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M/M/1/C QUEUING MODEL WITH REVERSE BALKING AND FEEDBACK CUSTOMERS

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Abstract: To retain existing customers and motivate to join new customers is a major issue in present global business environment. In this paper, we develop M/M/1/C queuing model with feedback and reverse balking behavior of customers. Reverse balking is a behavior of customers in which an arriving customer joins a system with high probability if they encounter large system size and vice-versa. This behavior of a customer can be observed in many businesses such as share market investment, policy purchasing, purchasing a product from supper market, taking admission in an institution etc. Feedback customer in queuing system refers to a customer who is unsatisfied with service. Here, we derive the steady-state solution of the model and obtain some performance measures.
Keywords: Reverse Balking, Feedback, Queuing Theory, Customer Impatience, System Size

1. INTRODUCTION & LITERATURE REVIEW

Waiting in lines or queues seems to be a general phenomenon in our day-to-day life. Many times persons person have to wait in many servicing system like railway ticket window, hospital counter, super market, in a pharmacy, stop light etc. Time and frustration are associated with these wait. Queueing theory is the formal study of waiting in line and is an entire discipline within the field of operational management. Queueing theory utilizes mathematical models and performance measures to assess and hopefully improve the service of customer and reduce the time of waiting of customers.

Queueing theory an important role in modeling real life problems involving congestion in wide areas of applied sciences. A customer decides to join the queue only when a short wait is expected and if the wait has been sufficiently small he tends to remain in the queue. If queue length is very large, then the customer leaves the system and then the customer is said to be impatient. Performance analysis of queueing model with balking and reneging behavior of customers has attracted many researchers working in the area of queueing theory. Due to their wide applications in real life problems like impatient customers in telephone switchboard system, critical patients handling in hospital and storage of goods in inventory systems.

Balking and reneging of customers are common problems in queueing system. When customers find long queue in the system or no waiting area, the customers either decide not to join the queue or depart after joining the queue without getting service due to impatience. The lost revenues due to balking and reneging in various industries can be enormous. While making decision for the number of servers required in the service facility to meet time varying demand the balking and reneging probabilities can be used to estimate the amount of loss in business analyzed by Liao [1].

Impatience is the most prominent characteristic of the customer as the customer always feel anxious and impatient during waiting for service in system. The customer's impatient behavior should be involved in the study of queuing system to model real time problem exactly. Intermittent operation of a service can be economically appealing whenever full time service would result in significant server idle time or would preclude the use of the server in some other productive capacity. On the other hand, having the server inoperative for periods of time may increase the probability of customer losses due to balking and reneging. A queue with customers balking and reneging has been discussed in Haight [2] and Haight [3], respectively. The combined impact of balking and reneging with finite capacity in a queue has been studied by Ancker and Gafarian [4]. Reynolds [5] gave the stationary solution of a multi-server model with discouragement. Discouraged arrival system was studied by many researchers. Boots and Tijms analyzed M/M/C queueing model with impatient customers and different performance measure discussed.

A finite capacity M/G/1 queueing model where the arrival and the service rates wer arbitrary functions of the current number of customers in the system was studied by Courtois and Georges [7]. Abou-El-Ata [8] discussed the finite buffer single server queueing system with balking and reneging. Analytical solutions of the single server Markovian overflow queue with balking, reneging and an additional server for longer queue were discussed in Abou-El-Ata and Shawky [9]. An additional repairman for machine repair problem with balking, and reneging and spares was incorporated in [10]. Recently, Jain and Singh [11] implemented additional servers for M/M/m queueing model with balking and reneging.Choudhury and Medhi [12] presented some aspects of balking and reneging in finite buffer queue. Recently, Rakesh and Sumeet [13] studied a queuing model with retention of reneged customers and balking. Sharma and Kumar [14] proposed a queueing model with single server, finite capacity Markovian feed-back queue with balking, reneging and retention of reneged customers in which the inter-arrival and service times follow exponential distribution.

In this paper, we study an M/M/1/C queuing system with reverse balking. We present the steady-state analysis and derive some important measures of performance. Rest of the paper is arranged as follows: In section 2, the Mathematical model for queueing system is described. In section 3, steady state solution of the systeml is presented. In section 4, performance measures of the queueing system are derived. Section 5 deals with the conclusion.

2. MATHEMATICAL MODEL FOR QUEUING SYSTEM

The most basic model of a queue is known as the M/M/1 system. This notation means that (persons, data packets, product units) arrive in the queue following a random Poisson distribution (also known as Markovian, First M), that the distribution of service intervals is first in first out and exponential Second M). In this model the service facility has only one server, so the queue processes or services only one item at a time.

The mathematical queueing model based on following assumptions:

- The arrival of customers in queueing system according to Poisson process and mean arrival rate of customers is λ.
- The queueing discipline is First Come First Service (FCFS).
- The no. of servers in the system is one.
- The capacity of the system is C.
- The balking probability of customers is β when the system is empty and may join the queue with probability $\beta' = (1-\beta)$. When there is at least one customer in the system, the customer balk with a probability $\left(1-\frac{k}{C-1}\right)$ and join the system with probability $\left(\frac{k}{C-1}\right)$. Such type of balking is known as reverse balking.
- Customers who have gotten service find the service unsatisfactory and rejoin the system as a feedback customer with probability c_f or leave the system with probability $c_1 = (1-c_f)$. Such types of customers are known as feedback customers.
- $P_k(t)$ be the probability that there are k customer in the system at time t.

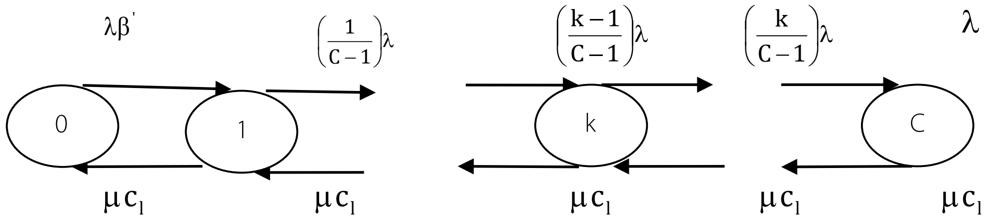

Figure 1: Transition rate diagram of the model

The governing differential difference equations of the system are

$$\frac{dP_0(t)}{dt} = -\lambda\beta' P_0(t) + \mu c_1 P_1(t); \text{ for } k = 0 \tag{1}$$

$$\frac{dP_1(t)}{dt} = \lambda\beta' P_0(t) - \left[\left(\frac{1}{C-1}\right)\lambda + \mu c_1\right] P_1(t) + \mu c_1 P_2(t); \text{ for } k = 1 \tag{2}$$

$$\frac{dP_k(t)}{dt} = \left(\frac{k-1}{C-1}\right)\lambda P_{k-1}(t) - \left[\left(\frac{k}{C-1}\right)\lambda + \mu c_1\right] P_k(t) + [\mu c_1] P_{k+1}(t); \quad 2 \le k \le C-1 \tag{3}$$

$$\frac{dP_C(t)}{dt} = \lambda P_{C-1}(t) - \mu c_1 P_C(t); \text{ for } k = C \tag{4}$$

3. STEADY STATE SOLUTIONS OF THE SYSTEM

After sufficient time has elapsed, the state of the system becomes essentially independent of the initial state and the elapsed time. Queueing theory has tended to focus largely on the steady state condition.

In the steady state position, $\lim_{k\to\infty} P_k(t) = P_k$.

Therefore

$$0 = -\lambda\beta' P_0(t) + \mu c_1 P_1(t); \text{ for } k = 0 \tag{5}$$

$$0=-\lambda\beta' P_0(t)-\left[\left(\frac{1}{C-1}\right)\lambda+\mu c_1\right]P_1(t)+\mu c_1 P_2(t);\ \text{for}\,k=1 \tag{6}$$

$$0=\left(\frac{k-1}{C-1}\right)\lambda P_{k-1}(t)-\left[\left(\frac{k}{C-1}\right)\lambda+\mu c_1\right]P_k(t)+[\mu c_1]P_{k+1}(t);\quad 2\le k\le C-1 \tag{7}$$

$$0=\lambda P_{C-1}(t)-\mu c_1 P_C(t);\ \text{for}\,k=C \tag{8}$$

Solving equations (5) to (8) iteratively, we get

$$P_k=\left[\frac{(k-1)!}{(C-1)^{k-1}}\prod_{i=1}^{k}\frac{\lambda}{\mu c_1}\right]\beta' P_0;\,1\le k\le C-1 \tag{9}$$

$$P_K=\left[\frac{(C-2)!}{(C-1)^{C-2}}\prod_{i=1}^{C}\frac{\lambda}{\mu c_i}\right]\beta' P_0;\ k=C \tag{10}$$

Using normalization condition

$$\sum_{k=0}^{C}P_k=1$$

$$\text{or }P_0+\sum_{k=1}^{C}P_k=1$$

$$\text{or }P_0+\sum_{k=1}^{C-1}P_k+P_C=1$$

$$\text{Or }P_0=\frac{1}{\left[1+\sum_{k=1}^{C-1}\left\{\frac{(k-1)!}{(C-1)^{k-1}}\prod_{i=1}^{C-1}\frac{\lambda}{\mu c_1}\right\}\beta'+\left\{\frac{(C-2)!}{(C-1)^{N-2}}\prod_{i=1}^{C}\frac{\lambda}{\mu c_1}\right\}\beta'\right]} \tag{11}$$

4. PERFORMANCE MEASURES OF THE SYSTEM

Here we derive some important measures of performance.

— Average number of customers in the system (Expected Size of the System)

Expected number of customers in the system is also known as system size. Therefore system size for given queueing model is given by

$$L_s=\sum_{k=1}^{C}kP_k$$

$$\text{or }L_s=\sum_{k=1}^{C-1}kP_k+CP_C$$

$$L_s=\sum_{k=1}^{C-1}k\left[\frac{(k-1)!}{(C-1)^{k-1}}\prod_{i=1}^{k}\frac{\lambda}{\mu P_i}\right]\beta' P_0+C\left[\frac{(C-2)!}{(C-1)^{C-2}}\prod_{i=1}^{K}\frac{\lambda}{\mu P_i}\right]\beta' P_0 \tag{12}$$

— Average Rate of Reverse Balking

Average rate of reverse baking means average number of customers after getting positive feedback of the company.

$$R_b=\beta\lambda P_0+\sum_{k=1}^{C-1}\left(1-\frac{k}{C-1}\right)\lambda P_k$$

$$R_b=\beta\lambda P_0+\sum_{k=1}^{C-1}\left(1-\frac{k}{C-1}\right)\lambda\left[\frac{(k-1)!}{(C-1)^{k-1}}\prod_{i=1}^{k}\frac{\lambda}{\mu P_i}\right]\beta' P_0 \tag{13}$$

— Expected queue length of the system

Expected queue length means average number of customer waiting for service in queue.

$$L_q=\sum_{k=1}^{C}(k-1)P_k$$

$$=\sum_{k=1}^{C}kP_k+CP_C+P_0-1$$

$$= \sum_{k=1}^{C-1} k \left\{ \frac{(k-1)!}{(C-1)^{k-1}} \prod_{i=1}^{k} \frac{\lambda}{\mu p_1} \right\} \beta' P_0 + C \left\{ \frac{(C-2)!}{(C-1)^{N-2}} \prod_{i=1}^{C} \frac{\lambda}{\mu p_1} \right\} \beta' P_0 + P_0 - 1 \tag{14}$$

5. CONCLUSION

In this paper, the concept of reverse balking is incorporated into an M/M/1/C queuing system with feedback customers. The steady-state analysis of the model is performed and some important measures of performance are derived for proposed queueing system. This model finds its application in investment sector facing the problem of customer impatience. The model may be extended to more server and infinite capacity of the system. Time-dependent analysis and sensitivity analysis can also be carried out.

References

[1] Liao, P.: 2007,Optimal staffing policy for queueing systems with cyclic demands: waiting cost approach, In Proceedings of the 18th Annual Conference of the Production and Operations Management Society (POMS '07), Dallas, Tex, USA, 2007.

[2] Haight, F. A.: Queueing with balking, Biometrika, vol. 44, pp. 360–369, 1957.

[3] Haight, F. A.: Queueing with reneging, Metrika, vol. 2, no. 1, pp. 186–197, 1959.

[4] Ancker Jr. C. J. and Gafarian, A. V.: Some queueing problems with balking and reneging: I, Operations Research, vol. 11, no. 1, pp. 88–100, 1963.

[5] Reynolds, J. F.: The Stationary solution of a multi-server model with discouragement, Operations Research vol. 16, 64–71, 1968.

[6] Boots, N. K. and Tijms, H.: An M/M/c queue with impatient customers, Top, vol. 7, no. 2, pp.213-220, 1999.

[7] Courtois, P. J. and Georges, J.: On a single server finite queueing model with state-dependent arrival and service processes, Operations Research, vol. 19, pp. 424-435, 1971.

[8] Abou-El-Ata, M. O.: The state-dependent queue: M/M/1/N with reneging and general balk functions, Microelectronics Reliability, vol. 31, no. 5, pp. 1001–1007, 1991.

[9] Abou-El-Ata, M. O. and Shawky, A. I.: The single-server Markovian overflow queue with balking, reneging and an additional server for longer queues, Microelectronics Reliability, vol. 32, no. 10, pp. 1389–1394, 1992.

[10] Jain, M., Singh, M. and Baghel, K. P. S.: M/M/c/K/N machine repair problem with balking reneging spares and additional repairman, Gujarat Statistical Review 26–27, pp. 49–60, 2000.

[11] Jain, M. and Singh, P.: M/M/m queue with balking reneging and additional servers, International Journal of Engineering vol.15, pp. 169–178, 2002.

[12] Choudhury, A. and Medhi, P.: Some aspects of balking and reneging in finite buffer queues, RAIRO-Operations Research, vol. 45, no. 3, pp. 223–240, 2011.

[13] Rakesh, K. and Sumeet, K. S.: An M/M/1/N queueing model with retention of reneged customers and balking, American Journal of Operational Research, vol. 2, no. 1, pp. 1–5, 2012

[14] Sharma, S. K. and Kumar, R. "A Markovian Feedback Queue with Retention of Reneged Customers and Balking," AMO-Advanced Modeling and Optimization, vol., no. 3, pp. 681-688, 2012.

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