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NUMERICAL SIMULATION OF UNSTEADY FLOW IN WATER SUPPLY PIPE NETWORKS

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Abstract: Water supplies of large urban and industrial centres consist of increasingly larger distribution networks that are necessary to ensure the greater uniformity and stability of pressure lines, with favourable economic and energy effects. The optimisation of pipe networks under steady flow conditions has been studied and various researchers have proposed the use of innovative non-linear and heuristic optimisation techniques in order to identify the optimal solution for water distribution systems. An unsteady flow in pipe networks is usually a transient state from one steady state to another, including to and from resting state. This paper presents the basic concepts associated with transient flow, discusses the theoretical background of water hammer, and introduces aspects of system design that should be considered during transient analysis. Additionally, several analysis models of transient flows are developed including the transient analysis and design optimisation for pipe networks using genetic algorithm method. Finally, the versatility of this approach is demonstrated solving a numerical example.
Keywords: Water distribution, pipe network, unsteady flow, transient analysis, design optimisation, application

1. INTRODUCTION

Attention towards efficient and eco-friendly management has been growing in recent years, especially in systems characterised by a large consumption of non-renewable energy. Supplying water resources for the growing population of urban areas has been a major challenge in past decade. The distribution network is an essential part of all urban water supply systems. Distribution system costs within any water supply scheme may be equal to or greater than 60% of the entire cost of the project [1,2]. As reported by Lingireddy and Wood [3] and Bene et al. [4], the electrical energy used to pump water is a significant portion of the total operational costs in water distribution systems. Water supplies of large urban and industrial centres consist of increasingly larger distribution networks that are necessary to ensure the greater uniformity and stability of pressure lines, with favourable economic and energy effects.

Attempts should be made to reduce the cost and energy consumption of the distribution system through optimisation in analysis and design. In water distribution systems, the optimisation process by trial and error methods can present difficulties due to the complexity of these systems such as multiple pumps, valves and reservoirs, head losses, large variations in pressure values, several demand loads, etc. For this reason, innovative non-linear and heuristic optimisation algorithms are becoming more widely explored in optimisation processes of the pipe networks under steady flow conditions [5].

An unsteady flow in pipe networks is usually a transient state from one steady state to another, including to and from resting state. Therefore, a hydraulic transient is the flow rate and pressure condition that occurs in a pipe network between an initial steady state condition and a final steady state condition. When velocity changes rapidly because a flow control component changes status (for example, a valve closing or pump turning off), the change moves through the system as a pressure wave.

The primary objectives of transient analysis are to determine the values of transient pressures that can result from flow control operations and to establish the design criteria for system equipment and devices (such as control devices and pipe wall thickness) so as to provide an acceptable level of protection against system failure due to pipe collapse or bursting. Because of the complexity of the equations needed to describe transients, numerical computer models are used to analyse transient flow hydraulics.

This paper presents the basic concepts associated with transient flow, discusses the theoretical background of water hammer, and introduces aspects of system design that should be considered during transient analysis. Additionally, several analysis models of transient flows are developed including the transient analysis and design optimisation for pipe networks using genetic algorithm method. Finally, the versatility of this approach is demonstrated solving a numerical example.

2. OVERVIEW OF TRANSIENT EVALUATION

Examples of system flow control operations include opening and closing valves, starting and stopping pumps, and discharging water in response to fire emergencies. These operations cause transient flow phenomena, especially if they are performed too quickly. Proper design and operation of a hydraulic system is necessary to minimise the risk of system damage or failure due to hydraulic transients. When a flow control operation is performed, the established steady state flow condition is altered. The values of the initial flow conditions of the system, characterised by the measured velocity V and pressure p at positions along the pipe x, change with time t until the final flow conditions are established in a new steady state condition.

The physical phenomenon that occurs during the time interval T_T between the initial and final steady state conditions is known as the hydraulic transient. In general, transients resulting from relatively slow changes in flow rate are referred to as *surges*, causing a *mass oscillation* and those resulting from more rapid changes in flow rate are referred to as *water hammer events* [2].

For typical water distribution main installation, transient analysis may be necessary even if velocities are low. System looping and service connections may amplify transient effects and need to be studied carefully. Transient analysis should be performed for large, high-value pipes, especially those with pumping stations.

Evaluating a system for potential transient impacts involves determining the values of head (H_{max} and H_{min}) at incremental positions in the system. These head values correspond to the minimum and maximum pressures of the transient pressure wave, depicted as p_{max} and p_{min} in Figure1. Computation of these head values through the system allows the engineer to draw the grade lines for the minimum and maximum hydraulic grades expected to occur due to the transient. If the elevation Z along the pipe is known, then the pipe profile can be plotted together with the hydraulic grades and used to examine the range of possible pressures throughout the system.

Figure 1 shows a pumping system in which an accidental or emergency pump shutdown has occurred. The extreme values indicated by the hydraulic grade lines were developed by reviewing the head versus time data at incremental points along the pipeline. The grade lines for H_{min} and H_{max}, which define the pressure envelope or head envelope, provide the basis for system design. If the H_{min} grade line drops significantly below the elevation of the pipe, as shown in a portion of the system in Figure 1, then the engineer is alerted to a vacuum pressure condition that could result in column separation and possible pipe collapse. Pipe failure can also result if the transient pressure in the pipe exceeds the pipe's pressure rating. Maximum (or minimum) transient pressure can be determined for any point in the pipe by subtracting the pipe elevation Z from H_{max} (or H_{min}) and converting the resulting pressure head value to the appropriate pressure units.

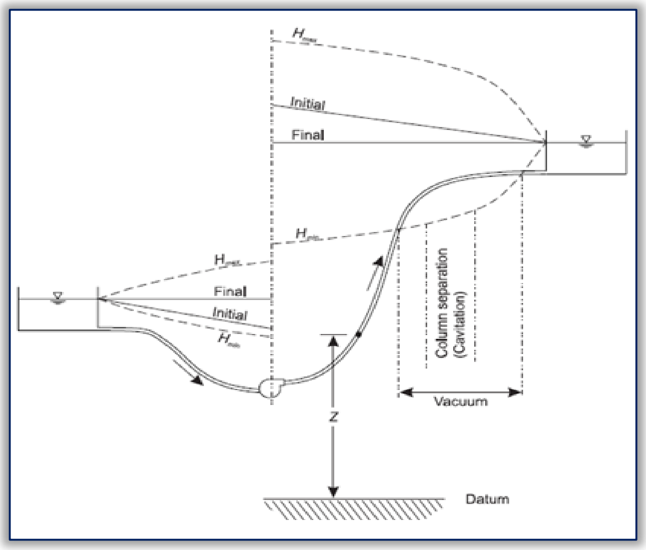

Figure 1. Grade lines for a pumping system during an emergency shutdown

Specialised programs are necessary to perform transient analysis in water distribution systems. Hydraulic transients can be analysed using one of two model types: a *rigid model* or an *elastic model* [5]. The rigid model has limited applications in hydraulic transient analysis because the resulting equation does not accurately interpret the physical phenomenon of pressure wave propagation caused by flow control operations, and because it is not applicable to rapid changes in flow.

Water hammer is considered as a hydraulic transient phenomenon and is defined as unsteady flow, which is transmitted as a pressure or water hammer wave in the pipe system. Water hammer can be generated by operating system devices including valves and pumps, and by events such as pipe rupture. The complete equations for water hammer are one-dimensional unsteady pressure flow equations given by [6,7]:

$$\frac{\partial H}{\partial t}+\frac{a^2}{gA}\frac{\partial Q}{\partial x}=0 \tag{1}$$

$$\frac{\partial Q}{\partial t}+gA\frac{\partial H}{\partial x}+\frac{\lambda Q|Q|}{2DA}=0 \tag{2}$$

where: H is the total head in a pipe, in m; t is the time, in s; a is the characteristic wave celerity of the liquid, in m/s; g is the gravitational acceleration, in m/s^2; A is the cross-sectional area of pipe, in m^2; Q is the flow rate, in m^3/s; λ is the Darcy-Weisbach friction factor; D is the pipe diameter, in m.

Transient modelling essentially consists of solving Eqs. (1) and (2) for a wide variety of boundary conditions and system topologies. The equations cannot be analytically solved, so various approximate methods have been developed over the years: arithmetic method [8]; graphical method [9]; method of characteristics [10,11]; algebraic method [6]; wave-plan analysis method [12]; implicit method [13]; perturbation method [14].

3. CONSIDERATIONS ON PIPE SYSTEM DESIGN

The emergency flow control scenarios should be analysed and tested during the design phase because they affect the pipe system design and the selection of system equipment. Steel, polyvinyl chloride (PVC), height density polyethylene (HDPE), and thin-wall ductile iron pipes are susceptible to collapse due to vapour separation, but any pipe that has been weakened by repeated exposure to these events may experience fatigue failure. A pipe weakened by corrosion may also fail.

Where very low pressures are possible during transient events, a more expensive material to preclude the chance of collapse can be used. For example, for large-diameter pipes under high pressures, steel is usually more economical than ductile iron. However, the engineer may select ductile iron because it is less susceptible to collapse. It is always best to avoid vapour pressure conditions through surge protection measures regardless of the type of pipe used. Pipe systems constructed above ground are more susceptible to collapse than are buried pipes. With buried pipes, the surrounding bedding material and soil provide additional resistance to pipe deformations and help the pipe resist structural collapse.

Another important consideration when designing a system to protect against hydraulic transients is the use of air valves. Using air valves to avoid vacuum conditions requires careful analysis of possible transient conditions to ensure that the air valve is adequately sized and designed. Other factors that influence extreme transient heads are wave celerity and liquid velocity. Selecting larger diameters to obtain lower velocities with the purpose of minimising transient heads is acceptable for short pipe systems delivering relatively low flows. However, for long pipe systems, the diameter should be selected to optimise construction and operating costs. Long pipe systems almost always require transient protection devices.

After considering these factors during the conceptual and preliminary designs of the system, the project should move into the final design phase. Any changes to the system during final design should be analysed with the transient model to verify that the previous analysis results and specifications are still appropriate.

4. TRANSIENT ANALYSIS IN PIPE NETWORKS

The process of obtaining an unsteady solution for a specific problem in which the water demands or pressure heads are specified functions of time consists of the following steps [15]:

1) The time T, over which the unsteady solution is to be obtained, is divided into $T/\Delta t$ time increments, where Δt is the time step.
2) The discharges in all pipes and the pressure heads at all nodes are assigned initial values that are chosen from a steady state solution that has the same demands, and all other data as the unsteady solution has at time zero.
3) All water demands over each time increment must be specified.
4) Over each new time increment, define and evaluate the functions and the Jacobean matrix of derivatives of these functions.
5) Solve the resulting linear equation system. The solution of this equation system is then subtracted from the set of unknown values, according to the Newton-Raphson method [16].
6) Steps 4) and 5) are repeated iteratively, until the specified convergence criterion has been satisfied.
7) Write the solution for the discharges and the nodal heads for this time increment, and then repeat steps 3) through 7) until the unsteady solution spans the entire time period.

The steps from 1) through 7) are the general method for analysing an unsteady flow in a pipe system. This system is consisted of pipes, reservoirs, pumps, tanks, etc. In the following sub-section, the governing equation for each component and some of their boundary conditions will be mentioned.

The unsteady flow inside the pipes is described in terms of the unsteady mass balance (continuity) equation and unsteady momentum equation, which define the state variables as the discharge Q or velocity V, and pressure head H.

Equations describing unsteady flow in pipes

Using the method of characteristics for analysing the unsteady flow in pipe networks, a pair of equations to find H and V in a pipe divided in n segments at the interior point P, starting from point 2 to point N (point 1 is related to the boundary condition) is developed [15]:

$$V_P = \frac{1}{2}\left[(V_L + V_R) + \frac{g}{a}(H_L - H_R) - \frac{\lambda \Delta t}{2D}(V_L|V_L| + V_R|V_R|)\right] \tag{3}$$

$$H_P = \frac{1}{2}\left[\frac{a}{g}(V_L - V_R) + (H_L + H_R) - \frac{a}{g}\frac{\lambda\Delta t}{2D}(V_L|V_L| - V_R|V_R|)\right] \quad (4)$$

where the subscripts *L* and *R* are considered as the left and right points on the characteristic grid with respect to certain point P and located at the same distance from it.

Reservoir boundary condition (upstream end of pipe)

For a pipe exiting from a reservoir and neglecting the entrance head losses, the *H* equation is the following:

$$H_{P_1} = H_0 \quad (5)$$

where H_0 is the head of the reservoir water surface. In addition, the velocity V_{P_1} can be calculated as

$$V_{P_1} = V_2 + \frac{g}{a}(H_0 - H_2) - \frac{\lambda\Delta t}{2D}V_2|V_2| \quad (6)$$

Three pipes connected in one junction

For a pipe junction with one inflow (pipe 1), two outflows (pipes 2 and 3) and an external demand *q* at the junction, the equations that describe the relationships between the six unknowns are:

$$V_{P_1} = V_2 + \frac{g}{a}(H_0 - H_2) - \frac{\lambda\Delta t}{2D}V_2|V_2| \quad (7)$$

Pipe 1, C^+:

$$V_{P_1} = C_1 - C_2 H_{P_1} \quad (8)$$

Pipe 2, C^-:

$$V_{P_2} = C_3 + C_4 H_{P_2} \quad (9)$$

Pipe 3, C^-:

$$V_{P_3} = C_5 + C_6 H_{P_3} \quad (10)$$

Conservation of mass:

$$V_{P1} A_1 = V_{P_2} A_2 + V_{P_3} A_3 + q \quad (11)$$

Work-energy:

$$H_{P1} = H_{P_2} = H_{P_3} \quad (12)$$

Solving this linear set of equations leads to:

$$H_{P1} = H_{P_2} = H_{P_3} = \frac{C_1 A_1 - C_3 A_2 - C_5 A_3 - q}{C_2 A_1 + C_4 A_2 + C_6 A_3} \quad (13)$$

where: H_P and V_P are the pressure head and velocity, respectively at a specific point P in a specific end of the three connected pipes; $C_1 - C_6$ are the pipes constants; and A_1, A_2, and A_3 are the pipe cross-sectional areas. In the same manner can be obtained equations for four or five pipes connected at the same junction.

Valve in the interior of a pipe

The internal boundary condition for equal cross-sectional areas of pipe on both sides of valve (Figure 2) is described by the equations:

Pipe 1, C^+:

$$V_{P1} = C_3 - C_4 H_{P_1} \quad (14)$$

Pipe 2, C^-:

$$V_{P2} = C_1 + C_2 H_{P_2} \quad (15)$$

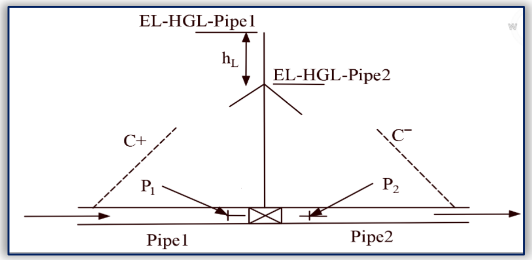

Figure 2. Valve in a pipe with constant diameter

Conservation mass:

$$V_{P1} = V_{P_2} \quad (16)$$

Work-energy:

$$H_{P1} = H_{P_2} + \zeta_v \frac{V_{P_2}^2}{2g} \quad (17)$$

where ζ_v is the valve minor loss coefficient.

The equation obtained by combining Eqs. (14) – (17) is:

$$V_{P_2}^2 + \frac{2g}{\zeta_v}\left(\frac{1}{C_4} + \frac{1}{C_2}\right)V_{P_2} - \frac{2g}{\zeta_v}\left(\frac{C_3}{C_4} + \frac{C_1}{C_2}\right) = 0 \quad (18)$$

While keeping ζ_v separate, definition of the coefficients:

$$C_5 = 2g\left(\frac{1}{C_4}+\frac{1}{C_2}\right); \quad C_6 = 2g\left(\frac{C_3}{C_4}+\frac{C_1}{C_2}\right) \tag{19}$$

leads to the velocity expression:

$$V_{P_1} = V_{P_2} = \frac{C_5}{2\zeta_v}\left[-1+\sqrt{1+\frac{4C_6\zeta_v}{C_5^2}}\right] \tag{20}$$

This equation is correct so long as the flow is in the original downstream direction. If the flow reverses, then the following equation is obtained:

$$V_{P_1} = V_{P_2} = \frac{C_5}{2\zeta'_v}\left[1-\sqrt{1-\frac{4C_6\zeta'_v}{C_5^2}}\right] \tag{21}$$

Source pumping station at upstream end of pipe

Discharge side, C^-:

$$V_{P_d} = C_3 + C_4 H_{P_d} \tag{22}$$

Conservation mass:

$$N_P Q = V_{P_d} A_d \tag{23}$$

Work-energy:

$$Z_{PS} + H_p = H_{P_d} \tag{24}$$

Pump characteristics:

$$\frac{H_p}{n_p^2} = n_{ss}\left[C_7\frac{Q}{n_p}+C_8\right] \tag{25}$$

with

$$C_7 = \frac{\left(H_p/n_p^2\right)_A - \left(H_p/n_p^2\right)_B}{\left(Q/n_p\right)_A - \left(Q/n_p\right)_B}; \quad C_8 = -C_7\left(Q/n_p\right)_B + \left(H_p/n_p^2\right)_B \tag{26}$$

where: N_P is the number of pumps in parallel; A_d is the area of delivery pipe; Z_{PS} is the pump elevation head; H_p is the head delivered by pump; n_p is the pump speed at the transient state; n_{ss} is the pump speed at the steady state; $C_3 - C_8$ are constants; and A, B are two points on the pump characteristic curve ($H_p - Q$).

From the previous equations, following solution is obtained for the pressure head at a specific point H_P:

$$H_P = \frac{Z_{PS} + \dfrac{n_{ss}n_p}{N_P}C_3C_7A_d + n_{ss}n_p^2C_8}{1-\dfrac{n_{ss}n_p}{N_P}C_4C_7A_d} \tag{27}$$

Nodal equations of looped networks

Looped networks are reduced to virtual branched networks by fictitious sectioning of pipes. Thus, there are created additional nodes called *apparent nodes* (A) in which, of course, the boundary and connection conditions at any moment t are reduced to the identity of the discharges and pressures on the left and right of the applied section (Figure 3).

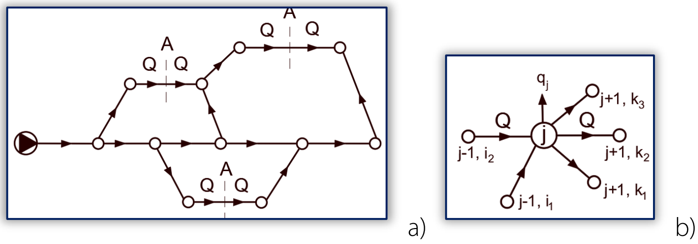

Figure 3. Transformation of a looped network into a virtual branched network. a) looped network with apparent nodes; b) junction node

Therefore, the following nodal equations result [10]:

– junction node:

$$\begin{aligned}
&H_{j,i}^{(t+1)} - H_{j-1,i}^{(t)} = -m_{j-1,i}\left(Q_{j,i}^{(t+1)} - Q_{j-1,i}^{(t)}\right) \quad (i=1,2,\ldots,I)\\
&H_{j,k}^{(t+1)} - H_{j+1,k}^{(t)} = m_{j,k}\left(Q_{j,k}^{(t+1)} - Q_{j+1,k}^{(t)}\right) \quad (k=1,2,\ldots,K)\\
&H_{j,i}^{(t+1)} = H_{j,k}^{(t+1)} = H_j^{(t+1)}\\
&\sum_{i=1}^{I} Q_{j,i}^{(t+1)} = \sum_{k=1}^{K} Q_{j,k}^{(t+1)} + q_j^{(t+1)}
\end{aligned} \tag{28}$$

where: t is the time of calculation; $m=a/(gA)$ is the wave resistance; I is the number of inflow pipes in node; K is the number of outflow pipes in node; q_j is the consumed discharge at node j, which may vary over time depending on the pressure head H_j from the node. For example, in the case of a hydrant or overpressure valve, the connection between q_j and H_j is precisely the characteristic curve of the device:

$$q_j = q_j(H_j) \tag{29}$$

– apparent node:

$$\begin{gathered} H_j^{(t+1)} - H_{j-1}^{(t)} = -m_{j-1}\left(Q_j^{(t+1)} - Q_{j-1}^{(t)}\right) \\ H_j^{(t+1)} - H_{j+1}^{(t)} = m_j\left(Q_j^{(t+1)} - Q_{j+1}^{(t)}\right) \\ Q_{j,i}^{(t+1)} = -Q_{j,k}^{(t+1)} \end{gathered} \tag{30}$$

Eqs. (30) result from Eqs. (28) substituting $I = K = 1$ and $q_j = 0$. A system of computer programs for the calculation of unsteady flows in single wire pipes and piping systems was developed by a team of specialists from the Technical University of Civil Engineering in Bucharest [17].

5. OPTIMISATION OF PIPE NETWORKS

Objective function and constraints

Water distribution network design problem is formulated and solved as a single-objective optimisation problem with the selection of pipe diameters as the decision variables. The main parameter is subject to minimisation which is the capital cost of the network. The optimisation problem is solved using a single-objective genetic algorithm (GA).

The objective of the optimal design model is to minimise total capital costs under the constraint of minimum pressure head requirements in steady state condition and minimum and maximum pressure heads requirements in transient condition (water hammer). The latter is included in order to protect the system from negative or positive transient pressures. More specifically, the optimisation problem is to minimise the objective function F_c. It is the summation of the network cost and penalty cost in both cases: steady state and water hammer (transient state):

$$F_c = C_n + C_{p-ST} + C_{p-TR} \rightarrow \min \tag{31}$$

with

$$C_n = \sum_{ij=1}^{T} c_{ij} L_{ij} = \sum_{ij=1}^{T} (a + bD_{ij}^{\alpha}) L_{ij} \tag{32}$$

where: C_n is the capital cost of the network; C_{p-ST} is the penalty cost in case of steady state; and C_{p-TR} is the penalty cost in case of transient condition; T is the number of pipes in a network; c_{ij} is the specific cost of pipe ij; a, b and α are the cost parameters depending on the network pipe material [18]; D_{ij}, L_{ij} are the diameter and the length of pipe ij, respectively.

Penalty cost in case of steady state C_{p-ST} is described as follows [19]:

$$C_{p-ST} = \begin{cases} 0 & \text{if} \quad H_{min-ST} - H_j \le 0 \\ \dfrac{C_n}{N} \sum_{j=1}^{N} \left(H_{min-ST} - H_j\right) & \text{if} \quad H_{min-ST} - H_j > 0 \end{cases} \tag{33}$$

where: N is the number of node in network; $H_{min\text{-}ST}$ is the minimum allowable pressure head for water hammer; and H_j is the pressure head at node j.

The total penalty cost in case of water hammer C_{p-TR} is described as follows:

$$C_{p-TR} = C_{p-TR,max} + C_{p-TR,min} \tag{34}$$

with

$$C_{p-TR,max} = \begin{cases} 0 & \text{if} \quad H_{j,max} - H_{max-TR} \le 0 \\ C_n \sum_{j=1}^{N} \left(H_{j.max} - H_{max-TR}\right) & \text{if} \quad H_{j,max} - H_{max-TR} > 0 \end{cases} \tag{35}$$

$$C_{p-TR,min} = \begin{cases} 0 & \text{if} \quad H_{min-TR} - H_{j,min} \le 0 \\ C_n \sum_{j=1}^{N} \left(H_{min-TR} - H_{j,min}\right) & \text{if} \quad H_{min-TR} - H_{j,min} > 0 \end{cases} \tag{36}$$

where: $C_{p\text{-}TR,max}$ is the penalty cost in case of water hammer when the pressure head exceeds the maximum allowable pressure head limit; $C_{p\text{-}TR,min}$ is the penalty cost in case of water hammer when the pressure head

decreases below the minimum allowable pressure head limit; $H_{max\text{-}TR}$ is the maximum allowable pressure head for water hammer; and $H_{min\text{-}TR}$ is the minimum allowable pressure head for water hammer
Generally, the penalty cost in case of steady state is a function of minimum allowable pressure head at each node, pressure head at each node, and number of nodes violating the criteria.
The minimisation of the objective function (31) is subject to:

(a) Discharge balance constraint, as described in Eq. (37):

$$\sum_{\substack{i=1\\ i\neq j}}^{N} Q_{ij} + q_j = 0 \quad (j = 1,...,N - N_{RP}) \tag{37}$$

where Q_{ij} is the discharge through pipe ij, with the sign (+) when entering node j and (–) when leaving it; q_j is the consumption discharge (demand) at node j, with the sign (+) for node inflow and (–) for node outflow.

(b) Energy balance constraint, as described in Eq. (38):

$$\sum_{\substack{ij\in m\\ ij=1}}^{T} \varepsilon_{ij} h_{ij} - f_m = 0 \quad (m = 1,...,M) \tag{38}$$

where h_{ij} is the hydraulic head loss of the pipe ij; ε_{ij} is the orientation of flow through the pipe, having the values (+1) if the water flow sense is the same, (–1) if the water flow sense is the opposite to the path sense of the loop m, or (0) if $ij \notin m$; and f_m is the pressure head introduced by the potential elements of the loop m, given by the relations:

» Simple closed–loops:

$$f_m = 0 \tag{39}$$

» Closed–loops containing booster pumps installed in the pipes:

$$f_m = \sum_{\substack{ij\in m\\ ij=1}}^{T} \varepsilon_{ij} H_{p,ij} \tag{40}$$

» Open–loops with pumps and/or reservoirs at nodes:

$$f_m = Z_I - Z_E \tag{41}$$

where Z_I and Z_E are the piezometric heads at the pressure devices at the entrance and the exit from the loop, respectively; $H_{p,ij}$ is the pumping head of the booster pump integrated on the pipe ij, for the discharge Q_{ij}, which is approximated by parabolic interpolation of the pump curve given by points:

$$H_{p,ij} = AQ_{ij}^2 + B\left|Q_{ij}\right| + C \tag{42}$$

The coefficients A, B, and C can be calculated, taking three points from the curve of the manufacturer.

(c) Design constraint is the pipe diameter bounds (maximum and minimum) and given as:

$$D_{min} \le D_{ij} \le D_{max} \qquad (ij = 1,...,T) \tag{43}$$

where D_{ij} is the discrete diameter of pipe ij, selected from the set of commercially available pipe sizes, and T is the total number of pipes.

(d) The hydraulic constraints for steady state and water hammer are given as:

$$H_j \ge H_{min-ST} \qquad (j = 1,...,N) \tag{44}$$

$$H_{min-TR} \le H_k \le H_{max-TR} \quad (k = 1,...,N_*) \tag{45}$$

where: H_j is the pressure head at node j; $H_{min\text{-}ST}$ is the minimum allowable pressure head at node j for the steady state: $H_{min\text{-}TR}$ and $H_{max\text{-}TR}$ are the minimum and maximum allowable pressure heads at node k for the transient state; and N_* is the number of segment into which the pipe is divided.
Using GA to solve the optimisation problem in Eq. (31), constraints (a), (b), (c), and (d) can be automatically satisfied by linking GA to the deterministic water distribution network solver such as Newton-Raphson method and transient analyser that has implemented the method of characteristics.

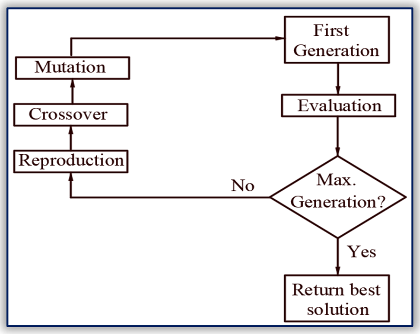

Figure 4. Genetic algorithm flow chart

Implementation of genetic algorithm over pipe network

The flow chart in Figure 4 shows the sequence of the basic operators used in GAs. The first generation is randomly selected from the start. Every string in this generation is evaluated according to its quality, and a fitness value is assigned. Next, a new generation is produced by applying the reproduction operator. Pairs of strings of the new generation are selected and crossover is performed. With a certain probability, genes are mutated before all solutions are evaluated again. This procedure is repeated until a maximum number of generations is reached. While doing this, the all-time best solution is stored and returned at the end of the algorithm.

The GA serves as a framework which provides the outer cycle of the search or optimisation process. The brief idea of GA is to select population of initial solution points scattered randomly in the optimised space, then converge to better solutions by applying in iterative manner the following three processes (reproduction/selection, crossover and mutation) until a desired criteria for stopping is achieved.

The optimisation program GASTnet (Genetic Algorithm Steady Transient network) was written in FORTRAN language and it links the GA, the Newton-Raphson simulation technique for the steady state hydraulic simulation and the transient analysis [20]. A brief description of the steps in using GA for pipe network optimisation, and including water hammer is as follows:

» *Generation of initial population.* The GA randomly generates an initial population of coded strings representing pipe network solutions of population size n. Each of the n strings represents a possible combination of pipe sizes.
» *Computation of network capital cost.* For each n string in the population, the GA decodes each substring into the corresponding pipe size and computes the total material cost. The GA determines the costs of each trial pipe network design in the current population.
» *Hydraulic analysis of each network.* A steady state hydraulic network solver computes the pressure heads and discharges under the specified demands for each of the network designs in the population. The actual nodal pressure heads are compared with the minimum allowable pressure heads, and any pressure deficits are noted. The Newton-Raphson technique is used.
» *Computation of penalty cost for steady state.* The GA assigns a penalty cost for each demand if a pipe network design does not satisfy the minimum pressure head constraints. The pressure violation at the node at which the pressure deficit is maximum, is used as the basis for computation of the penalty cost. The maximum pressure deficit is multiplied by a penalty factor (C_n/N), as described in Eq. (33).
» *Transient analysis of each network.* A transient analysis solver computes the transient pressure heads resulting from the pump power failure, sudden valve closure or sudden demand change as best described in the section 4 by Eqs. (3) – (27). The minimum and maximum pressure heads are estimated in each pipe of the network and compared with the minimum and maximum allowable pressure heads, and any pressure deficits are noted.
» *Computation of penalty cost for transient state.* The GA assigns a penalty cost if a pipe design does not satisfy the minimum and maximum allowable pressure heads constraints. The penalty cost is estimated as the pressure violation multiplied by a penalty factor equals to the cost of the pipes, as described by Eqs. (34) – (36).
» *Computation of total network cost.* The total cost of each network in the current population is taken as the sum of the network cost (Step 2), the penalty cost of steady state (Step 4), plus the penalty cost of transient state (Step 6). This step is an expression to Eq. (31).
» *Computation of the fitness.* The fitness of the coded string is taken as some function of the total network cost. For each proposed pipe network in the current population, it can be computed as the inverse or the negative value of the total network cost from Step 7.
» *Generation of a new population using the selection operator.* The GA generates new members of the next generation by a selection scheme.
» *The crossover operator.* Crossover occurs with some specified probability of crossover for each pair of parent strings selected in Step 9.
» *The mutation operator.* Mutation occurs with some specified probability of mutation for each bit in the strings, which have undergone crossover.
» *Production of successive generations.* The use of the three operators described above produces a new generation of pipe network designs using Steps 2 to 11. The GA repeats the process to generate successive generations. The last cost strings (e.g., the best 20) are stored and updated and cheaper cost alternatives are generated.

These steps for the optimisation of water networks considering both steady state and transient conditions are illustrated in the flow chart of the GASTnet program (Figure 5) [20].

After the model has been constructed and calibrated, it is ready to be used in design. To get the most benefit from the model, the designer should examine a broad range of alternatives.

6. NUMERICAL EXAMPLE

The water supply pipe network with the topology from Figure 6 is considered. The system comprises six nodes, six pipes and two reservoirs at nodes 5 and 6, with constant level, equal to 311 m and 305 m respectively. It is supplied with a flow rate of 0.136 m^3/s provided from two reservoirs ($Q_{5\text{-}1}$=0.108 m^3/s, $Q_{6\text{-}3}$=0.028 m^3/s).

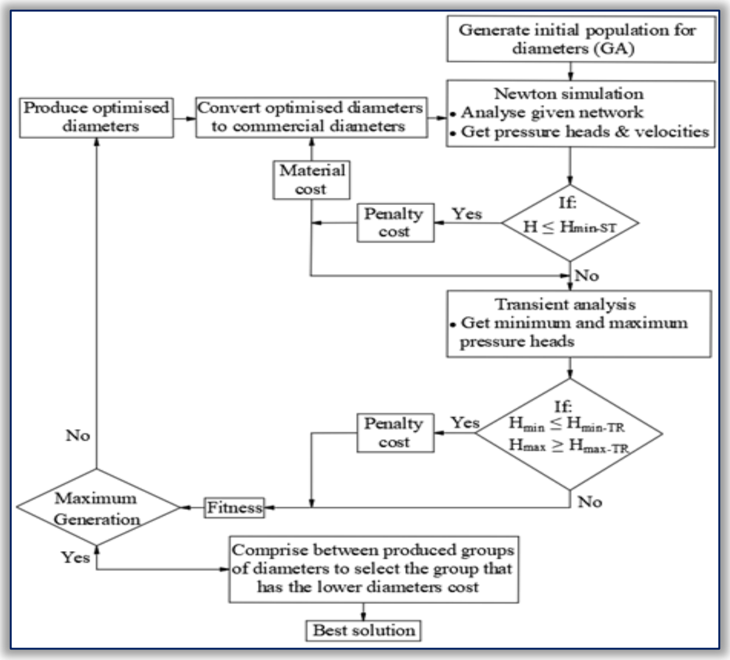

Figure 5. Flow chart of the GASTnet program

The following data are known: pipe length L_{ij}, in m; pipe diameter D_{ij}, in mm; elevation head ZT_j, in m; and the water demands at nodes q_j, in m^3/s. The roughness height of pipes and wave speed are 0.051 mm and 914 m/s, respectively.

The water demand was sudden increased at node 2 from 0.028 m^3/s to 0.057 m^3/s.

For the steady state, the required minimum pressure head at all nodes is 24 m and for the transient conditions, the minimum and maximum pressure heads are 24 m and 55 m, respectively.

Applying the GASTnet program in the transient-optimisation mode, the optimal diameters for the network against the original ones are summarised in Table 1.

The least cost is 267,000.00 units after optimisation against 348,000.00 units, which is equal 0.767 times the original cost.

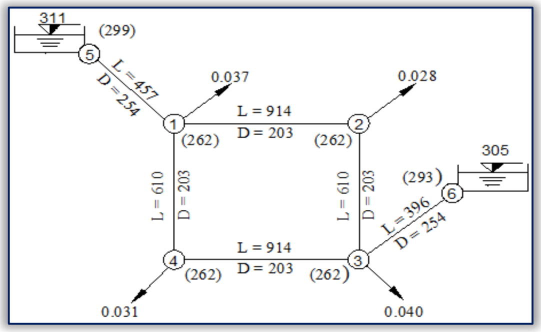

Figure 6. Typical pipe network for sudden water demand change

Table 2 presents the corresponding nodal pressure heads for the steady state. These heads fulfill the minimum pressure constraint of 24 m at all nodes except the reservoirs nodes. The two reservoirs at nodes 5 and 6 have pressure heads of 12 m.

Table 1. Optimal and original diameters for the sudden water demand change

Pipe *i–j*	Original diameter (mm)	Optimal diameter (mm)
5–1	254	305
1–2	203	203
3–2	203	152
6–3	254	152
3–4	203	152
1–4	203	152
Cost (units)	348,000.00	267,000.00
Run time (s)	30	150

Figure 7 illustrates the pressure head variation depending on time at all nodes before and after optimisation. The dashed curves represent the results for transient-simulation using the original network pipes diameters. It is clear that the pressure fluctuations are very significant and destructive as it crosses the pressure limits of 24–55 m.

The results in transient-optimisation mode (continuous curves) reveal that the pressure fluctuations have been contained within the predetermined pressure head limits 24–55 m.

Table 2. Pressure heads at nodes for the steady state using the optimal diameters

Node	Pressure head (m)
1	46.22
2	39.65
3	37.35
4	37.25
5	12.00
6	12.00

As observed from Figure 7, the convergence to steady state caused by sudden demand change is rapid. The choice of the time of the transient flow simulation as 60 seconds was sufficient to obtain nearly steady state condition at the end of this time.

7. CONCLUSIONS

When designing water distribution systems, the engineer needs to consider economic and technical factors such as acquisition of property, construction costs, site topography, and geological conditions of the land where the pipe system will be constructed.

The previous studies were concerned with the optimisation of networks under steady state conditions in spite of the fundamental importance of transients. The optimisation of a transient flow for water distribution systems is investigated relative recently. In this paper, the transient flow is introduced to the water network by the pump power failure, sudden valve closure and sudden demand change.

The technique of the optimal pipe diameter selection is very economical as the network design can be achieved without using anti-water hammer protection devices. This technique is not only crucial to water networks design and performance, but also effective in minimising costs.

The importance of the optimisation technique in pipe networks designs is concluded as follows:

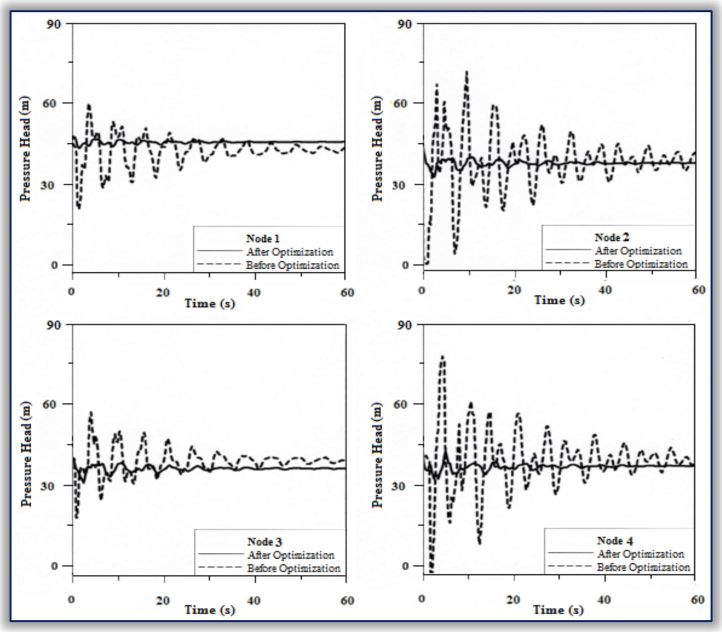

Figure 7. Pressure head versus time for various demand nodes

- Optimise the cost with respect to the operational conditions.
- Minimise the use of the water hammer arrestors.
- Increase the pipe network reliability under water hammer circumstances.
- Decrease the noise resulted from the water hammer phenomenon.

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