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## OPTIMUM POWER FLOW ANALYSIS BY NEWTON RAPHSON METHOD, A CASE STUDY

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**Abstract:** Optimal power flow (OPF) is a critical control task for reliable and efficient operation of power grids. OPF studies to minimize both the power distribution losses and the cost of power drawn from the substation, without affecting on the voltage regulation. This paper discusses the Newton Raphson method that's can uses with OPF control for the stability of the power systems. This method has a faster solution for load flow analysis with the optimized techno-economical and saving the stable system. The Newton Raphson method is requiring an initial condition and work well for heavily load system when compared to another method. The expected results for load flow are voltage magnitude, phase angle, real and reactive power. This paper simulates the Newton Raphson method for an optimal load flow analysis with IEEE-5 buses.

**Keywords:** OPF, Newton Raphson, power system and load flow

### 1. INTRODUCTION

At the end of 19Th century the electric power started by low generation voltage level for closed areas. With the increasing on the demand power, the electrical grid extended and classified to generation, transmission and distribution. This extension required to increase the transmission voltage, that's reached now to 1200kV. So, the electric networks became more complex, that's may cause many problems in power flow control [1-2]. Regarding this increasing, it's important to apply optimum plans for the power system to reach to minimum cost and without affecting the voltage in the system. The advanced development of power grid system for future will give immediate impacts of anew connection such as power flow direction, protection, voltage profile, power quality and stability [2-4]. The purpose of power flow studies is to plan ahead and account for various hypothetical situations. For example, if a transmission line is being taken off- line for maintenance, can the remaining lines in the system handle the required loads without exceeding their rated values. Smart Grid considered as future electrical power generation, uses calculation tools methods on flow of electricity and information to create a widely distributed automated energy delivery network. This concept is being widely accepted in power system today and now it presents some big challenges in integrating generation with additional of a communication network in more efficiently [4]. Optimal reactive power dispatch problem as a sub-problem of the OPF is a very important optimization problem in power systems as proper management of reactive power injection into the system can minimize real power loss and voltage profile deviations and improve voltage stability [2-6]. The algorithm for the power flow calculation based on the Newton's method in optimization allows to find a solution for the situation when initial data are outside the existence domain and to pull the operation point onto the feasibility boundary by an optimal path. Also, it is possible to estimate a static stability margin by utilizing Newton's method in optimization.

### 2. ACTIVE AND REACTIVE POWER CALCULATIONS

The formulation of the active and reactive power entering a bus, it's need to define the following quantities [5-8]. By assuming the voltage at the ith bus be denoted by

$$V_i = |V_i| \angle \delta_i = |V_i| (\cos \delta_i + j \sin \delta_i) \quad (1)$$

Also let us define the self-admittance at bus-i as

$$Y_{ii} = |Y_{ii}| \angle \theta_{ii} = |Y_{ii}| (\cos \theta_{ii} + j \sin \theta_{ii}) = G_{ii} + jB_{ii} \quad (2)$$

Similarly, the mutual admittance between the buses i and j can be written as

$$Y_{ij} = |Y_{ij}| \angle \theta_{ij} = |Y_{ij}| (\cos \theta_{ij} + j \sin \theta_{ij}) = G_{ij} + jB_{ij} \quad (3)$$

Also, assuming the power system contains a total number of n buses. So, the current injected at bus-i is given as

$$I_i = Y_{i1} V_1 + Y_{i2} V_2 + \dots + Y_{in} V_n = \sum_{k=1}^n Y_{ik} V_k \quad (4)$$

Also, assume the current entering a bus to be positive and that leaving the bus to be negative. As a consequence, the power and reactive power entering a bus will also be assumed to be positive. The complex power at bus-i is then given by

$$P_i - jQ_i = V_i^* I_i = V_i^* \sum_{k=1}^n Y_{ik} V_k = |V_i| \left( \cos \delta_i - j \sin \delta_i \right) \sum_{k=1}^n |Y_{ik} V_k| \left( \cos \theta_{ik} + j \sin \theta_{ik} \right) \left( \cos \delta_k + j \sin \delta_k \right) \quad (5)$$

$$= \sum_{k=1}^n |Y_{ik} V_i V_k| \left( \cos \delta_i - j \sin \delta_i \right) \left( \cos \theta_{ik} + j \sin \theta_{ik} \right) \left( \cos \delta_k + j \sin \delta_k \right)$$

Note that

$$\begin{aligned} \left( \cos \delta_i - j \sin \delta_i \right) \left( \cos \theta_{ik} + j \sin \theta_{ik} \right) \left( \cos \delta_k + j \sin \delta_k \right) &= \left( \cos \delta_i - j \sin \delta_i \right) \left[ \cos(\theta_{ik} + \delta_k) + j \sin(\theta_{ik} + \delta_k) \right] \\ &= \cos(\theta_{ik} + \delta_k - \delta_i) + j \sin(\theta_{ik} + \delta_k - \delta_i) \end{aligned}$$

Therefore, substituting in (5) we get the real and reactive power as

$$P_i = \sum_{k=1}^n |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \quad (6)$$

$$Q_i = - \sum_{k=1}^n |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \quad (7)$$

### 3. DATA FOR LOAD FLOW

The active and reactive power generated at bus-i be denoted by  $P_{Gi}$  and  $Q_{Gi}$  respectively, also, the real and reactive power consumed at the  $i$ th bus by  $P_{Li}$  and  $Q_{Li}$  respectively [4-9]. Then the net real power injected in bus-i is

$$P_{i,inj} = P_{Gi} - P_{Li} \quad (8)$$

By assume the injected power calculated by the load flow program be  $P_{i,calc}$ . Then the mismatch between the actual injected and calculated values is given by

$$\Delta P_i = P_{i,inj} - P_{i,calc} = P_{Gi} - P_{Li} - P_{i,calc} \quad (9)$$

In a similar way the mismatch between the reactive power injected and calculated values is given by

$$\Delta Q_i = Q_{i,inj} - Q_{i,calc} = Q_{Gi} - Q_{Li} - Q_{i,calc} \quad (10)$$

The purpose of the load flow is to minimize the above two mismatches. It is to be noted that equation (6) and equation (7) are used for the calculation of real and reactive power in equation (9) and equation (10). However, since the magnitudes of all the voltages and their angles are not known a priori, an iterative procedure must be used to estimate the bus voltages and their angles in order to calculate the mismatches. It is expected that mismatches  $\Delta P_i$  and  $\Delta Q_i$  reduce with each iteration and the load flow is said to have converged when the mismatches of all the buses become less than a very small number. [8-11]

For the load flow studies, by consider the system of figure (1), which has 2 generators and 3 load buses. We define bus-1 as the slack bus while taking bus-5 as the P-V bus. Buses 2, 3 and 4 are P-Q buses. The line impedances and the line charging admittances are given in table (1). Based on this data the  $Y_{bus}$  matrix is given in table (2). This matrix is to be noted here that the sources and their internal impedances are not considered while forming the  $Y_{bus}$  matrix for load flow studies which deal only with the bus voltages.

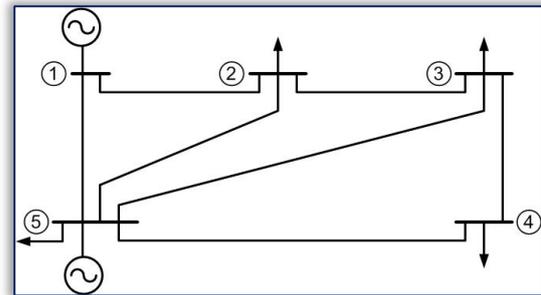


Figure 1. The simple power system used for load flow studies

Table (1). Line impedance and line charging data of the system of figure (1)

Line (bus to bus)	Impedance	Line charging (Y/2)
1-2	0.02 + j0.10	j0.030
1-5	0.05 + j0.25	j0.020
2-3	0.04 + j0.20	j0.025
2-5	0.05 + j0.25	j0.020
3-4	0.05 + j0.25	j0.020
3-5	0.08 + j0.40	j0.010
4-5	0.10 + j0.50	j0.075

Table (2)  $Y_{bus}$  matrix of the system of fig. 1

	1	2	3	4	5
1	2.6923 - j13.4115	-1.9231 + j9.6154	0	0	-0.7692 + j3.8462
2	-1.9231 + j9.6154	3.6538 - j18.1942	-0.9615 + j4.8077	0	-0.7692 + j3.8462
3	0	-0.9615 + j4.8077	2.2115 - j11.0027	-0.7692 + j3.8462	-0.4808 + j2.4038
4	0	0	-0.7692 + j3.8462	1.1538 - j5.6742	-0.3846 + j1.9231
5	-0.7692 + j3.8462	-0.7692 + j3.8462	-0.4808 + j2.4038	-0.3846 + j1.9231	2.4038 - j11.8942

The bus voltage magnitudes, their angles, the power generated and consumed at each bus are given in table (3). In this table, some of the voltages and their angles are given in boldface letters. This indicates that these are initial data used for starting the load flow program. The power and reactive power generated at the slack bus

and the reactive power generated at the P-V bus are unknown. Therefore, each of these quantities are indicated by a dash (-). Since we do not need these quantities for our load flow calculations, their initial estimates are not required. Also note from figure (1) that the slack bus does not contain any load while the P-V bus 5 has a local load and this is indicated in the load column.

Table (3) Bus voltages, power generated and load – initial data.

Bus no.	Bus voltage		Power generated		Load	
	Magnitude (pu)	Angle (deg)	P (MW)	Q (MVA <sub>r</sub> )	P (MW)	P (MVA <sub>r</sub> )
1	1.05	0	-	-	0	0
2	1	0	0	0	96	62
3	1	0	0	0	35	14
4	1	0	0	0	16	8
5	1.02	0	48	-	24	11

#### 4. OVERVIEW ABOUT NEWTON-RAPHSON METHOD

This part discusses the solution of a set of nonlinear equations through Newton-Raphson method. by consideration that the setting of n nonlinear equations of a total number of n variables  $x_1, x_2, \dots, x_n$ . Let these equations be given by

$$\begin{aligned} f_1(x_1, \dots, x_n) &= \eta_1 \\ f_2(x_1, \dots, x_n) &= \eta_2 \\ &\vdots \\ f_n(x_1, \dots, x_n) &= \eta_n \end{aligned} \tag{11}$$

where  $f_1, \dots, f_n$  are functions of the variables  $x_1, x_2, \dots, x_n$ . By define another set of functions  $g_1, \dots, g_n$ , as given below

$$\begin{aligned} g_1(x_1, \dots, x_n) &= f_1(x_1, \dots, x_n) - \eta_1 = 0 \\ g_2(x_1, \dots, x_n) &= f_2(x_1, \dots, x_n) - \eta_2 = 0 \\ &\vdots \\ g_n(x_1, \dots, x_n) &= f_n(x_1, \dots, x_n) - \eta_n = 0 \end{aligned} \tag{12}$$

And by assume that the initial estimates of the n variables are  $x_1(0), x_2(0), \dots, x_n(0)$ . By add corrections  $\Delta x_1(0), \Delta x_2(0), \dots, \Delta x_n(0)$  to these variables such that result is the correct solution of these variables defined by

$$\begin{aligned} x_1^* &= x_1^{(0)} + \Delta x_1^{(0)} \\ x_2^* &= x_2^{(0)} + \Delta x_2^{(0)} \\ &\vdots \\ x_n^* &= x_n^{(0)} + \Delta x_n^{(0)} \end{aligned} \tag{13}$$

The functions in (12) then can be written in terms of the variables given in (13) as

$$g_k(x_1^*, \dots, x_n^*) = g_k(x_1^{(0)} + \Delta x_1^{(0)}, \dots, x_n^{(0)} + \Delta x_n^{(0)}), \quad k = 1, \dots, n \tag{14}$$

We can then expand the above equation in Taylor's series around the nominal values of  $x_1(0), x_2(0), \dots, x_n(0)$ . Neglecting the second and higher order terms of the series, the expansion of  $g_k, k = 1, \dots, n$  is given as

$$g_k(x_1^*, \dots, x_n^*) = g_k(x_1^{(0)}, \dots, x_n^{(0)}) + \Delta x_1^{(0)} \left. \frac{\partial g_k}{\partial x_1} \right|^{(0)} + \Delta x_2^{(0)} \left. \frac{\partial g_k}{\partial x_2} \right|^{(0)} + \dots + \Delta x_n^{(0)} \left. \frac{\partial g_k}{\partial x_n} \right|^{(0)} \tag{15}$$

where  $\left. \frac{\partial g_k}{\partial x_i} \right|^{(0)}$  is the partial derivative of  $g_k$  evaluated at  $x_2(0), \dots, x_n(0)$ .

Equation (15) can be written in vector-matrix form as

$$\begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \dots & \frac{\partial g_1}{\partial x_n} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \dots & \frac{\partial g_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial g_n}{\partial x_1} & \frac{\partial g_n}{\partial x_2} & \dots & \frac{\partial g_n}{\partial x_n} \end{bmatrix}^{(0)} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \\ \vdots \\ \Delta x_n^{(0)} \end{bmatrix} = \begin{bmatrix} 0 - g_1(x_1^{(0)}, \dots, x_n^{(0)}) \\ 0 - g_2(x_1^{(0)}, \dots, x_n^{(0)}) \\ \vdots \\ 0 - g_n(x_1^{(0)}, \dots, x_n^{(0)}) \end{bmatrix} \tag{16}$$

The square matrix of partial derivatives is called the Jacobian matrix J with  $J(0)$  indicating that the matrix is evaluated for the initial values of  $x_2(0), \dots, x_n(0)$ . the solution of (16) can be write as

$$\begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \\ \vdots \\ \Delta x_n^{(0)} \end{bmatrix} = [J^{(0)}]^{-1} \begin{bmatrix} \Delta g_1^{(0)} \\ \Delta g_2^{(0)} \\ \vdots \\ \Delta g_n^{(0)} \end{bmatrix} \tag{17}$$

Since the Taylor's series is truncated by neglecting the 2nd and higher order terms, we cannot expect to find the correct solution at the end of first iteration. We shall then have

$$\begin{aligned} \mathbf{x}_1^{(1)} &= \mathbf{x}_1^{(0)} + \Delta\mathbf{x}_1^{(0)} \\ \mathbf{x}_2^{(1)} &= \mathbf{x}_2^{(0)} + \Delta\mathbf{x}_2^{(0)} \\ &\vdots \\ \mathbf{x}_n^{(1)} &= \mathbf{x}_n^{(0)} + \Delta\mathbf{x}_n^{(0)} \end{aligned} \quad (18)$$

These are then used to find  $J(1)$  and  $\Delta\mathbf{g}_k(1)$ ,  $k = 1, \dots, n$ . We can then find  $\Delta\mathbf{x}_2(1), \dots, \Delta\mathbf{x}_n(1)$  from an equation like (17) and subsequently calculate  $\mathbf{x}_2(1), \dots, \mathbf{x}_n(1)$ . The process continues till  $\Delta\mathbf{g}_k$ ,  $k = 1, \dots, n$  becomes less than a small quantity.

### 5. SIMULATION OF POWER FLOW BY NEWTON RAPHSON METHOD

Let us assume that an  $n$ -bus power system contains a total number of  $n_p$  P-Q buses while the number of P-V (generator) buses be  $n_g$  such that  $n = n_p + n_g + 1$ . Bus-1 is assumed to be the slack bus. We shall further use the mismatch equations of  $\Delta P_i$  and  $\Delta Q_i$  given in (9) and (10) respectively [9-12]. The approach to Newton-Raphson load flow is similar to that of solving a system of nonlinear equations using the Newton-Raphson method: at each iteration, we have to form a Jacobian matrix and solve for the corrections from an equation of the type given in (16). For the load flow problem, this equation is of the form

$$J \begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \Delta\delta_n \\ \frac{\Delta|V_2|}{|V_2|} \\ \vdots \\ \frac{\Delta|V_{1+n_p}|}{|V_{1+n_p}|} \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \vdots \\ \Delta Q_{1+n_p} \end{bmatrix} \quad (19)$$

where the Jacobian matrix is divided into submatrices as

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \quad (20)$$

It can be seen that the size of the Jacobian matrix is  $(n + n_p - 1) \times (n + n_p - 1)$ . For example, for the 5-bus problem of figure (1) this matrix will be of the size  $(7 \times 7)$ . The dimensions of the submatrices are as follows:

$J_{11}$ :  $(n - 1) \times (n - 1)$ ,  $J_{12}$ :  $(n - 1) \times n_p$ ,  $J_{21}$ :  $n_p \times (n - 1)$  and  $J_{22}$ :  $n_p \times n_p$

The submatrices are

$$J_{11} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \dots & \frac{\partial P_2}{\partial \delta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_2} & \dots & \frac{\partial P_n}{\partial \delta_n} \end{bmatrix} \quad (21)$$

$$J_{12} = \begin{bmatrix} |V_2| \frac{\partial P_2}{\partial |V_2|} & \dots & |V_{1+n_p}| \frac{\partial P_2}{\partial |V_{1+n_p}|} \\ \vdots & \ddots & \vdots \\ |V_2| \frac{\partial P_n}{\partial |V_2|} & \dots & |V_{1+n_p}| \frac{\partial P_n}{\partial |V_{1+n_p}|} \end{bmatrix} \quad (22)$$

$$J_{21} = \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_2} & \dots & \frac{\partial Q_2}{\partial \delta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial Q_{1+n_p}}{\partial \delta_2} & \dots & \frac{\partial Q_{1+n_p}}{\partial \delta_n} \end{bmatrix} \quad (23)$$

$$J_{22} = \begin{bmatrix} |V_2| \frac{\partial Q_2}{\partial |V_2|} & \dots & |V_{1+n_p}| \frac{\partial Q_2}{\partial |V_{1+n_p}|} \\ \vdots & \ddots & \vdots \\ |V_2| \frac{\partial Q_{1+n_p}}{\partial |V_2|} & \dots & |V_{1+n_p}| \frac{\partial Q_{1+n_p}}{\partial |V_{1+n_p}|} \end{bmatrix} \quad (24)$$

### — Load Flow Algorithm

The Newton-Raphson procedure is as follows:

Step-1: Choose the initial values of the voltage magnitudes  $|V|(0)$  of all  $np$  load buses and  $n - 1$  angles  $\delta(0)$  of the voltages of all the buses except the slack bus.

Step-2: Use the estimated  $|V|(0)$  and  $\delta(0)$  to calculate a total  $n - 1$  number of injected real power  $P_{calc}(0)$  and equal number of real power mismatch  $\Delta P(0)$ .

Step-3: Use the estimated  $|V|(0)$  and  $\delta(0)$  to calculate a total  $np$  number of injected reactive power  $Q_{calc}(0)$  and equal number of reactive power mismatch  $\Delta Q(0)$ .

Step-3: Use the estimated  $|V|(0)$  and  $\delta(0)$  to formulate the Jacobian matrix  $J(0)$ .

Step-4: Solve (19) for  $\Delta\delta(0)$  and  $\Delta|V|(0) \div |V|(0)$ .

Step-5: Obtain the updates from

$$\delta^{(1)} = \delta^{(0)} + \Delta\delta^{(0)} \quad (25)$$

$$|V|^{(1)} = |V|^{(0)} \left[ 1 + \frac{\Delta|V|^{(0)}}{|V|^{(0)}} \right] \quad (26)$$

Step-6: Check if all the mismatches are below a small number. Terminate the process if yes. Otherwise go back to step-1 to start the next iteration with the updates given by (25) and (26).

### — Formation of the Jacobian Matrix

Formation of the submatrices of the Jacobian matrix can be simulated by use the active and reactive power equations of (6) and (7) can be rewrite them with the help of (2) as

$$P_i = |V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \quad (27)$$

$$Q_i = -|V_i|^2 B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \quad (28)$$

A. Formation of  $J_{11}$

By define  $J_{11}$  as

$$J_{11} = \begin{bmatrix} L_{22} & \cdots & L_{2n} \\ \vdots & \ddots & \vdots \\ L_{n2} & \cdots & L_{nn} \end{bmatrix} \quad (29)$$

From (21) that  $M_{ik}$ 's are the partial derivatives of  $P_i$  with respect to  $\delta_k$ . The derivative  $P_i$  (27) with respect to  $k$  for  $i \neq k$  is given by

$$L_{ik} = \frac{\partial P_i}{\partial \delta_k} = -|Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i), \quad i \neq k \quad (30)$$

Also, the derivative  $P_i$  with respect to  $k$  for  $i = k$  is given by

$$L_{ii} = \frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i)$$

Comparing the above equation with (28) we can write

$$L_{ii} = \frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2 B_{ii} \quad (31)$$

B. Formation of  $J_{21}$

Let us define  $J_{21}$  as

$$J_{21} = \begin{bmatrix} M_{22} & \cdots & M_{2n} \\ \vdots & \ddots & \vdots \\ M_{n,2} & \cdots & M_{n,n} \end{bmatrix} \quad (32)$$

From (23) it is evident that the elements of  $J_{21}$  are the partial derivative of  $Q$  with respect to  $\delta$ . From (28) we can write equation (33)

$$M_{ik} = \frac{\partial Q_i}{\partial \delta_k} = -|Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i), \quad i \neq k \quad (33)$$

Similarly, for  $i = k$  we have

$$M_{ii} = \frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) = P_i - |V_i|^2 G_{ii} \quad (34)$$

The last equality of (34) is evident from (27).

C. Formation of J12

By define J12 as

$$J_{12} = \begin{bmatrix} N_{22} & \cdots & N_{2n_p} \\ \vdots & \ddots & \vdots \\ N_{n_2} & \cdots & N_{nn_p} \end{bmatrix} \quad (35)$$

As evident from (22), the elements of J21 involve the derivatives of real power P with respect to magnitude of bus voltage |V|. For  $i \neq k$ , we can write from (27)

$$N_{ik} = |V_k| \frac{\partial P_i}{\partial |V_k|} = |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) = -M_{ik} \quad i \neq k \quad (36)$$

For  $i = k$  we have

$$\begin{aligned} N_{ii} &= |V_i| \frac{\partial P_i}{\partial |V_i|} = |V_i| \left[ 2|V_i| G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik} V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \right] \\ &= 2|V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) = 2|V_i|^2 G_{ii} + M_{ii} \end{aligned} \quad (37)$$

D. Formation of J22

For the formation of J22 let us define

$$J_{22} = \begin{bmatrix} O_{22} & \cdots & O_{2n_p} \\ \vdots & \ddots & \vdots \\ O_{n_p 2} & \cdots & O_{n_p n_p} \end{bmatrix} \quad (38)$$

For  $i \neq k$  can write from (4.39)

$$O_{ik} = |V_i| \frac{\partial Q_i}{\partial |V_k|} = -|V_i| |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) = L_{ik}, \quad i \neq k \quad (39)$$

Finally, for  $i = k$  we have

$$\begin{aligned} O_{ii} &= |V_i| \frac{\partial Q_i}{\partial |V_i|} = |V_i| \left[ -2|V_i| B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik} V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \right] \\ &= -2|V_i|^2 B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) = -2|V_i|^2 B_{ii} - L_{ii} \end{aligned} \quad (40)$$

So, the submatrices J11 and J21 are computed, the formation of the submatrices J12 and J22 is fairly straightforward. For large system, this will result in considerable saving in the computation time.

## 6. SOLUTION OF NEWTON-RAPHSON LOAD FLOW

The Newton-Raphson load flow program is tested on the system of figure (1) with the system data and initial conditions given in tables (1) to (3) [11-14]. From (30) can write

$$L_{23}^{(0)} = -|Y_{23} V_2^{(0)} V_3^{(0)}| \sin(\theta_{23} + \delta_3 - \delta_2) = -|Y_{23}| \sin \theta_{23} = -B_{23} = -4.8077$$

Similarly, from (28) we have

$$\begin{aligned} Q_2^{(0)} &= -|V_2^{(0)}|^2 B_{22} - \sum_{\substack{k=1 \\ k \neq 2}}^n |Y_{2k} V_2^{(0)} V_k^{(0)}| \sin(\theta_{2k} + \delta_k - \delta_2) \\ &= -B_{22} - 1.05B_{21} - B_{23} - B_{24} - 1.02B_{25} = -0.6327 \end{aligned}$$

Hence from (31)

$$L_{22}^{(0)} = -Q_2^{(0)} - |V_2^{(0)}|^2 B_{22} = -0.6327 - B_{22} = 18.8269$$

In a similar way, the rest of the components of the matrix J11(0) are calculated. This matrix is given by

$$J_{11}^{(0)} = \begin{bmatrix} 18.8269 & -4.8077 & 0 & -3.9231 \\ -4.8077 & 11.1058 & -3.8462 & -2.4519 \\ 0 & -3.8462 & 5.8077 & -1.9615 \\ -3.9231 & -2.4519 & -1.9615 & 12.4558 \end{bmatrix}$$

For forming the off diagonal elements of J21 we note from (33) that

$$M_{23}^{(0)} = -|Y_{23} V_2^{(0)} V_3^{(0)}| \cos(\theta_{23} + \delta_2 - \delta_3) = -G_{23} = 0.9615$$

Also from (27) the real power injected at bus-2 is calculated as

$$P_2^{(0)} = |V_2^{(0)}|^2 G_{22} + \sum_{\substack{k=1 \\ k \neq 2}}^n |Y_{2k} V_2^{(0)} V_k^{(0)}| \cos(\theta_{2k} + \delta_k - \delta_2) = G_{22} + 1.05G_{21} + G_{23} + G_{24} + 1.02G_{25} = -0.1115$$

Hence from (34) we have

$$M_{22} = P_2^{(0)} - |V_2^{(0)}|^2 G_{22} = -3.7654$$

Similarly, the rest of the elements of the matrix J21 are calculated. This matrix is then given as

$$J_{21}^{(0)} = \begin{bmatrix} -3.7654 & 0.9615 & 0 & 0.7846 \\ 0.9615 & -2.2212 & 0.7692 & 0.4904 \\ 0 & 0.7692 & -1.1615 & 0.3923 \end{bmatrix}$$

For calculating the off diagonal elements of the matrix J12 we note from (36) that they are negative of the off diagonal elements of J21. However, the size of J21 is (3 × 4) while the size of J12 is (4×3). Therefore to avoid this discrepancy we first compute a matrix M that is given by

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}$$

he elements of the above matrix are computed in accordance with (33) and (34). can define

$$J_{21} = M(1:3, 1:4) \text{ and } J_{12} = -M(1:4, 1:3)$$

Furthermore, the diagonal elements of J12 are overwritten in accordance with (37). This matrix is then given by

$$J_{12}^{(0)} = \begin{bmatrix} 3.5423 & -0.9615 & 0 \\ -0.9615 & 2.2019 & -0.7692 \\ 0 & -0.7692 & 1.1462 \\ 0.7846 & -0.4904 & -0.3923 \end{bmatrix}$$

Finally, it can be noticed from (39) that J22 = J11(1:3, 1:3). However, the diagonal elements of J22 are then overwritten in accordance with (40). This gives the following matrix

$$J_{22}^{(0)} = \begin{bmatrix} 17.5615 & -4.8077 & 0 \\ -4.8077 & 10.8996 & -3.8462 \\ 0 & -3.8462 & 5.5408 \end{bmatrix}$$

From the initial conditions the power and reactive power are computed as

$$P_{\text{calc}}^{(0)} = [-0.1115 \quad -0.0096 \quad -0.0077 \quad -0.0098]^T$$

$$Q_{\text{calc}}^{(0)} = [-0.6327 \quad -0.1031 \quad -0.1335]^T$$

Consequently, the mismatches are found to be

$$\Delta P^{(0)} = [-0.8485 \quad -0.3404 \quad -0.1523 \quad 0.2302]^T$$

$$\Delta Q^{(0)} = [0.0127 \quad -0.0369 \quad 0.0535]^T$$

Then the updates at the end of the first iteration are given as

$$\begin{bmatrix} \delta_2^{(0)} \\ \delta_3^{(0)} \\ \delta_3^{(0)} \\ \delta_4^{(0)} \end{bmatrix} = \begin{bmatrix} -4.91 \\ -6.95 \\ -7.19 \\ -3.09 \end{bmatrix} \text{ deg} \quad \begin{bmatrix} |V_2^{(0)}| \\ |V_3^{(0)}| \\ |V_4^{(0)}| \end{bmatrix} = \begin{bmatrix} 0.9864 \\ 0.9817 \\ 0.9913 \end{bmatrix}$$

The load flow converges in 7 iterations when all the power and reactive power mismatches are below 10<sup>-6</sup>.

## 7. CONCLUSIONS

Today's electric grid was designed to operate as a vertical structure consisting of generation, transmission and distribution and advanced control support devices for reliability, stability and efficiency. Newton Raphson

method can be used to solve the power flow problems to save the stability system with the increasing on the electric network. Newton Raphson method calculation steps by minimizing the square sum of discrepancies of nodal capacities. During the power flow calculation, the determinant of matrix is positive around zero and negative value of the Jacobian matrix determinant. The main properties of Newton Raphson method are easy to handle P-V bus difficulties. The Newton's method in optimization for power flow calculation the method computational costs on each iteration will be several times greater. Each row of Jacobian matrix corresponding to any bus contains nonzero elements corresponding to all incident buses of the scheme.

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