted to the additive is larger than the convective heat supplied from the bath. The excess conductive heat is balanced by the latent heat of fusion evolved due to freezing of the bath material onto the additive. Once these two heats become equal, no further freezing takes place. After this time, the conductive heat to the additive is less than the convective heat causing the frozen layer to melt and allowing the interface temperature, T_e, and both the heat penetration and the melt thicknesses increase. Finally, the frozen layer melts completely, and the melt portion of the additive gets assimilated in the bath leaving the rest portion of the additive at a raised temperature.

This event is assumed to be regulated by unsteady conjugated radial heat conduction, and the temperature field sets in the frozen layer and the additive axi-symmetry. The dimensionless heat conduction equation in the integral form for the melt region of the additive with the initial and boundary conditions can be written as

$$\gamma\left[\frac{d}{d\tau}\int_{R_{am}}^{1}\theta_{af}R_{af}dR_{af}-\left[\theta_{af}R_{af}\right]_{Raf=1}\frac{d1}{d\tau}+\left[\theta_{af}R_{af}\right]_{Raf=Ram}\frac{dR_{am}}{d\tau}\right]=\left[R_{af}\frac{\partial\theta_{af}}{\partial R_{af}}\right]_{Raf=1}-\left[R_{af}\frac{\partial\theta_{af}}{\partial R_{af}}\right]_{Raf=Ram} \quad (1)$$

$$\theta af=0,\quad 0<R_{af}<1, R_{am}=1,\qquad \tau=0 \quad (2)$$

$$\theta_{af}=\theta_{e}>\theta_{ab}, R_{af}=1 \qquad \tau>0 \quad (3)$$

$$\theta_{af}=\theta_{ab}, R_{af}=R_{am} \qquad \tau>0 \quad (4)$$

Physically, the first term on the right hand side of Eq.(1) denotes the rate of heat conducted to the melt region of the additive from the freezing layer through contact interface at R_{af}=1, whereas the second term of the right hand side is indicative of the rate of heat conducted out of the melt layer to the heated additive at $R_{af}= R_{am}$. The difference of these two causes the net rate of increase in the internal thermal energy of the melt layer represented by left hand side of Eq.(1). Its first term corresponds to the rate of increase in thermal energy, and the combination of the second and third terms relates to rate of internal thermal energy available from increase in the melt depth of the additive. In the heated portion of the additive, the heat conduction equation assumes the following integral format

$$\gamma\left[\frac{d}{d\tau}\int_{Rai}^{Ram}\theta_{ah}R_{ah}dR_{ah}-\left[\theta_{ah}R_{ah}\right]_{Rah=Ram}\frac{dR_{am}}{d\tau}+\left[\theta_{ah}R_{afh}\right]_{Rah=Rai}\frac{dR_{ai}}{d\tau}\right]=\left[R_{ah}\frac{\partial\theta_{ah}}{\partial R_{ah}}\right]_{Rah=Ram}-\left[R_{ah}\frac{\partial\theta_{ah}}{\partial R_{ah}}\right]_{Rah=Rai} \quad (5)$$

Like Eq.(1), Eq.(5) can also be physically explained. Initial and boundary conditions associated with Eq.(5) are

$$\theta_{ah}=0, 0<R_{ah}<R_{am},\quad R_{ah}=1 \qquad \tau=0 \quad (6)$$

$$\theta_{ah}=\theta_{ab}, R_{ah}=R_{am}, \qquad \tau>0 \quad (7)$$

$$\frac{\partial\theta_{ah}}{\partial R_{ah}}=0, \theta_{ah}=0, R_{ah}=R_{ai} \qquad \tau>0 \quad (8)$$

The coupling conditions at the interface between the melt layer and the heated region of the additive are:

$$\frac{\partial\theta_{af}}{\partial R_{af}}+\frac{\gamma}{S_{ta}}\frac{dR_{am}}{d\tau}=\frac{\partial\theta_{ah}}{\partial R_{ah}};\ \theta_{ah}=\theta_{af}=\theta_{ab},\ R_{ah}=R_{af}=R_{am}\quad \tau>0 \quad (9)$$

Surrounding the cylindrical additive, the bath is highly agitated. It provides large value of heat transfer coefficient between the additive and the bath resulting in high bath convective heat. Since the heat conducted to the additive needs to be balanced by the sum of the latent heat of fusion evolved due to freezing of the bath material and the convective heat supplied by the bath, only a small amount of latent heat of fusion is required due to availability of the large convective heat. It leads to formation of a very small thickness of the frozen layer [13,14] around the additive having its thermal resistance negligible with respect to the thermal convective resistance of the bath permitting establishment of a uniform temperature [20,21,22] in the frozen layer. Because the moving front of this layer is always at the freezing temperature, T_{bm} of the bath material, the temperature of the entire frozen layer is also at T_{bm}, due to the above fact. The temperature at the interface T_e between the additive and the frozen layer owing to this becomes at T_{bm}. Consequently, the frozen layer acts as a lump [20,21,22] that does not liberate or absorb the sensible heat. Application of the conservation of energy to such a lump leads to a balance between the heat conducted to the additive and the bath convective heat, $B_{im}(\theta_b-1)$, plus the latent heat of fusion released by the freezing. It takes the following mathematical form. In terms of conduction factor, C_{of}, it can be written as

$$\frac{1}{\gamma C_{of}}+\frac{1}{S_{tb}}\frac{dR_{bm}}{d\tau}=-Q_{bn},\quad R_{bf}=R_{bm}, \qquad \tau>0 \quad (10)$$

where, $\gamma C_{of} = B_{im}(\Theta_b-1)$. Its initial condition is

$$R_{bm} = C_r = 1, \qquad \tau = 0 \tag{11}$$

The conjugating conditions at the interface between the additive and the frozen layer can be cast as:

$$\frac{1}{\gamma}\frac{\partial\theta_{af}}{\partial R_{af}} = -Q_{bn};\ \theta_{af} = \theta_{bf} = \theta_e = 1,\ R_{af}=1, R_{bf}=C_r,\ \tau > 0 \tag{12}$$

It may be noted that the non-dimensional temperature in either additive or frozen layer denotes the temperature above the initial temperature of the additive with reference to the temperature difference, (T_{bm}-T_{ai}) between the freezing temperature of the bath material and the additive initial temperature.

The mathematical model of the present problem comprising Eqs. (1) to (12) is based on the assumptions of the thermo-physical properties of the freezing bath material and the additive uniform but different, whereas, the melt layer and the heated part of the additive have uniform and the same thermo-physical properties. Moreover, the change in the volume of the additive upon melting and heating is zero and the surface of the additive is in perfect contact with the surface of the frozen layer in spite of the fact that the interfacial resistance during imperfect contact between the frozen layer and the additive occurring in manufacturing gives negligible effect with respect to the thermal resistance of the additive to heat transfer due to its very small values ranging from 1.9×10^{-4} m^2sK/J to 2.1×10^{-4} m^2sK/J [27]. In the past this assumption was also employed in the model of such problems by previous investigators [1-19]. They stated that their solutions were accurate and realistic. Additionally, the low melting temperature additive remains within the frozen layer of the high melting temperature bath material until the frozen layer completely melts. This frozen layer is symmetric about the central axis of the additive [33]

3. SOLUTIONS

The model just developed indicates that it is nonlinear owing to the presence of the moving phase-change boundary as a result of freezing of the bath material, Eq.(10), and melting of the additive, Eq.(9), and coupled because of conjugating conditions at the interface between the heated region and the melt layer of the additive, Eq.(9) and between the melt layer of the additive and the frozen layer of the bath material, Eq.(12). The coupling and nonlinearity present in the model prohibit exact solutions of the problem employing the analytical methods. In such a situation a semi-analytical method, known as the integral method that yielded closed-form expressions for several heating and phase-change problems [23-25] in the past is employed. The governing equation for the melting, Eq. (1) and that for the heating, Eq.(5) of the additive are already written in the integral forms.

The integral Eq.(1) related to melting of the additive is reduced to

$$\gamma\left[\frac{d}{d\tau}\int_{Ram}^{1}\theta_{af}R_{af}dR_{af} + \theta_{ab}R_{am}\frac{dR_{am}}{d\tau}\right] = \left[R_{af}\frac{\partial\theta_{af}}{\partial R_{af}}\right]_{Raf=1} - \left[R_{af}\frac{\partial\theta_{af}}{\partial R_{af}}\right]_{Raf=Ram} \tag{13}$$

when the boundary conditions, Eqs.(3) and (4) are employed, whereas Eq.(5) associated with heating of the additive is simplified as

$$\gamma\left[\frac{d}{d\tau}\int_{Ram}^{Rai}\theta_{ah}R_{ah}dR_{ah} - \theta_{ab}R_{am}\frac{dR_{am}}{d\tau}\right] = \left[R_{ah}\frac{\partial\theta_{ah}}{\partial R_{ah}}\right]_{Rah=Ram} \tag{14}$$

once Eqs.(7) and (8) are applied. Combining Eqs.(13) and (14) leads to a single integral equation, called global integral equation. It assumes the form

$$\gamma\left[\left\{\frac{d}{d\tau}\int_{Rai}^{Ram}(\theta_{ah}R_{ah})dR_{ah}\right\} + \left\{\frac{d}{d\tau}\int_{Ram}^{1}(\theta_{af}R_{af})dR_{af}\right\}\right] =$$
$$\left[R_{ah}\frac{\partial\theta_{ah}}{\partial R_{ah}}\right]_{Rah=Ram} - \left[R_{af}\frac{\partial\theta_{af}}{\partial R_{af}}\right]_{Raf=Ram} + \left[R_{af}\frac{\partial\theta_{af}}{\partial R_{af}}\right]_{Raf=1} \tag{15}$$

To find the solution of the globalized integral equation, Eq.(15), the temperature distributions both in the heated region and the melt region of the additive need to be prescribed . A linear temperature distribution in the melt.

$$\theta_{af} = \theta_e - (\theta_e - \theta_{ab})((1 - Ra_f)/(1 - R_{am})) \tag{16}$$

and a cubic temperature profile in the heated region of the additive,

$$\theta_{ah} = \theta_{ab}((R_{ah} - R_{ai})/(R_{am} - R_{ai}))^3 \tag{17}$$

are assumed. Note that Eq.(16) satisfies the boundary conditions, Eqs.(3) and (4), whereas Eq.(17) fulfills the boundary conditions, Eqs.(7) and (8). In the previous studies, the choice of cubic temperature distribution in Eq.(17) in the heating problems [25,31] provided results close to exact solutions [32] whereas a linear temperature profile, Eq.(16) in the melting [24,28-30] and freezing problems [1,17,19] yielded accurate results. Substituting Eqs.(16) and (17) and coupling condition (Eq.9), Eq.(15) takes the form

$$\frac{d}{d\tau}\left[\begin{array}{l}\theta_{ab}\left\{\frac{1}{4}R_{am}(R_{am}-R_{ai})-\frac{1}{20}(R_{am}-R_{ai})^2\right\}+\frac{\theta_e}{2}\left(1-R^2{}_{am}\right)\\ -(\theta_e-\theta_{ab})\left\{\frac{1}{2}R_{am}(1-R_{am})+\frac{1}{6}(1-R_{am})^2\right\}-\frac{1}{2S_{ta}}R^2{}_{am}\end{array}\right]=\frac{(\theta e-\theta ab)}{\gamma(1-R_{am})} \tag{18}$$

The conjugating condition, Eq.(12) at the interface between the melt of the additive and the frozen layer becomes

$$\frac{\theta_e-\theta_{ab}}{\gamma(1-R_{am})}=-Q_{bm} \tag{19}$$

once Eq.(16) is applied whereas its substitution in Eq.(10) gives

$$\frac{1}{\gamma C_{of}}+\frac{1}{S_{tb}}\frac{dR_{bm}}{d\tau}=\frac{\theta_e-\theta_{ab}}{\gamma(1-R_{am})} \tag{20}$$

As stated earlier, the temperature θ_e at the interface between the frozen layer and the melt of the additive is one, ($\theta_e=1$). Its substitution in Eqs.(18) and (20) makes them, respectively,

$$\frac{d}{d\tau}\left[\begin{array}{l}\theta_{ab}\left\{\frac{1}{4}R_{am}(R_{am}-R_{ai})-\frac{1}{20}(R_{am}-R_{ai})^2\right\}+\frac{1}{2}\left(1-R^2{}_{am}\right)\\ -(1-\theta_{ab})\left\{\frac{1}{2}R_{am}(1-R_{am})+\frac{1}{6}(1-R_{am})^2\right\}-\frac{1}{2S_{ta}}R^2{}_{am}\end{array}\right]=\frac{(1-\theta ab)}{\gamma(1-R_{am})} \tag{21}$$

and

$$\frac{1}{\gamma C_{of}}+\frac{1}{S_{tb}}\frac{dR_{bm}}{d\tau}=\frac{1-\theta_{ab}}{\gamma(1-R_{am})} \tag{22}$$

Combination of Eq.(21) and Eq.(22) leads to closed- form solution for the frozen layer R^*_{bm} at any time τ

$$R_{bm}{}^*=-\frac{\tau}{\gamma C_{of}}+\left[\begin{array}{l}\theta_{ab}\left\{\frac{1}{4}R_{am}(R_{am}-R_{ai})-\frac{1}{20}(R_{am}-R_{ai})^2\right\}+\frac{1}{2}\left(1-R^2{}_{am}\right)\left(1+\frac{1}{S_{ta}}\right)-\\ (1-\theta_{ab})\left\{\frac{1}{2}R_{am}(1-R_{am})+\frac{1}{6}(1-R_{am})^2\right\}\end{array}\right] \tag{23}$$

Note that $R^*{}_{bm}=(R_{bm}-C_r)/S_{tb}$, frozen layer thickness per unit Stefan number of the bath material.
It satisfies the initial conditions τ=0, $R_{am}=1$, Eq.(2) $R_{ai}=1$, Eq.(6) and $R_{bm}=C_r$, Eq. (11).
To derive the solutions for R_{am} and R_{ai}, Eqs. (16), (17) and $\theta_e=1$ are substituted in Eq.(9) giving

$$\frac{1-\theta_{ab}}{\gamma(1-R_{am})}+\frac{1}{S_{ta}}\frac{dR_{am}}{d\tau}=\frac{3\theta_{ab}}{\gamma(R_{am}-R_{ai})} \tag{24}$$

Since it is in the form of first order differential in time, its use may not provide close-form solutions for R_{am} and R_{ai}. To overcome this difficulty, the total differential of the temperature, θ_{ab} at the interface between the melt and the heated region of the additive becomes.

$$\Delta\theta_{af}=\frac{\partial\theta_{af}}{\partial R_{af}}\frac{dR_{af}}{d\tau}+\frac{\partial\theta_{af}}{\partial R_{af}}=0 \tag{25}$$

due to $\Delta\theta_{af}=\Delta\theta_{ab}=0$ at the interface $R_{af}=R_{am}$. Combination of Eq.(25) and differential form of Eq.(1)

$$\gamma\frac{\partial\theta_{af}}{\partial\tau}=\frac{1}{R_{af}}\frac{\partial}{\partial R_{af}}\left(R_{af}\frac{\partial\theta_{af}}{\partial R_{af}}\right) \tag{26}$$

leads to

$$dR_{am}/d\tau=-1/R_{am} \tag{27}$$

whereas its application transforms Eq.(24) to

$$R_{am}-R_{ai}=\frac{3\theta_{ab}S_{ta}R_{am}\left(1-R_{am}\right)}{S_{ta}\left(1-\theta_{ab}\right)R_{am}-\left(1-R_{am}\right)}=\frac{3\theta_{ab}S_{ta}R_{am}\left(1-R_{am}\right)}{\left\{S_{ta}\left(1-\theta_{ab}\right)+1\right\}R_{am}-1} \tag{28}$$

Note that the application of Eq.(28) makes expression for R^*_{bm}, Eq.(23) only in terms of the melt layer, R_{am} of the additive. Employing Eq.(28), Eq.(23) can be written as

$$R_{bm}^{\ *}=-\frac{\tau}{\gamma C_{of}}+\left[\left[\left\{\frac{A}{4}\frac{R^2{}_{am}\left(1-R_{am}\right)}{DR_{am}-1}\right\}-\frac{C}{20}\left\{\frac{R_{am}\left(1-R_{am}\right)}{DR_{am}-1}\right\}^2+\frac{1}{2}\left(1-R^2{}_{am}\right)-\right.\right.$$
$$\left.\left(1-\theta_{ab}\right)\left\{\frac{1}{2}R_{am}\left(1-R_{am}\right)+\frac{1}{6}\left(1-R_{am}\right)^2\right\}+\frac{1-R^2{}_{am}}{2S_{ta}}\right] \tag{29}$$

and Eq.(18) concerning heating and melting of the additive after substitution of $\theta_e = 1$ takes the form

$$\frac{d}{d\tau}\left[\left\{\frac{A}{4}\frac{R^2{}_{am}\left(1-R_{am}\right)}{\left(DR_{am}-1\right)}-\frac{C}{20}\frac{R^2{}_{am}\left(1-R_{am}\right)^2}{\left(DR_{am}-1\right)^2}\right\}+\frac{1}{2}\left(1-R^2{}_{am}\right)-\right.$$
$$\left.\left(1-\theta_{ab}\right)\left\{\frac{1}{2}R_{am}\left(1-R_{am}\right)+\frac{1}{6}\left(1-R_{am}\right)^2\right\}-\frac{R^2{}_{am}}{2S_{ta}}\right]=\frac{\left(1-\theta_{ab}\right)}{\gamma\left(1-R_{am}\right)} \tag{30}$$

Here, A=$3S_{ta}\theta_{ab}{}^2$, C=$9S_{ta}{}^2\theta_{ab}{}^3$ and $D=1+\left(1-\theta_{ab}\right)S_{ta}$

Since Eq.(30) is only in terms of the melt layer, R_{am} of the additive, it readily results in a close-form solution, which, when employed in Eq.(28) leads to the solution of the heated region of the additive. In terms of $\frac{dR_{am}}{d\tau}$, Eq.(30) can be expressed as

$$\frac{dR_{am}}{d\tau}\left[\frac{A}{2}\frac{R_{am}}{DR_{am}-1}-\frac{3A}{4}\frac{R^2{}_{am}}{DR_{am}-1}-\frac{C}{10}\frac{R_{am}}{\left(DR_{am}-1\right)^2}-\left(\frac{5AD-6C}{20}\right)\frac{R^2{}_{am}}{\left(DR_{am}-1\right)^2}\right.$$
$$+\left(\frac{5AD-4C}{20}\right)\frac{R^3{}_{am}}{\left(DR_{am}-1\right)^2}+\frac{CD}{10}\frac{R^2{}_{am}}{\left(DR_{am}-1\right)^3}-\frac{CD}{5}\frac{R^3{}_{am}}{\left(DR_{am}-1\right)^3}+\frac{CD}{10}\frac{R^4{}_{am}}{\left(DR_{am}-1\right)^3}$$
$$\left.-\left\{\left(1-\frac{1}{S_{ta}}\right)-\frac{2}{3}\left(1-\theta_{ab}\right)\right\}R_{am}-\frac{\left(1-\theta_{ab}\right)}{6}\right]=\frac{\left(1-\theta_{ab}\right)}{\gamma\left(1-Ram\right)} \tag{31}$$

Its leads to closed form solution that satisfies the initial condition $\tau^*=0$, $R_{am}=1$.

$$\tau^*=\frac{1}{\left(1-\theta_{ab}\right)}\left[b_1\left(R_{am}-1\right)-b_2\left(R^2{}_{am}-1\right)+b_3\left(R^3{}_{am}-1\right)+b_4\left\{\ln\left(\frac{DR_{am}-1}{D-1}\right)\right\}-b_5\left\{\left(DR_{am}-1\right)-\left(D-1\right)\right\}\right.$$
$$-b_6\left\{\left(DR_{am}-1\right)^2-\left(D-1\right)^2\right\}+b_7\left\{\frac{1}{\left(DR_{am}-1\right)}-\frac{1}{\left(D-1\right)}\right\}+b_8\left\{\frac{1}{\left(DR_{am}-1\right)^2}-\frac{1}{\left(D-1\right)^2}\right\}$$
$$+b_9\left\{\frac{R^3{}_{am}}{\left(DR_{am}-1\right)}-\frac{1}{D-1}\right\}+b_{10}\left\{\frac{R^3{}_{am}}{\left(DR_{am}-1\right)^2}-\frac{1}{\left(D-1\right)^2}\right\}-b_{11}\left\{\frac{R^4{}_{am}}{\left(DR_{am}-1\right)}-\frac{1}{\left(D-1\right)}\right\}$$
$$\left.+b_{12}\left\{\frac{R^4{}_{am}}{\left(DR_{am}-1\right)^2}-\frac{1}{\left(D-1\right)^2}\right\}-b_{13}\left\{\frac{R^5{}_{am}}{\left(DR_{am}-1\right)^2}-\frac{1}{\left(DR_{am}-1\right)^2}\right\}\right] \tag{32}$$

where, $\tau^*=\frac{\tau}{\gamma}$, $b_1=\left(\frac{a_1}{D}-a_{14}\right)$, $b_2=\frac{a_{13}}{2}$, $b_3=\left(\frac{a_3}{3D}+\frac{a_{12}}{3}\right)$,

$$b_4=\left[\left(\frac{a_1-a_4}{D^2}\right)-\left(\frac{a_2-2a_5-a_8}{D^3}\right)+\left(\frac{a_3+3a_6-3a_9}{D^4}\right)-\left(\frac{4a_7+6a_{10}}{D^5}\right)-\frac{10a_{11}}{D^6}\right],$$

$b_5=\left[\frac{2a_2}{D^3}-\frac{2a_3}{D^4}-\frac{a_5}{D^3}-\frac{3a_6}{2D^4}+\frac{2a_7}{D^5}\right]$, $b_6=\left(\frac{a_2}{2D^3}-\frac{a_3}{2D^4}\right)$, $b_7=\left[\frac{a_4}{D^2}-\frac{a_5}{D^3}-\frac{3a_6}{2D^4}+\frac{2a_7}{D^5}-\frac{2a_8}{D^3}+\frac{6a_9}{D^4}-\frac{12a_{10}}{D^5}+\frac{20a_{11}}{D^6}\right]$,

$b_8=\left[\frac{a_8}{2D^3}-\frac{3a_9}{2D^4}+\frac{3a_{10}}{D^5}-\frac{5a_{11}}{D^6}\right]$, $b_9=\left[\frac{a_6}{2D}-\frac{2a_7}{3D^2}\right]$, $b_{10}=\left[\frac{2a_{10}}{D^2}-\frac{a_9}{D}-\frac{10a_{11}}{3D^3}\right]$, $b_{11}=\frac{a_{11}}{3D}$,

$b_{12}=\left[\frac{a_{10}}{2D}-\frac{5a_{11}}{6D^2}\right]$, $b_{13}=\frac{a_{11}}{3D}$, $a_1=\frac{A}{2}$, $a_2=\left(\frac{A}{2}+\frac{3A}{4}\right)$, $a_3=\frac{3A}{4}$, $a_4=\frac{C}{10}$, $a_5=\left[\frac{C}{10}-\left(\frac{5AD-6C}{20}\right)\right]$,

$$a_6=\left[\left(\frac{5AD-6C}{20}\right)+\left(\frac{5AD-4C}{20}\right)\right],\ a_7=\left(\frac{5AD-4C}{20}\right),\ a_8=\frac{CD}{10},\ a_9=\left(\frac{CD}{10}+\frac{CD}{5}\right),\ a_{10}=\left(\frac{CD}{5}+\frac{CD}{10}\right),$$

$$a_{11}=\frac{CD}{10},\ a_{12}=\left[\left(1-\frac{1}{S_{ta}}\right)-\frac{2}{3}(1-\theta_{ab})\right],\ a_{13}=\left[\left(1-\frac{1}{S_{ta}}\right)-\frac{5}{6}(1-\theta_{ab})\right],\ a_{14}=\frac{(1-\theta_{ab})}{6}$$

4. MAXIMUM FROZEN LAYER DEVELOPMENT

For the growth of the frozen layer of the bath material onto the additive to be maximum, $\frac{dR_{bm}^{*}}{d\tau}=0$ and $\frac{d^2R_{bam}^{*}}{d\tau^2}<0$. Using Eq.(23), $\frac{dR_{bm}^{*}}{d\tau}$ takes the following expression

$$\frac{dR_{bm}^{*}}{d\tau}=-\frac{1}{\gamma C_{of}}+\frac{d}{d\tau}\left[\theta_{ab}\left\{\frac{1}{4}R_{am}(R_{am}-R_{ai})-\frac{1}{20}(R_{am}-R_{ai})^2\right\}\right.$$
$$\left.-(1-\theta_{ab})\left\{\frac{1}{2}R_{am}(1-R_{am})+\frac{1}{6}(1-R_{am})^2\right\}+\frac{1}{2}(1-R_{am})^2\left(1+\frac{1}{S_{ta}}\right)\right] \tag{33}$$

Application of Eq.(21) converts Eq.(33) to

$$\frac{dR_{bm}^{*}}{d\tau}=-\frac{1}{\gamma C_{of}}+\frac{(1-\theta_{ab})}{\gamma(1-R_{am})} \tag{34}$$

which leads to

$$1-R_{am}=C_{of}(1-\theta_{ab})=C_{ofm} \tag{35}$$

for $dR_{am}/d\tau=0$, where C_{ofm} is modified conduction factor. Note that Eq.(35) gives the requisite condition for the frozen layer to be maximum, since it satisfies $d^2R_{am}/d\tau^2<0$. The corresponding R_{ai} is obtained once Eq.(35) is applied to Eq.(28), having $E=3S_{ta}\theta_{ab}$, $D=S_{ta}(1-\theta_{ab})+1$

$$1-R_{ai}=C_{ofm}+\frac{E(1-C_{ofm})C_{ofm}}{D(1-C_{ofm})-1} \tag{36}$$

The maximum frozen layer formed, MR_{bm}^{*} can be obtained from Eq.(29) when Eq.(35) is employed

$$MR_{bm}^{*}=-\frac{\tau_{max}^{*}}{C_{of}}+\left[\left\{\frac{A}{4}\frac{(1-C_{ofm})^2C_{ofm}}{D(1-C_{ofm})-1}-\frac{C}{20}\left(\frac{(1-C_{ofm})C_{ofm}}{D(1-C_{ofm})-1}\right)^2\right\}-\right.$$
$$\left.(1-\theta_{ab})\left\{\frac{1}{2}(1-C_{ofm})C_{ofm}+\frac{C^2_{ofm}}{6}\right\}+\frac{C_{ofm}(2-C_{ofm})}{2}\left(1+\frac{1}{S_{ta}}\right)\right] \tag{37}$$

Note that τ_{max}^{*} is found from Eq.(32),when Eq.(35) is substituted in it.

5. TOTAL TIME OF FREEZING AND MELTING OF THE BATH MATERIAL, T^{*}_{T}

In this situation $R_{bm}^{*}=0$. Its substitution in Eq.(29) gives

$$\tau_t^{*}=C_{of}\left[\left\{\frac{A}{4}\frac{R^2_{amt}(1-R_{amt})}{DR_{am}-1}\right\}-\frac{C}{20}\left\{\frac{R_{amt}(1-R_{amt})}{DR_{amt}-1}\right\}^2+\frac{1}{2}(1-R^2_{amt})-\right.$$
$$\left.(1-\theta_{ab})\left\{\frac{1}{2}R_{amt}(1-R_{amt})+\frac{1}{6}(1-R_{amt})^2\right\}+\frac{1-R^2_{amt}}{2S_{ta}}\right] \tag{38}$$

Where, R_{amt} is the radius of the melting boundary corresponding to $R_{bm}^{*}=0$ and $\tau_t^{*}=\tau_t/\gamma$,. For this condition, radius of the heating boundary R_{ai} now called R_{ait} can be obtained by Eq.(28)

$$R_{ait}=\frac{R_{amt}[R_{amt}(D+E)-(1+E)]}{DR_{amt}-1} \tag{39}$$

If the heat penetration approaches the central axis of the cylindrical additive, R_{ait}=0. Its substitution in Eq.(39) leads to

$$R_{amt} = R_{amo} = \frac{E+1}{D+E} = \frac{3S_{ta}\theta_{ab}+1}{2\theta_{ab}S_{ta}+S_{ta}+1} \quad (40)$$

when Eq.(28) is employed. It may be noted that Eq.(40) provides R_{amt} in terms of the melting temperature, θ_{ab} and the phase-change parameter S_{ta} of the additive in situation of complete melting of the frozen layer of the bath material formed onto the additive with simultaneous reaching of the heat penetration at the central axis of the cylindrical additive. In this case, Eq.(38) readily gives the total time τ^*_t of the freezing and melting of the bath material R_{bm}, once Eq.(40) is employed.

$$\tau_t^* = C_{of}\left[\left(\frac{1+E}{D+E}\right)^2\left(\frac{A}{4E}-\frac{C}{20E^2}\right)+\frac{1}{2}\left(1-\frac{1}{S_{ta}}\right)\left\{1-\left(\frac{1+E}{D+E}\right)^2\right\} - \frac{(1-\theta_{ab})}{(D+E)^2}\left\{\frac{1}{2}(1+E)(D-1)+\frac{1}{6}(D-1)^2\right\}\right] \quad (41)$$

It signifies that the total time of freezing and melting of the bath material is functions of C_{of}, θ_{ab}, and S_{ta}.

6. BATH KEPT AT FREEZING TEMPERATURE OF BATH MATERIAL OR FREEZING WITHOUT MELTING

In case of the bath kept at the freezing temperature, T_{bm} of the bath material, the convective heat of the bath becomes zero. Here, only the latent heat of fusion librated due to the freezing of the bath material onto the additive meets the heat requirement that is conducted to the additive. It results in only freezing onto the additive, and makes the conduction factor, C_{of} infinity $(C_{of} \to \infty)$. Substituting it, Eq.(29) reduces to

$$R_{bm}^* = \left[\left\{\frac{A}{4}\frac{R^2_{am}(1-R_{am})}{DR_{am}-1}\right\} - \frac{C}{20}\left\{\frac{R_{am}(1-R_{am})}{DR_{am}-1}\right\}^2 + \frac{1}{2}(1-R^2_{am}) - (1-\theta_{ab})\left\{\frac{1}{2}R_{am}(1-R_{am})+\frac{1}{6}(1-R_{am})^2\right\} + \frac{1-R^2_{am}}{2S_{ta}}\right] \quad (42)$$

The corresponding heat penetration thickness is still obtained from Eq.(28), whereas time for development of R_{bm}^* is derived from Eq.(31).

When the heat penetration boundary, R_{ai} reaches the central axis of the additive $(R_{ai} \to 0)$. Eq.(28) leads to Eq.(40), which, when applied to Eq.(42) gives

$$R_{bm}^* = \left[\left(\frac{1+E}{D+E}\right)^2\left(\frac{A}{4E}-\frac{C}{20E^2}\right)+\frac{1}{2}\left(1+\frac{1}{S_{ta}}\right)\left\{1-\left(\frac{1+E}{D+E}\right)^2\right\} - \frac{(1-\theta_{ab})}{(D+E)^2}\left\{\frac{1}{2}(1+E)(D-1)+\frac{1}{6}(D-1)^2\right\}\right] \quad (43)$$

It states that the growth of the frozen layer thickness, Eq.(42) under the freezing without melting is always greater than that of the freezing and melting, Eq.(29) that occurs when the convective heat is supplied from the bath.

7. VALIDITY

To validate the current problem with those of the literature, it is transformed to melting of the cylindrical additive subjected to constant temperature heat injection at the freezing temperature of the bath material. The additive is initially at its material melting temperature. It leads to $R_{ai} = R_{am}$, $\theta_e = 1$, $\theta_{ab} = 0$, and $\gamma = 1$. Their substitution in Eq.(21) leads to

$$\frac{d}{d\tau}\left[\frac{1}{2}(1-R_{am})-\frac{1}{6}(1-R_{am})^2+\frac{R^2_{am}}{2S_{ta}}\right] = \frac{1}{1-R_{am}} \quad (44)$$

whereas the temperature distribution in the melt region, Eq.(16) changes to

$$\theta_{af} = 1-(1-R_{af}/1-R_{am}) \quad (45)$$

Note that Eq.(44) can also be directly obtained once Eq.(45) is employed in Eq.(15) with $\gamma = 1$.

It yields a close-form solution that appear in Eq.(44) that is transform to

$$\tau = \frac{1}{4}(1-R_{am})^2\left(1+\frac{2}{S_{ta}}\right)-\frac{1}{9}(1-R_{am})^3\left(1+\frac{3}{S_{ta}}\right) \quad (46)$$

It satisfies the initial condition $\tau = 0$, $R_{am}=1$. In case of the complete melting of the cylinder, $R_{am}=0$. Its substitution in Eq.(46) reduces to

$$\tau_{cm} = \frac{1}{4}\left(1+\frac{2}{S_{ta}}\right)-\frac{1}{9}\left(1+\frac{3}{S_{ta}}\right) \quad (47)$$

In the past , Zhang and Faghri, [26] found a close-form solution for the same problem but for solidification instead of melting and obtained

$$\tau\left(\sqrt{(1-2S_{ta})}-1\right)=\frac{1}{4}\left(1-R^{2}{}_{am}\right)-\frac{1}{2}R^{2}{}_{am}\left(\ln R_{am}\right) \tag{48}$$

when they employed a temperature distribution, different than that of Eq.(46). For complete melting of the cylinder R_{am}=1, Eq.(48) gives

$$\tau_{cm}=\frac{1}{4\left(\sqrt{1-2S_{ta}}-1\right)} \tag{49}$$

The current solution, Eq.(48) is exactly the same as that of Eq.(49) for $S_{ta}=3/2$ validating the present problem. For S_{ta} > 3/2, it underestimates and for S_{ta} < 3/2 overestimates the total time of the literature, Eq.(49).

8. RESULTS AND DISCUSSIONS

This investigation devises a non-dimensional lump-integral model for axi-symmetric freezing and melting of agitated bath material onto a low melting temperature cylindrical solid additive. The model exhibits the dependence of this event upon non-dimensional independent parameters: the property-ratio, γ, the heat-capacity-ratio, C_r , and the melting temperature-ratio, θ_{ab}, of the additive-bath system, the phase-change parameters , the Stefan number of the bath material, S_{tb}, and that of the additive, S_{ta} and the bath condition represented by the conduction factor, C_{of}. It yields closed-form solutions for the freezing and melting of the bath material onto the additive and the associated heating and melting of the additive in terms of these parameters but when the frozen layer, R^{*}_{bm}, taken as per unit Stefan number, S_{tb} of the bath material, the time τ^{*} per unit property-ratio, γ they become only in terms of C_{of}, S_{ta} and θ_{ab}. Their values for different additive-bath systems employed in industrial applications are presented in Table.1 and their physical significance is stated.

Table 1. Based on thermo physical properties of the Steel bath [12]; T_b = 1873 K (1600°C), T_{bm} = 1804 K (1531°C), C_b=0.69 KJ/KgK, L_{bm} = 277000 J/Kg, K_{bm}= 29.3W/mK, ρ_{bm} =7820Kg/m³, S_{tb} = 3.751

Additive	Thermo-physical properties of low melting temperature cylindrical solid additive [34] [T_{ai}=298K(25°C), r_o=0.025m]						Non-dimensional parameters		
	T_{am} [K(°C)]	C_{pa} KJ/KgK	ρ_a Kg/m³	L_a KJ/Kg	K_a * W/mK	h w/m²K	S_{ta}	θ_{ab}	C_{of}
Tin	504.9(231.9)	0.226	7300	60.71	60.47	20x10⁴	5.60	0.137	0.288
Bismuth	544.4(271.4)	0.13	9780	51.9	8.0	20x10⁴	3.51	0.16	0.034
Selenium	494(221)	0.352	4816	68.62	0.52	0.25x10⁴	7.72	0.13	0.181
Sulphur	388.2(115.2)	0.71	2000	115.6	0.205	1.2x10⁴	9.25	0.059	0.071

Note: Data for Ka are taken from Ref [20] and [35].

The property-ratio, γ is the ratio of the effusivity of the bath, CbKb , and that of the additive, C_aK_a , and signifies the thermal force. When its value is low, it generates large thermal force in the bath owing to which the smaller thickness of the bath material freezes onto the additive. The melting temperature-ratio, θ_{ab}<1 of the additive and the bath material is indicative of the additive melting temperature lower than that of the material of the bath. In this situation, both melting and heating of the additive take place. The heat-capacity ratio, C_r, is taken as the heat capacity of the bath material divided by the heat capacity of the additive. C_r < 1 signifies large heat storage in the additive with reference to that stored in the frozen layer. The Stefan number is defined as the ratio of the sensible heat and the latent heat of fusion. Its high value denotes the phase-change material of small latent heat of fusion permitting the formation of large thickness of the frozen layer of bath material onto the additive for the same convective heat provided by the bath or allowing the melting of the larger thickness of the low melting temperature additive. The conduction factor, $C_{of,}$ denotes the ratio of the heat conducted to the additive as a result of difference of the freezing temperature of the bath material and the initial temperature of the additive and the convective heat available from the bath and ranges from zero to infinity $\left(0\le C_{of}\le\infty\right)$. $C_{of}\to 0$ signifies that no heat conduction to the additive takes place due to which freezing does not occur. It results in the absence of the freezing and melting event. This situation is attained once the additive before immersion in the bath is heated to the freezing temperature of the bath material. $C_{of}\to 0$ is also achieved for the bath that gives extremely high heat transfer co-efficient, $h\to\infty$ by making the bath highly agitated, $C_{of}\to\infty$ is indicative of absence of the bath convective heat. It is due to the maintenance of the bath at the bath material freezing temperature. In this condition, the conductive heat to the additive is met by only latent heat of fusion generated as a result of the freezing of the bath material onto the additive. It continues to grow till the conductive heat gets balanced by the latent heat of fusion. From these, it is inferred that the unavoidable freezing and melting is completed in much less time if the conduction factor C_{of} is made near zero value for a prescribed additive-bath system. It is achieved by only increasing the bath agitation.

Displayed in Figure (2) are the behavior of the frozen layer thickness, R_{bm}^*, melt layer thickness, R_{am}^*, and heat penetration depth, R_{ai}^* with time for various C_{of}, θ_{ab} and S_{ta} are assumed to be parameters whereas Figure (4) exhibits these behavior for different S_{ta} with C_{of} and θ_{ab} taken as parameters. Figure (6) shows these behavior for different θ_{ab}, when C_{of} and S_{ta} are considered as parameters. It is observed that in all these figures the freezing and melting assumes a parabolic shape. Its apex height denotes the growth of the maximum thickness of the frozen layer. Its left represents the domain of the freezing and time taken to buildup the frozen layer thickness to its maximum extent, whereas its right is the melting region and is indicative of the time for the melting of the frozen layer. The freezing is faster than the melting. Combination of these times is the total time of the freezing and melting, which can directly be obtained from the start of the freezing to the end of the melting of the frozen layer. It is further observed that this ratio remains unchanged with respect to the total time of the freezing and melting for all values of C of considered. Both melting and heating depth of the additive increase fast in the beginning, but as the time passes, they follow almost a linear behavior with time. Moreover, the frozen layer takes 25.48% of the total time of the freezing and melting and the melting of this layer is completed in 74.51% of this total time. These lead to time ratio of the growth of this frozen layer and its melting is approximately $1/4 : 1/3$. It is exactly the same that was obtained earlier [14] but of the additive of higher melting temperature than that of the bath material.

Influence of conduction factor, C_{OF}

For particular values of θ_{ab} and S_{ta}, Figure (2) shows the time variant freezing and melting, R_{bm}*of the bath material for different C_{of}. As stated above, its behavior for each C_{of} is parabolic, but its size reduces with decreasing C_{of} indicating the reduction in time of the freezing and melting, and smaller thickness of the frozen layer grown. Corresponding melt depth and heat penetration depth, Figure (2) in this reduced time diminish. Physically, these predictions appear to be realistic since for a certain additive-bath system, decrease in C_{of} is indicative of increase in the bath agitation and, in turn, convective heat transfer co-efficient and associated bath convective heat. As the heat conducted to the additive is sum of the bath convective heat and the latent heat of fusion evolved due to freezing of the bath material onto the additive, the increased convective heat reduces the requirement of the latent heat of fusion owing to which formation of the smaller thickness of the frozen layer takes place, Figure (2).

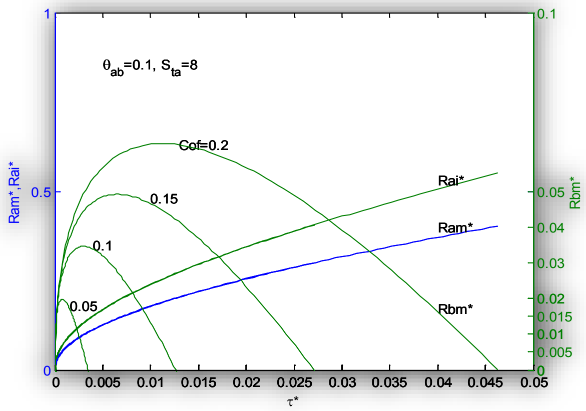

Figure 2: Time dependent freezing and melting of the bath material and the corresponding melt depth and heat penetration depth in the additive for different conduction factors, C_{of}. θ_{ab} and S_{ta} of the additive are taken as parameters

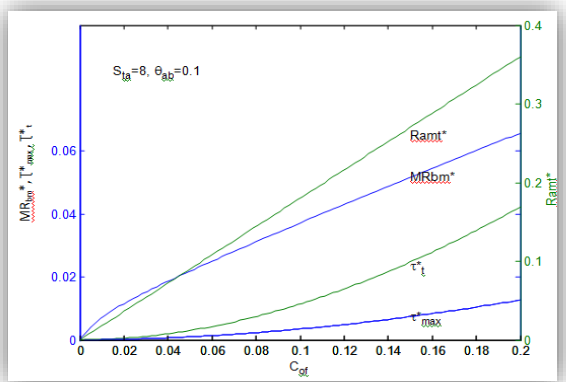

Figure 3: Conduction factor, C_{of} variant maximum frozen layer thickness, MR_{bm}^*, its growth time τ_{max}/γ and the total time of the freezing and melting of the bath material τ_t/γ, along with melt depth, R_{amt}^* when heating front reaches the central axis of the additive for prescribed θ_{ab} and S_{ta}

For those values of θ_{ab} and S_{ta}, stated in Figure (2), Figure (3) displays the plots for the growth of the maximum frozen layer thickness, MR_{bm}*, the time for its formation, τ_{max}* and total time of the freezing and melting, τ_t*, with C_{of}. The corresponding melt depth, R_{amt}^* when the heat penetration depth, R_{ai}^* reaches the central axis of the additive is also exhibited in the same Fig.(3). It is observed that MR_{bm}^* increases with a faster rate for Cof<0.02, whereas it rises almost linearly with C_{of} when C_{of} >0.02. Its time of formation follows a non-linear behavior with C_{of}. The total time of freezing and melting with respect to C_{of} is also non- linear but its value increases with much faster rate as compared with the rate of increase in τ_{max}^{*} when C_{of} increases, Figure (3). However, the melt depth R_{amt}^{*} varies linearly with increase in C_{of}.

Effect of stefan number, S_{TA} of the additive

Displayed in Figure (4), variation of the freezing and melting of the bath material and related melting and heating of the additive with time for different S_{ta}. They are for certain C_{of} and θ_{ab}. Here also, the freezing and melting of the bath material follows parabolic shape for each of S_{ta}, whereas the melt depth and heat penetration depth increase fast during onset of the freezing but their rate of increase becomes slow with passing of the time. Decreasing S_{ta} increases the size of the parabola, the maximum thickness of the frozen layer, the time taken in its growth, the time of its melting and the total time of the freezing and melting. In this time period, both the melt depth and the heat penetration depth increase. These findings can be corroborated from the facts that decreasing S_{ta} is due to increase in the latent of fusion of the additive resulting in increase in the requirement of the total heat which is to be conducted to the additive through the interface. As for the prescribed C_{of}, the convective heat from the bath remains unaltered, the increased total heat is balanced only by the increase in the latent heat of fusion generated through freezing the larger thickness of the bath material onto the additive, Figure (4). S_{ta} variant the maximum frozen layer thickness, MR_{bm}^{*}, its time of formation, τ_{max}^{*}, the total time of the freezing and melting, τ_t^{*} and the melt depth, R_{amt}^{*} in the additive in case of heat penetration depth reaching the central axis of the additive $(R_{ai} \rightarrow 0)$ are plotted in Figure (5) for given C_{of} and θ_{ab}. It is observed that MR_{bm}^{*} decreases and R_{amt}^{*} increases fast, whereas reduction in τ_t^{*} and τ_{max}^{*} is small when S_{ta} resides below the value of 5. For $S_{ta} > 5$, slow decrease in MR_{bm}^{*} is observed, but τ_t^{*} and τ_{max}^{*} remain almost invariant. The rise in R_{amt}^{*} is slow Figure (5).

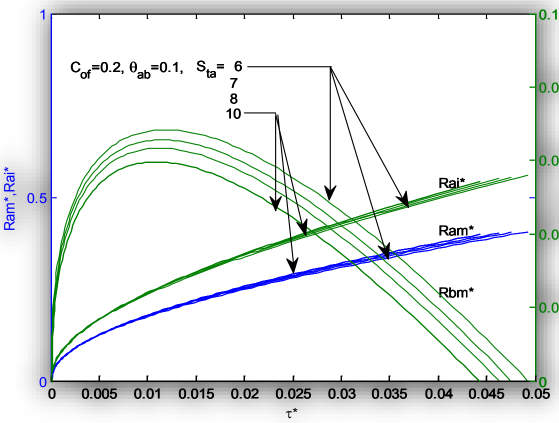

Figure 4: Behavior of freezing and melting of the bath material and the corresponding melt depth and heat penetration depth in the additive for different S_{ta} conduction factor, C_{of} and melting temperature-ratios, θ_{ab} are assumed to be parameters

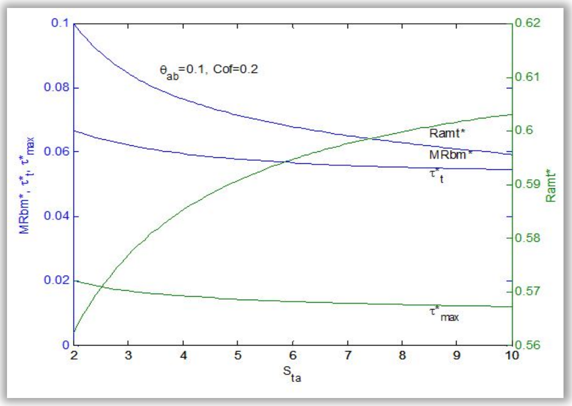

Figure 5: S_{ta} variant maximum frozen layer thickness, MR_{bm}^{*}, its growth time τ_{max}^{*} and the total time of the freezing and melting of the bath material τ_t^{*} along with related melt depth when the heat penetration depth in the additive approaches its central axis. C_{of} and θ_{ab} are employed as parameters

Impact of temperature-ratio, Θ_{ab}

Fig.(6) exhibits time dependent freezing and melting of the bath material around the additive along with melting and heating of the additive for various θ_{ab}. They are for prescribed C_{of} and S_{ta}. The features of the graphs are similar to those appeared in Fig.(2). When θ_{ab} is allowed to decrease, the parabola denoting freezing and melting gets smaller. It leads to reduction in the maximum thickness of the frozen layer, its time of formation, the total time of the freezing and melting and the time of the melting of the frozen layer. Also, this diminished time reduces both the melt and the heat penetration depths in the additive. Physically, they are true since decreasing θ_{ab} decreases the melting temperature of the additive or increasing the initial temperature of the additive. It results in the reduction in sensible heat and, consequently, decrease in the total heat requirement to be conducted to the additive through the interface. Because for a given C_{of} the convective heat available from the bath is unchanged, this decreased heat requirement is met by less amount of latent heat of fusion, which becomes available, once a smaller thickness of the bath material freezes onto the additive, Fig.(6). The effect of θ_{ab} on the total time of freezing and melting τt^{*}, the maximum growth of the frozen layer thickness, MR_{bm}^{*}, the time of its formation $\tau^{*}max$ and the melt depth R_{amt}^{*} in the additive in case of certain C_{of} and S_{ta} is depicted in Fig.(7). It shows that τ_t^{*}, MR_{bm}^{*} and τ^{*}_{max} increase almost linearly whereas R_{amt}^{*} decreases fast but linearly once θ_{ab} is allowed to increase from 0.05 to 0.1 $(0.05 \le \theta_{ab} \le 0.1)$.

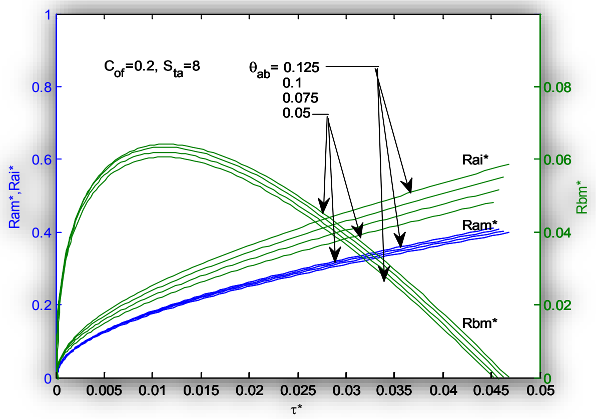

Figure 6: Time variant freezing and melting of the bath material and the corresponding melt depth and heat penetration depth in the additive for different Stefan number, S_{ta}, C_{of}. and θ_{ab} are taken as parameters

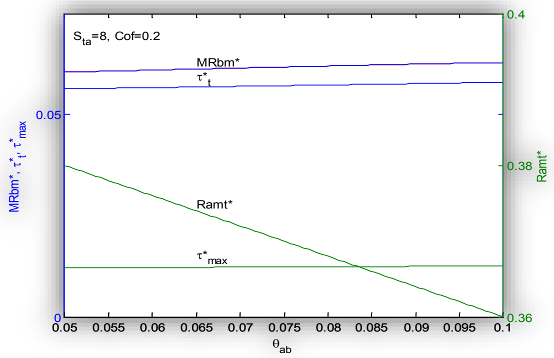

Figure 7: Variation of maximum frozen layer thickness, MR_{bm}^*, its growth time τ^*_{max} and total time of freezing and melting of the bath material τ^*_t along with the corresponding melt depth of the additive with different θ_{ab} in case of heat penetration reaches the central axis of the additive, C_{of} and S_{ta} act as parameters

⧉ Model application to manufacturing processes

The model just presented gives closed-form solutions for the freezing and melting of the bath material onto the additive and the heating and melting of the additive. These solutions are applied to predict the freezing and melting behavior of the steel bath material [12] onto additives made of different materials, Table-1, employed in manufacturing process. Each of these additives is of 0.025 m diameter. The figure exhibits that the freezing and melting of the bath material on to Bismuth takes much less time than Tin for their same diameter. Fig.8.

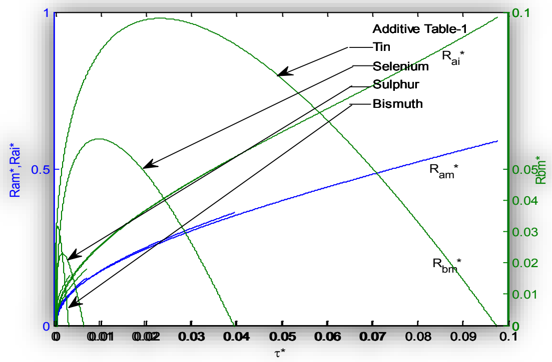

Figure 8: Time dependent freezing and melting of the bath materil onto the additive often employed in steel manufacturing along with associated time variant melting and heating of additive

9. CONCLUSIONS

A non-dimensional lump-integral model is evolved for the freezing and melting of agitated bath material onto a low melting temperature solid additive of cylindrical type when the thermal resistance of the frozen layer developed is negligible with respect to that of the additive. It gives closed-form expressions for the freezing and melting of the bath material and the associated heating and melting of the additive in terms of independent non-dimensional parameters, the melting temperature ratio, θ_{ab}, the Stefan number, S_{ta} of the additive and the conduction factor, C_{of}. The maximum frozen layer thickness grew, its time of growth and the freezing and melting time are obtained in terms of these θ_{ab}, S_{ta} and C_{of}. They are reduced by increasing S_{ta} for prescribed C_{of} and θ_{ab}, or decreasing C_{of} for given θ_{ab} and S_{ta}, or decreasing θ_{ab} for certain S_{ta} and C_{of} . The model is tested by validating it after its transformation to only melting of the additive with that of the literature. When there is no bath convective heat, only freezing occurs. Its close-form expression is also found.

NOMENCLATURE

B_i	Biot number, hr_a/K_a
B_{im}	Modified Biot number, $(hr_a/K_a)*(K_aC_a/K_bC_b)$
C	heat capacity (ρ C_p), $Jm^{-3}K^{-1}$
C_{of}	conduction factor, $1/\gamma B_{im}(\theta_b-1)$, $1/B_i(\theta_b-1)$,
$C_{of\,m}$	modified conduction factor, C_{of} $(\theta_{ab}-1)$
C_p	specific heat, $(J\ Kg^{-1}K^{-1})$
C_r	heat capacity ratio, C_b/C_a
h	heat transfer coefficient, $Wm^{-2}K^{-1}$
K	thermal conductivity, $Wm^{-1}K^{-1}$
L	latent heat of fusion, JKg^{-1}
r	radius, m
R_{ah}	non-dimensional radius in the heat penetration region of the additive, (r_{ah}/r_a)
R_{ai}	non-dimensional radius of the heat penetration front in the additive at any time, (r_{ai}/r_a)
R_{bf}	non-dimensional radius within the frozen layer region, (C_br_{bf}/C_ar_a)
R_{bm}	non-dimensional radius of the frozen layer front at anytime, (C_br_{bm}/C_ar_a)
S_{ta}	Stefan number of the additive, $C_a(T_{bm}-T_{ai})/L_a\ \rho_a$
S_{tb}	Stefan number of the bath material, $C_b(T_{bm}-T_{ai})/L_b\ \rho_b$
t	time, s
T	temperature, K
T_b	bulk temperature of the bath material, K
T_e	instant equilibrium temperature at the interface between the additive and the frozen layer, K

GREEK LETTERS

α thermal diffusivity, $m^2 s^{-1}$
γ property ratio, (K_bC_b/K_aC_a)
ρ density, (Kgm^{-3})
θ non-dimensional temperature, $(T-T_{ai}/T_{bm}-T_{ai})$
θ_{ab} ratio of melting or freezing temperature of the additive that of the bath, $(T_{am}-T_{ai})/(T_{bm}-T_{ai})$
τ non-dimensional time, $(K_bC_b/C_a^2 r_0^2)t = \frac{\gamma\alpha_a t}{r_a^2}$
τ^* non-dimensional time per unit property ratio

SUBSCRIPTS

a cylindrical additive,
ai initial condition of additive,
af within melting or freezing region of additive,
ah within heating region of additive,
am melting or freezing of additive,
b frozen bath material or bulk condition of bath material,
bf within melting or freezing region of bath material,
bm melting or freezing condition of bath material,
e interface condition,
max for maximum frozen layer development,
t for total time of freezing and melting

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ISSN 1584 - 2665 (printed version); ISSN 2601 - 2332 (online); ISSN-L 1584 - 2665