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A STUDY ON EFFECTS OF WALL SLIP CONDITION AND MAGNETIC FIELD ON SQUEEZING AXISYMMETRIC FLOW OF FIRST-GRADE NANOFLUID THROUGH A POROUS MEDIUM

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Abstract: The influences of wall slip condition and magnetic field on steady two-dimensional axisymmetric squeezing flow of nanofluid through a porous medium are studied using homotopy perturbation method. The approximate analytical method used in this work is verified by comparing the results of the approximate analytical method with the results of numerical method using Runge-Kutta coupled with shooting method. Thereafter, the analytical solution is used to carry out parametric studies of the flow process. The results show that the velocity of the fluid increases with increase in the magnetic parameter under slip condition while the velocity of the fluid decreases with increase in the magnetic field parameter under the no slip condition. By increasing the slip parameter, the velocity of the fluid increases and the fluid velocity decrease as the Reynolds number increases. This study is useful in the flow analysis of fluid such as found in biological and engineering applications.
Keywords: nanofluid, first grade fluid, squeezing flow, magnetic field, slip boundary, homotopy perturbation method

1. INTRODUCTION

The various applications of squeezing flow in biological and engineering processes have continued to generate renewed research interests in the study and understanding of the flow phenomena. The numerous applications of such flow process are evident in biomedical, industrial and engineering applications such as flow inside syringes and nasogastric tubes, moving pistons, chocolate fillers, hydraulic lifts, electric motors, compression, injection, power transmission squeezed film and polymers processing. In such applications, the flows of fluid are performed as a result of the moving apart or the coming together of two parallel plates. Following the pioneer work and the basic formulations of on squeezing flows under lubrication assumption by Stefan [1], there have been increasing research interests and many scientific studies on these types of flow. In a past work over few decades, Reynolds [2] analyzed the squeezing flow between elliptic plates while Archibald [3] investigated the same problem for rectangular plates. The earlier studies on squeezing flows were based on Reynolds equation which insufficiency for some cases has been shown by Jackson [4] and Usha and Sridharan [5]. Therefore, there have been several attempts and renewed research interests by different researchers to proper analyze and understand the squeezing flows [5-15]. In the past efforts to analyze such flow process, Rashidi et al. [16] used homotopy analysis method (HAM) to develop analytical approximate solutions to study the unsteady two dimensional axisymmetric squeezing flow between parallel plates while Duwairi et al. [17] investigated effects of squeezing on heat transfer of a viscous fluid between parallel plates. Qayyum et al. [18] studied the squeezing flow of non-Newtonian second grade fluids and micro-polar models presenting effect on velocity profiles. Hamdan [19] analyzed the effect of squeezing flow on dusty fluids discussing squeeze effect on fluid flow. Mahmood et al. [20] investigated the effects of Prandtl's number and Nusselt number on the squeezed flow and heat transfer over a porous surface for viscous fluids. Hatami and Jing [21] applied differential transformation method to study the natural convection of a non–Newtonian nanofluid between two vertical plates and Newtonian nanofluid between horizontal plates. Mohyud-Din et al. [22] investigated on heat and mass transfer analysis for the flow of a nanofluid between rotating parallel plates while Mohyud-Din and Khan [23] analyzed the nonlinear radiation effects on squeezing flow of a Casson fluid between parallel disks. Qayyum et al. [24] modeled and applied homotopy perturbation method to analyze the unsteady axisymmetric squeezing fluid flow through porous medium channel with slip boundary. Qayyum and Khan [25] presented the behavioral study of unsteady squeezing flow through porous medium using homotopy perturbation method. Mustafa et al. [26] presented their study on the heat and mass transfer in unsteady fluid flow under squeezed flow between two parallel plates using homotopy analysis method. In order to study the influence of magnified on the squeezing flow of non-Newtonian fluid, Siddiqui et al. [27] adopted homotopy perturbation method investigated the magnetic effect of squeezing viscous magneto-hydrodynamics (MHD) fluid flow. Few years later, Domairry and Aziz [28] used homotopy perturbation method (HPM) to study the MHD squeezed flow between two parallel disks with suction or injection. Also, the effect of squeeze on Copper-water and Copper-Kerosene nanofluid between two parallel plates subjected to magnetic field was studied by Acharya et al [29] using the differential transformation method (DTM). Ahmed et al. [30] analyzed magneto hydrodynamic (MHD) squeezing flow of a Casson fluid between parallel disks. A year later, Ahmed et al. [31] investigated on MHD flow of an incompressible fluid through porous medium between dilating and squeezing permeable walls. The same year, Khan et al. [32] studied unsteady two-dimensional and axisymmetric squeezing flow between parallel plates. The same authors Khan et al. [33] MHD squeezing flow between two infinite plates while Hayat et al. [34] had earlier investigated the effect of squeezing flow of second grade fluid between two parallel disks. Khan et al. [35] analyzed unsteady squeezing flow of Casson fluid with magnetohydrodynamic effect and passing through porous medium while Ullah et al. [36] used homotopy perturbation method to present analytical solution of squeezing flow in porous medium with MHD effect. Thin Newtonian liquid films squeezing between two plates were studied by Grimm [37]. Squeezing flow under the influence of magnetic field is widely applied to bearing with liquid-metal lubrication [38–41]. Islam et al [42] studied squeezing fluid flow between the two infinite parallel plates in a porous medium channel. In case of many polymeric liquids when the weight of molecule is high, then they show slip at the boundary. The no-slip boundary condition is not applicable in this case. In many cases such as thin film problems, rarefied fluid problems, fluids containing concentrated suspensions, and flow on multiple interfaces, the no-slip boundary condition fails to work. Navier [43], for the first time, proposed the general boundary condition which demonstrates the fluid slip at the surface. The difference of fluid velocity and velocity of the boundary is proportional to the shear stress at that boundary. The proportionality constant is named the slip parameter having length as its dimension. The slip condition is of great importance especially when fluids with elastic character are under consideration [44]. Newtonian fluid was considered by Ebaid [45] to study the effects of magnetic field and wall slip conditions on the peristaltic transport in an asymmetric channel. It has great importance in medical sciences, particularly in polishing artificial heart valves and internal cavities in many manufactured parts achieved by embedding such fluids with abrasives [46]. The influence of slip on the peristaltic motion of third-order fluid in asymmetric channel is studied by Hayat et al. [47]. The effects of slip condition on the rotating flow of a third grade fluid in a nonporous medium are investigated by Hayat and Abelman [48]. Their work was extended to a porous medium and obtaining the numerical solutions for the steady magneto-hydrodynamics flow of a third grade fluid in a rotating frame is presented by Abelman et al. [49]. Ullah et al. [50] presented approximation of first grade MHD squeezing fluid flow with slip boundary condition using DTM and OHAM. The past efforts in analyzing the squeezing flow problems have been largely based on the applications approximate analytical methods such as HAM, DTM, ADM, VIM, OHAM etc. In the paper, magnetohydrodynamic squeezing flow of first-grade fluid with slip boundary condition between two infinite plates is analyzed using homotopy perturbation method. The study is carried out to further study and analyze the applications and limitations of the HPM to the fluid flow problem. Also, effects of pertinent flow, magnetic field and slip parameters are studied. By comparing the results of approximate analytical methods in this work with the numerical method using Runge-Kutta coupled with shooting method, the verification and the accuracy of approximate analytical solution is established.

2. PROBLEM FORMULATION

Consider a squeezing flow of nanofluid squeezed between two parallel plates which are at distance $2h$ apart and they approach each other with slowly with a constant velocity under in the presence of a magnetic field as shown in Figure 1. Assuming that the fluid is incompressible, the flow is laminar and isothermal, the governing equations of motion for the quasi steady flow of the nanofluid are given as: Assume that the flow is quasi steady, and the Navier-Stokes equations governing such flow when inertial terms, the equations of motion governing the flow are:

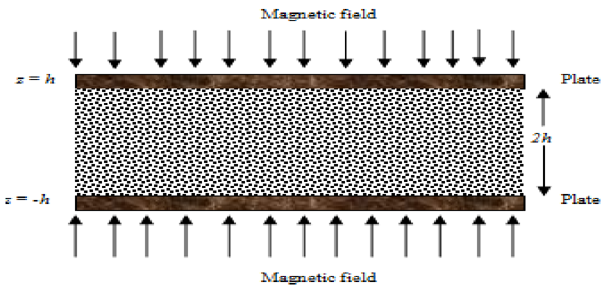

Figure 1. Model of the MHD squeezing flow of nanofluid between two parallel paltes separated by distance 2h

$$\nabla.\tilde{v} = 0 \quad (1)$$

$$\left[\rho_f(1-\phi)+\rho_s\phi\right]\left[\frac{\partial \tilde{v}}{\partial t}+(\nabla.\tilde{v})\tilde{v}\right]=\left[\rho_f(1-\phi)+\rho_s\phi\right]f-\nabla.p+\left\{\frac{\mu_f}{(1-\phi)^{2.5}}\left(\nabla^2-\frac{1}{k}\right)-\sigma B_0^2\right\}\tilde{v} \quad (2)$$

Neglecting the body force, the continuity and Navier-Stokes' equation for the problem is given as

$$\nabla.\tilde{v} = 0 \quad (3)$$

$$-\left[\rho_f(1-\phi)+\rho_s\phi\right](\tilde{v}\times\tilde{w})+\tilde{\nabla}\left(\frac{\left[\rho_f(1-\phi)+\rho_s\phi\right]}{2}|\tilde{v}|^2+p\right)=\frac{\mu_f}{(1-\phi)^{2.5}}\left(\tilde{\nabla}\times\tilde{w}-\frac{1}{k}\tilde{v}\right)-\sigma B_0^2\tilde{v} \quad (4)$$

Introducing the stream function $\psi(r,z)$, vorticity function $\Omega(r,z)$ and a generalized pressure for the cylindrical coordinate system as follows

$$u=\frac{1}{r}\frac{\partial\psi}{\partial z},\quad v=-\frac{1}{r}\frac{\partial\psi}{\partial r},\quad \Omega(r,z)=-\frac{1}{r}\lambda^2\psi,\quad p=\frac{\left[\rho_f(1-\phi)+\rho_s\phi\right]}{2}\left(u^2+v^2\right) \tag{5}$$

Eliminating the pressure term from Eqs. (3) and (4), we have

$$\left[\rho_f(1-\phi)+\rho_s\phi\right]\left[\frac{\partial\left(\psi,\lambda^2\psi/r^2\right)}{\partial(r,z)}\right]=-\frac{1}{r}\frac{\mu_f}{(1-\phi)^{2.5}}\lambda^4\psi+\frac{1}{r}\left(\frac{\mu_f}{k(1-\phi)^{2.5}}+\sigma B_0^2\right)\frac{\partial^2\psi}{\partial z^2} \tag{6}$$

where

$$\lambda^2=\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2} \tag{7}$$

The boundary conditions are given as

$$z=0,\ v=0\ \ and\ \frac{\partial v}{\partial z}=0$$
$$z=H,\ \ v=-V_w\ \ and\ \frac{\partial v}{\partial z}=\beta v \tag{8}$$

Applying a transformation $\psi(r,z)=r^2 f(z)$, the compatibility Eq. (6) reduces to Eq. (9) as

$$f^{iv}(z)-\left(\frac{1}{k}+\frac{\sigma B_0^2(1-\phi)^{2.5}}{\mu_f}\right)f''(z)+\frac{2\left[\rho_f(1-\phi)+\rho_s\phi\right](1-\phi)^{2.5}}{\mu_f}f(z)f'''(z)=0 \tag{9}$$

And the slip boundary conditions as

$$f(0)=0,\ f''(0)=0,$$
$$f(h)=\frac{v}{2},\ f'(h)=\gamma f''(h) \tag{10}$$

Using the following dimensionless parameters in Eq. (11)

$$F^*=\frac{f}{v/2},\ z^*=\frac{z}{h},\ R=\frac{\rho_f Hv}{\mu_f},\ G=h\sqrt{\left(\frac{1}{k}+\frac{\sigma B_0^2}{\mu_{nf}}\right)}=\sqrt{\left(Da+m^2\right)}. \tag{11}$$

The dimensionless form of Eq. (9) is given as

$$F^{(iv)}(z)+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}F(z)F'''(z)-G^2F''(z)=0 \tag{12}$$

And the dimensionless boundary conditions in Eq. (10) as

$$F(0)=0,\quad F''(0)=0$$
$$F(1)=1,\quad F'(1)=\gamma F''(1) \tag{13}$$

where the asterisk, * has been omitted in Eqs. (12) and Eq. (13) for the sake of conveniences.

3. METHOD OF SOLUTION BY HOMOTOPY PERTURBATION METHOD

It is very difficult to develop a closed-form solution for the above non-linear equation (19). Therefore, recourse has to be made to either approximation analytical method, semi-numerical method or numerical method of solution. In this work, homotopy perturbation method is used to solve the equation.

— The basic idea of homotopy perturbation method

In order to establish the basic idea behind homotopy perturbation method, consider a system of nonlinear differential equations given as

$$A(U)-f(r)=0,\quad r\in\Omega \tag{17}$$

with the boundary conditions

$$B\left(u,\frac{\partial u}{\partial\eta}\right)=0,\qquad r\in\Gamma \tag{18}$$

where *A* is a general differential operator, *B* is a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω

The operator *A* can be divided into two parts, which are *L* and *N*, where *L* is a linear operator, *N* is a non-linear operator. Eq.(24) can be therefore rewritten as follows

$$L(u)+N(u)-f(r)=0 \tag{19}$$

By the homotopy technique, a homotopy $U(r,p):\Omega\times[0,1]\rightarrow R$ can be constructed, which satisfies

$$H(U,p)=(1-p)\left[L(U)-L(U_o)\right]+p\left[A(U)-f(r)\right]=0,\quad p\in[0,1] \tag{20}$$

or

$$H(U,p)=L(U)-L(U_o)+pL(U_o)+p\left[N(U)-f(r)\right]=0 \tag{21}$$

In the above Eqs. (20) and (21), $p\in[0,1]$ is an embedding parameter, u_o is an initial approximation of equation of Eq.(17), which satisfies the boundary conditions.
Also, from Eqs. (20) and (21), we will have

$$H(U,0)=L(U)-L(U_o)=0 \tag{22}$$

$$H(U,0)=A(U)-f(r)=0 \tag{23}$$

The changing process of *p* from zero to unity is just that of $U(r,p)$ from $u_o(r)$ to $u(r)$. This is referred to homotopy in topology. Using the embedding parameter *p* as a small parameter, the solution of Eqs. (20) and (21) can be assumed to be written as a power series in p as given in Eq. (24)

$$U=U_o+pU_1+p^2U_2+\ldots \tag{24}$$

It should be pointed out that of all the values of *p* between 0 and 1, *p=1* produces the best result. Therefore, setting $p=1$, results in the approximation solution of Eq.(17)

$$u=\lim_{p\to 1}U=U_o+U_1+U_2+\ldots \tag{25}$$

The basic idea expressed above is a combination of homotopy and perturbation method. Hence, the method is called homotopy perturbation method (HPM), which has eliminated the limitations of the traditional perturbation methods. On the other hand, this technique can have full advantages of the traditional perturbation techniques. The series Eq.(25) is convergent for most cases.

3.2 Application of the homotopy perturbation method to the present problem

According to homotopy perturbation method (HPM), one can construct an homotopy for Eq. (16) as

$$H(z,p)=(1-p)\tilde{F}^{(iv)}+p\left[\tilde{F}^{(iv)}+RR\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}\tilde{F}'''-G^2\tilde{F}''\right] \tag{33}$$

Using the embedding parameter *p* as a small parameter, the solution of Eqs. (16) can be assumed to be written as a power series in p as given in Eq. (33)

$$\tilde{F}=\tilde{F}_o+p\tilde{F}_1+p^2\tilde{F}_2+p^3\tilde{F}_3+\ldots \tag{34}$$

On substituting Eqs. (34) and into Eq.(33) and expanding the equation and collecting all terms with the same order of p together, the resulting equation appears in form of polynomial in p. On equating each coefficient of the resulting polynomial in *p* to zero, we arrived at a set of differential equations and the corresponding boundary conditions as

$$p^0:\quad \tilde{F}_0^{(iv)}=0,\qquad \tilde{F}_0(0)=0,\ \tilde{F}_0''(0)=0,\quad \tilde{F}_0(1)=1,\ \tilde{F}_0'(1)=\gamma\tilde{F}_0''(1) \tag{35}$$

$$p^1:\quad \tilde{F}_1^{(iv)}-G^2\tilde{F}_0''+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_0\tilde{F}_0'''=0, \tag{36}$$

$$\tilde{F}_1(0)=0,\ \tilde{F}_1''(0)=0,\quad \tilde{F}_1(1)=0,\ \tilde{F}_1'(1)=\gamma\tilde{F}_1''(1)$$

$$p^2:\quad \tilde{F}_2^{(iv)}-G^2\tilde{F}_1''+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_1\tilde{F}_0'''+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_0\tilde{F}_1'''=0, \tag{37}$$

$$\tilde{F}_2(0)=0,\ \tilde{F}_2''(0)=0,\quad \tilde{F}_2(1)=0,\ \tilde{F}_2'(1)=\gamma\tilde{F}_2''(1)$$

$$p^3:\tilde{F}_3^{(iv)}-G^2\tilde{F}_2''+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_2\tilde{F}_0'''+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_1\tilde{F}_1'''$$

$$+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_0\tilde{F}_2'''=0, \tag{38}$$

$$\tilde{F}_3(0)=0,\ \tilde{F}_3''(0)=0,\tilde{F}_3(1)=0,\ \tilde{F}_3'(1)=\gamma\tilde{F}_3''(1)$$

$$p^4:\quad \tilde{F}_4^{(iv)}-G^2\tilde{F}_3''+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_3\tilde{F}_0'''+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_3\tilde{F}_0'''$$

$$+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_2\tilde{F}_1'''+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_1\tilde{F}_2''' \tag{39}$$

$$+R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_0\tilde{F}_3'''=0$$

$$\tilde{F}_4(0)=0,\quad \tilde{F}_4''(0)=0, \tilde{F}_4(1)=0,\quad \tilde{F}_4'(1)=\gamma\tilde{F}_4''(1)$$

$$\begin{aligned} p^5: \quad & \tilde{F}_5^{(iv)} - G^2\tilde{F}_4'' + R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_3\tilde{F}_0''' + R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_4\tilde{F}_0''' \\ & + R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_3\tilde{F}_1''' + R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_2\tilde{F}_2''' \\ & + R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_1\tilde{F}_3''' + R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\tilde{F}_0\tilde{F}_4''' = 0 \end{aligned} \tag{40}$$

$$\tilde{F}_5(0)=0,\quad \tilde{F}_5''(0)=0, \tilde{F}_5(1)=0,\quad \tilde{F}_5'(1)=\gamma\tilde{F}_5''(1)$$

On solving the above Eqs. (35-40), we arrived at

$$\tilde{F}_0(z)=\frac{3(2\gamma-1)z+z^3}{2(3\gamma-1)} \tag{41}$$

$$\begin{aligned} \tilde{F}_1(z)= & \left\{\frac{3G^2}{3\gamma-1}+\frac{9R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}(2\gamma-1)}{2(3\gamma-1)^2}\right\}z^5+\frac{3R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}}{2(3\gamma-1)^2}z^7 \\ & -\frac{1}{3(2\gamma+1)}\left\{\begin{aligned} &\gamma\left(\frac{60G^2}{3\gamma-1}+\frac{90R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}(2\gamma-1)}{(3\gamma-1)^2}+\frac{63R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}}{(3\gamma-1)^2}\right) \\ &-\left(\frac{12G^2}{3\gamma-1}+\frac{36R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}(2\gamma-1)}{(3\gamma-1)^2}+\frac{9R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}}{(3\gamma-1)^2}\right)\end{aligned}\right\}z^3 \\ & +\frac{1}{3(2\gamma+1)}\left\{\begin{aligned} &\gamma\left(\frac{60G^2}{3\gamma-1}+\frac{90R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}(2\gamma-1)}{(3\gamma-1)^2}+\frac{63R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}}{(3\gamma-1)^2}\right) \\ &-\left(\frac{12G^2}{3\gamma-1}+\frac{36R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}(2\gamma-1)}{(3\gamma-1)^2}+\frac{9R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}}{2(3\gamma-1)^2}\right)\end{aligned}\right\}z \\ & -\left\{\frac{3G^2}{3\gamma-1}+\frac{9R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}(2\gamma-1)}{(3\gamma-1)^2}+\frac{3R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}}{2(3\gamma-1)^2}\right\}z \end{aligned} \tag{42}$$

In the same manner, the expressions for $\tilde{F}_2(z),\tilde{F}_3(z),\tilde{F}_4(z),\tilde{F}_5(z),\tilde{F}_6(z)...$ were obtained. However, they are too large expressions to be included in this paper.

Setting $p=1$, results in the approximation solution of Eq.(24)

$$F(z)=\lim_{p\to 1}\tilde{F}(z)=\tilde{F}_o(z)+\tilde{F}_1(z)+\tilde{F}_2(z)+\tilde{F}_3(z)+\tilde{F}_4(z)+... \tag{43}$$

4. RESULTS AND DISCUSSION

The above analysis shows the development of approximate analytical methods of differential transformation and homotopy perturbation methods for the analysis of a steady two-dimensional axisymmetric flow of an incompressible viscous fluid under the influence of a uniform transverse magnetic field with slip boundary condition. Using HPM, a series solution (10 terms) is obtained as it provides excellent approximations to the solution of the non-linear equation with good accuracy. Although, the other approximate analytical methods such as might seem somehow easier and straight-forward as compared to HPM, the search for included unknown parameter that will satisfy second the boundary condition lead to additional computational cost in the generation of the solution to the problem using the other approximate analytical methods. Moreover, they have their own operational restrictions that severely narrow their functioning domain as they are limited to small domain. Using DTM, HAM, ADM, VIM for large or infinite domain is done with either the application of before-treatment techniques such as domain transformation techniques, domain truncation techniques and conversion of the boundary value problems to initial value problems or the use of after-treatment techniques such as Pade-approximants, basis functions, cosine after-treatment technique, sine after-treatment technique and domain decomposition technique. This is because they were initially established for initial value problems. Amending the methods to boundary value problems especially for large or infinite domains boundary value problems leads to search for unknown parameter(s) that will satisfy the end boundary condition (s). This drawback in the other approximation analytical methods is not experienced in HPM as such tasks of before- and after-treatment techniques are not necessarily required in HPM as it easily applied to the boundary value problems without any included unknown parameter in the solution as found in DTM, HAM, ADM, VIM.

Table 1: Comparison of results

The results of VIM and Numerical method (NM) for F(z)			
z	NM	HPM	|NM-VIM|
0.00	0.000000	0.000000	0.000000
0.10	0.075739	0.075739	0.000000
0.20	0.152935	0.152935	0.000000
0.30	0.233046	0.233045	0.000001
0.40	0.317540	0.317540	0.000000
0.50	0.407893	0.407892	0.000001
0.60	0.505591	0.505592	0.000001
0.70	0.612134	0.612314	0.000000
0.80	0.729034	0.729035	0.000001
0.90	0.857813	0.857813	0.000000
1.00	1.000000	1.000000	0.000000

In order to get an insight into the problem, the effects of pertinent flow, magnetic field and slip parameters on the velocity profile of the fluid are investigated. Figure 2 shows the effects of magnetic field parameter, Hartmann number m on the velocity of the fluid under the influence of slip condition, while Figure 3 depicts the influence of the magnetic field parameter on the velocity of the fluid under no-slip condition. It could be inferred from the figures that the velocity of the fluid increases with increase in the magnetic parameter under slip condition while an opposite trend was recorded during no-slip condition as the velocity of the fluid decreases with increase in the magnetic field parameter under the no slip condition. The magnetic field plays the role of a resistance contributed to by the magnetic pressure field component of Lorentz force. The observed decrease in the velocity of the fluid as magnetic field increases under no-slip condition is due to fact that the applied transverse magnetic field produces a damping or retarding force in the form of Lorentz force. As the value of magnetic parameter M increases, the retarding body force enhances and consequently the velocity reduces. Physical significance of this behavior is, the Lorentz force is a frictional resistive force which opposes the fluid motion and consequently, reduces the velocity of fluid flow. Under this scenario, the boundary layer thickness becomes thicker for stronger magnetic field.

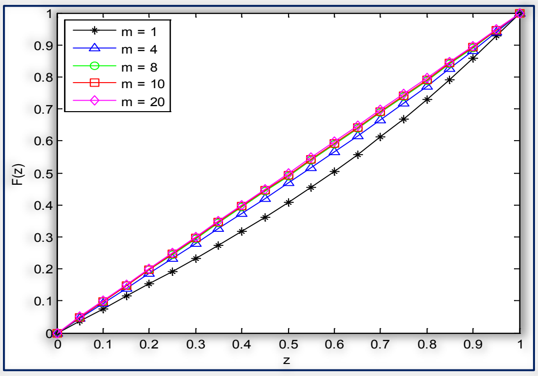

Figure 2: Effects of magnetic parameter on the flow behavior of the fluid under the influence of slip condition, γ=0.5

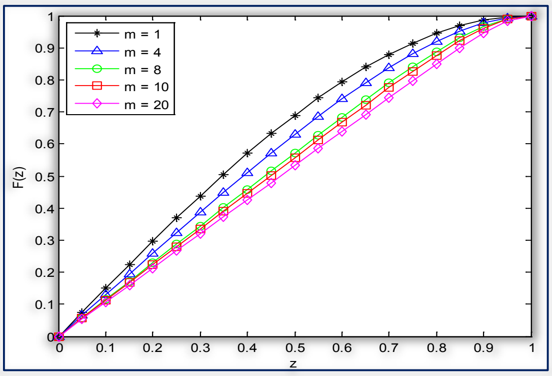

Figure 3: Effects of magnetic field parameter on the flow behavior of the fluid for no-slip condition

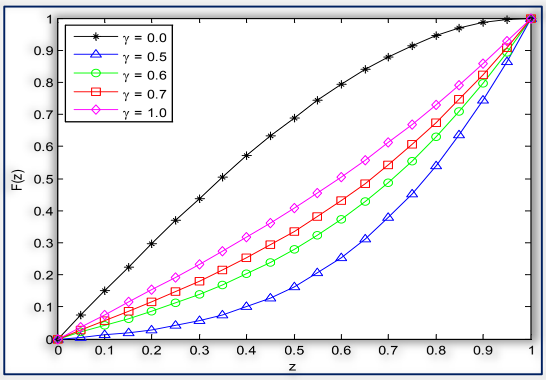

Figure 4: Effects of slip parameter on the flow behavior of the fluid

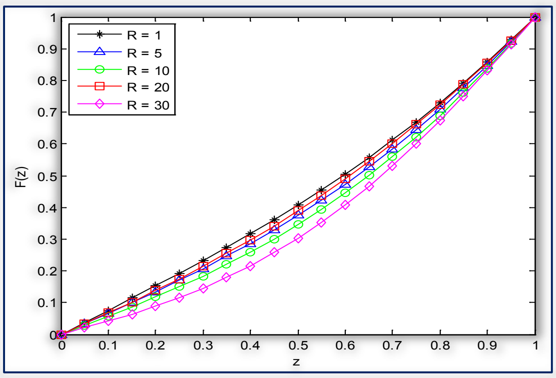

Figure 5: Effects of Reynolds number on the flow behavior of the fluid under the influence of slip condition

Figure 4 shows the influence of the slip parameter γ on the fluid velocity. By increasing γ, it is observed that the velocity of the fluid increases. The impact of slip conditions significantly enhances the velocity profile in the presence and absence of Hartmann number.

Figure 5 presents the effects of Reynolds number on the velocity of the fluid. It is observed from the figure that by increasing the value R, the velocity of the fluid decreases. Also, the flow increases significantly towards the center of the pipe as depicted in the figure. The observed behavior of fluid velocity for different Reynolds number is because the flow toward the center becomes greater to make up for the space and consequently, the fluid velocity also becomes greater near the center.

5. CONCLUSIONS

In this work, homotopy perturbation method has been used to analyze steady two-dimensional axisymmetric flow of an incompressible viscous fluid under the influence of a uniform transverse magnetic field with slip boundary condition. Effects of pertinent flow, magnetic field and slip parameters have been investigated. It was established from the results that, the velocity of the fluid increases with increase in the magnetic parameter under slip condition while the velocity of the fluid decreases with increase in the magnetic field parameter under the no slip condition. By increasing the slip parameter, the velocity of the fluid increases and the fluid velocity decrease as the Reynolds number increases. The approximate analytical solution has been verified by comparing the results of the approximate analytical methods with the numerical method using Runge-Kutta coupled with shooting method.

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ISSN 1584 - 2665 (printed version); ISSN 2601 - 2332 (online); ISSN-L 1584 - 2665