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# EQUIVALENT CIRCUIT PARAMETERS OF A SYNCHRONOUS MACHINE FROM MANUFACTURER'S DATA

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**Abstract:** The power plant engineer is in possession of the manufacturer's data of an alternator. The equivalent circuit parameters are to be computed, if one desires to make detailed performance analysis. The equivalent circuit normally used is on the basis of Park-Gorev transformation. The detailed mathematical model can be constructed if the equivalent circuit parameters are to hand. If some parameters be missing in the manufacturer's data suitable approximation has to be made. Also, a programme has to be constructed for equivalency the parameters of a coherent group. This paper is intended to give the procedure for the same. Case-studies have been made on real world machines of a thermal power station.

**Keywords:** d-axis and q-axis, steady state, transient, sub-transient, time constants, equivalent circuit, coherent group, infinite bus

## 1. INTRODUCTION

The most important study, which is to be conducted for a synchronous machine, operating in a grid system is its various stability analysis- transient, dynamic and voltage. A more or less accurate analysis can be made provided the equivalent circuit parameters in terms of d-q axis representation are known. Many researchers worked in this field and advanced bulk of papers right from the first half of 19<sup>th</sup> Century. Some of the recent publications are being mentioned. D.C. Aliprantis et al tried to include saturation in the equivalent circuit model with arbitrary rotor network [1]. The same attempt was taken by Kingsley, Frolick and Anderson [2]. G. R. Slemon also tried to include saturation in presence of saliency and saturation [3]. Later on it was extended by him along with Awad [4]. Relationship between parameter sets of equivalent synchronous machines was examined by Verbeeck [5]. Earlier analog measurement techniques for estimating parameters and time constants from S.C. current was replaced by more accurate digital method by M. Al-Kandari et al [6]

In the earlier regimes, the computing facility was very limited- there was no digital computer available for fast computing and data storage. So, the then engineers, had to make recourse to either grapho-analytical methods (e.g. equal area criterion) or approximate methods (the voltage behind the transient reactance assumed constant for the period of swing) [7-9]. But now-a-days, after the advent of fast-acting digital computer with enormous memory, much more accurate analysis can be made using state space models with equivalent circuit parameters as the base [2, 10, 11]. However, the equivalent circuit parameters are not readily available- they have to be computed from manufacturer's data. The paper presents the techniques for finding them out. Some of the parameters may be missing in the manufacturer's data e.g. the q-axis data or data for sub-transient operation. So, approximate methods to account for them have also been given [2].

## 2. PARK'S TRANSFORMATION AND GENERALIZED MACHINE THEORY

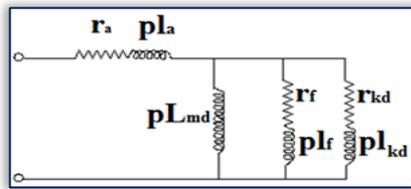
A 3-phase alternator has three space-distributed windings, mutually at 120°, and a rotor with a field winding, also dampers to counteract hunting. The self and mutual inductances have time-varying components due to speed action of the rotor. For this reason, transient problems could not be solved in terms of the space phase model in the earlier days. Now it is possible though due to advent of fast-acting computers, the space phase model is very complex and the solution is time-taking. Hence in general recourse is made to generalized machine theory in terms of Park's transformation (1929). As generation of e.m.f. depends on relative motion of the stator and the rotor, in generalized machine theory the 3-phase armature is assumed to be rotating, the field and the damper windings are assumed to be stationary. A 2-pole structure is assumed in consideration of the symmetry between poles [12].

R.H. Park advanced an ingenious method to convert the rotating phase windings to two pseudo-stationary coils, one along the polar or direct axis and another at right angle to it, the interpolar or quadrature axis. The transformed coils are stationary in space but are having the property of inducing e.m.f. by the speed action of

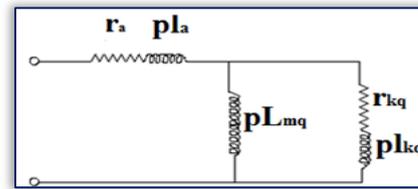
the rotor (somewhat similar to armature of the d.c. machine.). The transformation reduces the time-varying inductances to fixed inductances along the d-q axes, thus drastically reducing the bulk of computation [12,13].

### 3. DIRECT AND QUADRATURE AXIS EQUIVALENT CIRCUITS

After transformation, the direct (D) axis consists of three mutually coupled coils: armature (D), field (F), d-axis damper (KD). They are assumed to have a common mutual inductance (i.e. neglecting partial leakage). The quadrature (Q) axis consists of two mutually coupled coils: armature (Q), q-axis damper with a common mutual inductance. The transformed armature coils D and Q are having pseudo-stationary property. The damper has been approximated as one coil per axis, neglecting the distribution effect [14]. Against these simplifying assumptions, the equivalent circuits are given as in Figure 1a and 1b.



a)



b)

Figure 1a. Equivalent circuit, d-axis

Figure 1b. Equivalent circuit, q-axis

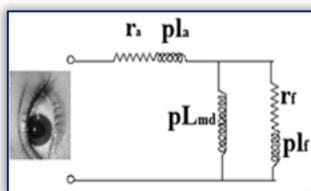
### 4. THE CONCEPT OF TRANSIENT AND SUB-TRANSIENT REACTANCE

Against the conceptual framework of Gabriel Kron, three reactance can be visualized for the d-axis and two for the q-axis. In case of a sudden symmetrical short-circuit, the d-axis component of the armature current will jump to a very high value and will have a fast exponential decay in the first stage due to the presence of the damper winding and then a comparatively slower exponential decay for the second stage due to the highly inductive field winding. The second stage is denoted as transient state. The effective reactance during this stage is called transient reactance and is very important for determining the electromechanical oscillation. The first stage is called sub-transient- the effective reactance during this stage is called sub-transient reactance. The rate of decay during the transient and the sub-transient states are determined by the transient short-circuit and sub-transient short-circuit time-constants. During open-circuit faults, the corresponding open circuit time-constants will come into action. For the quadrature axis, the behavior will be almost same, but in this case the transient part is left off, as there is no field winding in the q-axis. So there will be only a sub-transient reactance. Steady state or synchronous reactance for the two axes will come into play after all the transients die down [7-9].

However, further research in this area has revealed that the presence of solid iron rotor adds a transient part in the q-axis. Suitable modifications are to be made to take care of it. We shall stick to our basic assumptions and will neglect this effect and also other secondary effects arising from partial linkage, saturation etc.

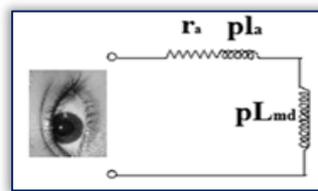
### 5. VISUALIZATION

If we look into the d-axis equivalent circuit from the left in Figure 1a, what we get is the d-axis sub-transient reactance, provided we neglect the resistances. We also get the sub-transient time constant, provided we neglect all resistances except that of d-axis damper. If we look into the q-axis equivalent circuit similarly in Figure 1b, we get the q-axis sub-transient reactance and the q-axis sub-transient time-constant. If we omit the damper element in the d-axis, as shown in Figure 2a, we get the transient reactance; also the transient time constant neglecting all resistances except that of the field winding. If we omit both damper and field and look into the circuits, we get the d and q axes steady state or synchronous reactance. With this idea in view, we proceed to find out the expressions for transient and sub-transient reactance and the time constants [9, 12].



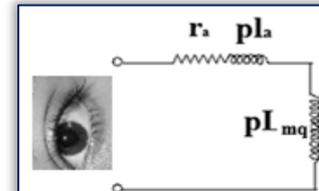
a)

Figure 2a. Transient condition



b)

Figure 2b. Steady state condition, d-axis



c)

Figure 2c. Steady state condition, q-axis

### 6. DEVELOPMENT OF MATHEMATICAL EXPRESSIONS FOR TRANSIENT AND SUB-TRANSIENT CONDITIONS

Under the transient condition, the reactance is given as [2]:

$$X'_d = x_a + X_{md} \parallel x_f, \text{ from which we get: } \frac{1}{x_f} = \frac{1}{X'_d - x_a} - \frac{1}{X_{md}} = \frac{X_d - X'_d}{X_{md}(X'_d - x_a)}$$

Therefore,  $x_f = \frac{X_{md}(X'_d - x_a)}{X_d - X'_d}$  = Field leakage reactance.

Again, the open circuit transient time constant is given as:  $T'_{do} = T'_d \times \frac{X_d}{X'_d} = \frac{X_{md} + x_f}{\omega r_f} \therefore r_f = \frac{X_{md} + x_f}{\omega T'_{do}}$ ;  $\omega = 2\pi f$ .  $r_f$  is the field resistance.

Proceeding similarly, we get the set of equations.

### 7. SET OF EQUATIONS FOR CONVERSION

The following set of equations for conversion (the parameters are given in p.u. and time-constants in seconds).

$$\begin{aligned} X_{md} &= X_d - x_a ; X_{mq} = X_q - x_a ; 1/X_f = 1/(X'_d - x_a) - 1/X_{md} ; X_f = x_f + X_{md} \\ 1/X_{kd} &= 1/(X''_d - x_a) - 1/x_f - 1/X_{md} ; X_{kd} = x_{kd} + X_{md} \quad 1/X_{kq} = 1/(X''_q - x_a) - 1/x_f - 1/X_{mq} ; X_{kq} = x_{kq} + X_{mq} \\ r_f &= X_f / (\omega T'_{do}) ; T'_{do} = T'_d (X_d / X'_d) \quad r_{kd} = (x_{kd} + X_{md} \parallel x_f) / (\omega T'_{do}) = (x_{kd} + X_{md} x_f / X_f) / (\omega T'_{do}) ; \\ T''_{do} &= T''_d (X'_d / X''_d) ; \quad r_{kq} = X_{kq} / (\omega T''_{qo}) ; T''_{qo} = T''_q (X'_q / X''_q) \end{aligned}$$

For the saturated condition, the saturated values of the parameters are to be used. It is assumed that the leakage reactance do not undergo any saturation as the flux-path is not through iron.

### 8. EQUIVALENCING

In many cases, under fault condition, a group of generators tend to oscillate similarly. This group of machines is called a coherent group. The group is treated as a single machine. In Kolaghat TPS, all the generators are of same rating and same make. So the equivalent parameters are same as the p.u. parameters of the individual machines. But in Bandel TPS there are four Westinghouse machines, originally of rating 89.25 MW and one 210 MW machine of BHEL. In such cases, data of the equivalence machine is obtained by the following manipulative process. Let there be  $n$  number of machines of rating  $S_1$  and  $m$  number of machines of rating  $S_2$  in the power house. The first task is to convert the parameters to a common base. In the earlier days for Indian power system, a 100 MVA base was chosen. Now, it may be chosen as 500 MVA, considering the rating of the individual machines. This conversion is made using (except for inertia constants):

$$\text{Impedance: } parameters_{newbase} = parameters_{oldbase} \times \frac{new\ MVA}{old\ MVA}$$

$$\begin{aligned} 1/X_{de} &= n/X_{de1} + m/X_{de2} ; 1/X'_d = n/X'_{de1} + m/X'_{de2} ; 1/X''_d = n/X''_{de1} + m/X''_{de2} \\ 1/X_{qe} &= n/X_{qe1} + m/X_{qe2} ; 1/X''_q = n/X''_{qe1} + m/X''_{qe2} ; H = nH_1 + mH_2 \end{aligned}$$

where,  $X_{de}$  is the equivalent d-axis synchronous reactance,  $X'_{de}$  is the equivalent d-axis transient reactance,  $X''_{de}$  is the equivalent d-axis sub transient reactance,  $X_{qe}$  is the equivalent q-axis synchronous reactance,  $X''_{qe}$  is the equivalent q-axis sub transient reactance and  $H$  is the equivalent inertia constant. The effect of resistance may be neglected except for calculating losses. After equivalency, the coherent group may be idealized as one machine of larger rating on infinite bus as shown in Figure 3.

### 9. CASE STUDY

#### ≡ Equivalent circuit parameters

The manufacturer's data is given below for a 210 MW turbo-generator set: parameters are in p.u. and time-constants in sec.

- D-axis Synchronous Reactance,  $X_d = 2.225$
- Q-axis Synchronous Reactance,  $X_q = 2.11$
- D-axis Transient Reactance,  $X'_d = 0.305$
- D-axis Sub-transient Reactance,  $X''_d = 0.214$
- Negative sequence Reactance,  $X_2 = 0.26$
- Q-axis Sub-transient Reactance,  $X''_q = 0.306$
- Armature Leakage Reactance,  $x_a = 0.156$
- Armature Resistance,  $r_a = 0.0019$
- D-axis transient time constant,  $T'_d = 0.96$
- D-axis sub-transient time constant,  $T''_d = 0.125$
- Q-axis sub-transient time constant,  $T''_q = 0.25$

The equivalent circuit parameters are given below (Symbols have their usual meanings  $X_{md}$ ,  $X_{mq}$  are mutual inductances. Capital 'X' in other places indicates self-inductance.)

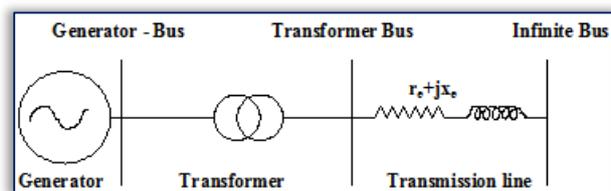


Figure 3. One machine on infinite bus (coherent group)

$$\begin{aligned}
 X_{md} &= 2.0690 & x_f &= 0.1606 & X_f &= 2.2296 & x_{kd} &= 0.0950 \\
 X_{kd} &= 2.1640 & X_{mq} &= 1.9540 & x_{kq} &= 0.1625 & X_{kq} &= 2.1165 \\
 r_f &= 0.00101 & r_{kd} &= 0.00436 & r_{kq} &= 0.00782
 \end{aligned}$$

### ≡ Equivalency of a coherent group

There are four TG-sets of Westinghouse and one of BHEL at Bandel TPS as shown in Figure 4. If there be a fault in some near-by place, (e.g. Jeerut Sub-station, but not in the power house itself), these machines are expected to form a coherent group.

Then, equivalent circuit parameters of the group can be found out to a reasonable approximation, using equations already given. The manufacturer's data for the 210 MW set has already been given. The data for the 89.25 MW, 0.85 lag set is given below:

- D-axis Synchronous Reactance,  $X_d = 1.6$
- Q-axis Synchronous Reactance,  $X_q = 1.53$
- D-axis Transient Reactance,  $X'_d = 0.181$
- D-axis Sub-transient Reactance,  $X''_d = 0.121$
- Q-axis Sub-transient Reactance,  $X''_q = 0.121$
- Armature Leakage Reactance,  $x_a = 0.08$
- External reactance to infinite bus,  $X_e = 0.11$
- d-axis short circuit transient time-constant = 0.87 s
- d-axis short circuit sub-transient time-constant = 0.0335 s
- q-axis short circuit sub-transient time-constant = 0.1 s

The equivalent circuit parameters of the machine are given below:

$$\begin{aligned}
 X_{md} &= 1.52; & x_f &= 0.1082; & X_f &= 1.6282; & x_{kd} &= 0.0690; & X_{kd} &= 1.5890; & X_{mq} &= 1.4500; \\
 x_{kq} &= 0.0422; & X_{kq} &= 1.4922; & r_f &= 0.00067; & r_{kd} &= 0.01025; & r_{kq} &= 0.00376
 \end{aligned}$$

The parameters of the 210 MW set has already been given. Parameters of equivalence machine calculated as per equations are given below:

- Total MVA taken as new base value = 667;
- total reactance to infinite bus= 0.2175
- d-axis synchronous reactance= 1.7858 ;
- d-axis transient reactance = 0.21306
- d-axis subtransient reactance = 0.14421 ;
- q-axis synchronous reactance = 1.7034
- q-axis subtransient reactance = 0.15591 ;
- d-axis S.C. transient time constant= 0.9139 sec.
- d-axis S.C. subtransient time constant = 0.0647 sec.;
- q-axis S.C. subtransient time constant= 0.15811sec.

The parameters may be expressed as per unit on a power system common base of 500 MVA. The equivalent circuit parameters can be found out for the equivalence machine for stability analysis by the same method.

### 10. CONCLUSION

Most of the alternators in our power system are synchronously coupled. If the synchronous coupling is disrupted due to some sudden impact the phenomenon is known as transient instability. On the other hand if the system oscillation gradually grows in amplitude against a small perturbation, eventually leading to de-synchronization, the phenomenon is called dynamic instability. On the other hand, if there be a voltage swing due to reactive power mismatch, there may be voltage instability leading to cascaded failure of the system. Understanding the phenomena under such conditions and predicting the performance (whether the oscillations will damp out and the system will retain stability) can be made accurately using state space representation based on the equivalent circuit parameters of the faulty machine.

The paper shows the techniques to find out such parameters from given manufacturer's data. De-synchronization may sometimes be between groups of machines, each group swinging similarly. Such a group is named as a coherent group. To reduce the no. of equations and the dimension of the resulting model it is expedient to reduce the coherent group to a single machine having equivalence parameters. Such a group, to a reasonable degree of approximation, may be thought of as a single machine on infinite bus. The paper has also shown the usual practice to equivalence a coherent group and find out its parameters. Case-studies have been made on real world machines of our system.

### SYMBOLS

- |   |   |
|---|---|
| $r_a, x_a$ Armature resistance, leakage reactance                 | $T''_d, T''_{do}$ D-axis S.C./ O.C. sub-transient time constant |
| $X_d, X_{ds}$ D-axis unsaturated/ saturated synchronous reactance | $T''_q, T''_{qo}$ Q-axis S.C./ O.C. sub-transient time constant |
|   | $\omega$ Angular speed in r/s                                   |

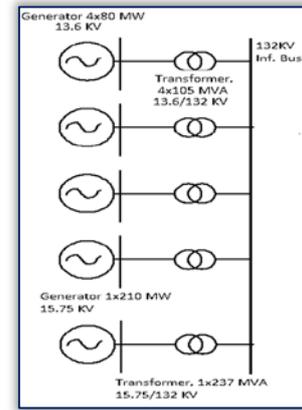


Figure 4. The generators of Bandel TPS

$X_q, X_{qs}$  Q-axis unsaturated/ saturated synchronous reactance

$X'_d, X'_{ds}$  D-axis unsaturated/ saturated transient reactance

$X''_d, X''_q$  D-axis/Q-axis sub-transient reactance

$T'_d, T'_{do}$  D-axis S.C./ O.C. transient time constant.

$r_f, x_f, X_f$  Resistance, Leakage reactance, Self-reactance of field

$r_{kd}, x_{kd}, X_{kd}$  Resistance, Leakage reactance, Self-reactance of d-axis damper

$r_{kq}, x_{kq}, X_{kq}$  Resistance, Leakage reactance, Self-reactance of q-axis damper

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