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DEEP BEAMS REINFORCEMENT, NATIONAL AND EUROCODE 2 DESIGN PROVISIONS

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Abstract: Deep beam or shear walls are fairly common structural elements in the building construction industry of countries like France, Germany, Switzerland or Belgique, and thus, since the introduction of concrete structures in the 20th century. Nevertheless, in Portugal and generally in southern European countries, reinforced concrete frames with a brick masonry infill is the foremost used building construction technique, with some punctually deep beams applications in recent years. Beyond that, Eurocode 2 is now the prevailing code in Europe for the concrete elements design. In the present paper, it is intended to compare the steel reinforcement amount defined for deep beams by the Eurocode 2, which is based in a finite element design approach or in the strut-and-tie model application, with the provisions defined in the French and Portuguese national codes. For the finite element design, a displacement based approach considering an isotropic linear elastic behavior is considered. For each code provisions, the reinforcement steel ratios are obtained and drawings are presented. The results allow understanding the economic impact of Eurocode 2 application in deep beams design.
Keywords: Deep beams, Eurocode 2, REBAP, BAEL91, strut-and-tie model, finite element model

1. INTRODUCTION

France, Germany, Switzerland or Belgique are countries where the concrete building industry applies deep beams and shear walls since the advent of concrete in the 20th century. Nevertheless in Portugal, as in south European countries, the concrete building industry is essentially based in concrete frames structures with a brick masonry infill. Nevertheless, and may be due to the European market, since recent years it is possible to observe in Portugal several cases of buildings where concrete deep beams are incorporated. In both countries, France and Portugal, the national codes define specific reinforcement provisions for deep beams. The Portuguese national code, reinforced concrete and prestressed structures Portuguese code (REBAP), [1], was published in 1983 and the French national code, reinforced concrete at the limit states (BAEL 91), [2], date from 1992. Furthermore, the Eurocodes are the standards applied nowadays in the European community, [3] and [4]. In this article, we compare the steel reinforcement amount defined the national French and Portuguese codes with the two methodologies adopted by the Eurocode 2, [4]. The results allow evaluating and compare the economic consequence of Eurocode application. A standard continuous deep beam example is defined and the reinforcement design is performed in order to obtain the total steel reinforcement amount following each one of three codes provisions.

This article is structured as follow. First, deep beams elements are defined. Secondly, a continuous deep beam design example is defined. Thirdly, as results, the total steel reinforcement amount is defined in the application of the Portuguese code, the French code and Eurocode 2. Finally, in the conclusions, the overall results are compared.

2. DEEP BEAMS

The theory used for the design of slender beams is based on the fundamental hypothesis that stress distribution across the section is proportional to the distance from the neutral axis of bending, i.e., plane sections, through the cross-section of a beam, perpendicular to its axis remain plane after the beam is subjected to bending. Nevertheless, this hypothesis has a limited applicability to deep beams, resulting in designs that are generally not conservative. In fact, a deep beam is a beam having a depth comparable to the span length and shear warping of the cross-section and a combination of diagonal and flexural tension stresses in the body of a deep beam require that deep beam theory is used for the design of such elements. Warpage of the sections are important for beams with small span-to-depth ratios [5] and smaller the span-to-depth ratio the more pronounced the deviation from the linear stress hypothesis. The deviation from the fundamental linear stress hypothesis should be considered for beams having span-to-depth ratios less than 2.5, [6]. According to deep beam theory, [7, 8], the load is carried to the supports by a compression force combining the load and the reaction, Figure 1. As a result, the strain distribution is no longer considered linear and the shear deformations become significant when compared to pure bending. The recognition that a deep beam behaves differently than a slender beam has led many countries to include design provisions for these elements into their design codes, [1], [2] and [4].

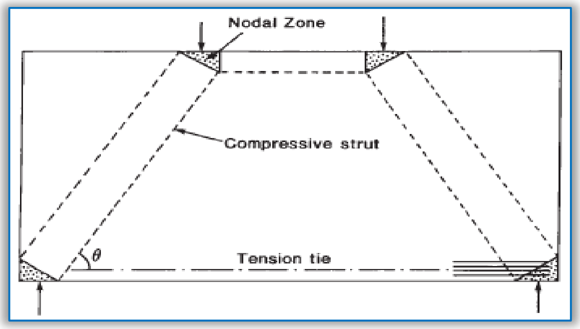


Figure 1. Deep beam structural system

3. DESIGN EXAMPLE

In order to apply design calculations provisions, a design example is defined. A continuous deep beam with two spans submitted to a uniform load. The deep beam as an overall depth, h, assumed to be 2.50 meters and a constant thickness equal to 20 centimeters, both spans are equal to 5 meters being the overall beam span 10 meters. The materials are C25/30 concrete and S500 steel reinforcement. The compressive strength for design is taken to be 16.67 MPa and the design yield strength of the reinforcing steel is taken as 435 MPa. The uniform loading is 240kN/m, applied in the top surface of the beam, assumed to be the fundamental combination of dead load and live load, including the self-weight of the beam and corresponding to the ultimate limit state defined in Eurocode 1, [1], Figure 2 represent the deep beam dimensions and the loading.

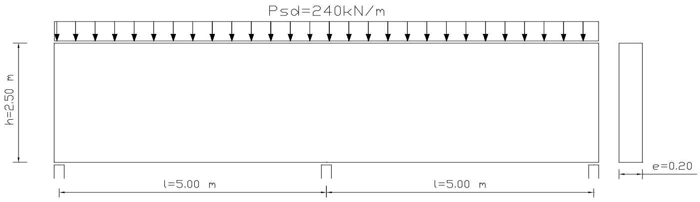


Figure 2. Deep beam example

4. FRENCH AND PORTUGUESE DESIGN CODES PROVISONS

— Internal forces e and flexural reinforcement

According to Portuguese and French national provisions, the reinforcement is based on the knowledge of the bending moment and shear forces internal forces such obtained by the traditional resistance of materials theory (Annex E.5.4 of BAEL 91 and Article 130 of REBAP), in the hypothesis that the deep beam is a linear element with a linear elastic behavior. For the design example, those internal forces were obtained through the calculation program FTOOL, [9], the bending moment and shear forces are represented in Figure 3. In those calculations, the shear deformation is neglected.

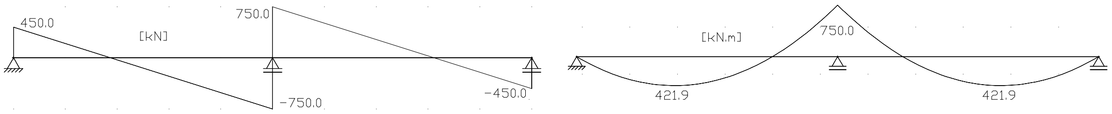


Figure 3. Bending moment and shear forces diagrams

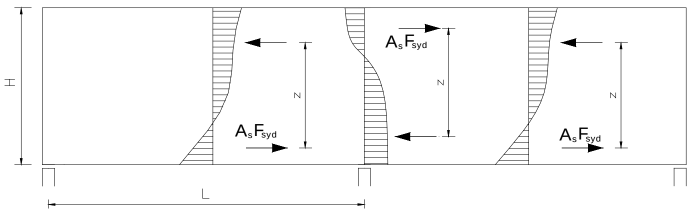


Figure 4. Normal stress diagram at mid-span and above supports

The bending moment is introduced in Eq. (1), [3], [4]. Thought this equation it is possible to obtain the reinforcement amount for the deep beam tension zone, Figure 4.

$$A_S = \frac{M_{sd}}{z \cdot f_{syd}} \tag{1}$$

where M_{sd} is the largest design bending moment in the span, z is the internal forces lever arm, f_{yd} is the reinforcement design strength and A_s is the design reinforcement amount.

— Portuguese code provisions

The steel reinforcement for continuous deep beams is obtained considering a lever arm z satisfying Eq. (2) along the span and over supports, Article 130 of REBAP, Figure 4.

$$z = 0.10\,(2.5\,h + 2\,l) \text{ se } 1 < l/h < 2.5 \tag{2}$$

The main longitudinal steel in the span should be detailed as for simply supported beams, thus extending without curtailment across the full span length. Over the support, half the steel should extend across the full length of the adjacent span; the remaining half is stopped at 0.4l or 0.4h, whichever is smaller, from the face of the support [7] and Figure 5. Furthermore, this reinforcement should be distributed along the beam height, according to two horizontal zones and measured from the tension face of the beam. Eq. (3) and Eq. (4) and Figure 5.

$$A_{s1} = 0.5\,(\frac{l}{h} - 1)\,A_S \tag{3}$$

$$A_{s2} = A_S - A_{s1} \tag{4}$$

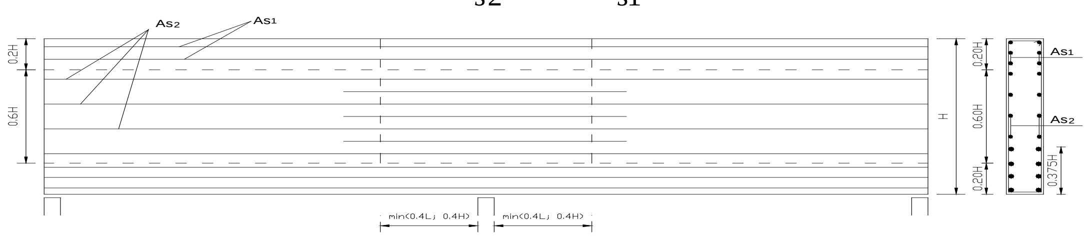


Figure 5. Steel reinforcement over interior support

Applying equations (1), (2), (3) and (4), the total amount of flexural reinforcement at mid-span and over the interior support are obtained and represented in Table 1.

Table 1. Steel reinforcement amount

M_{sd} (kN.m)	z (m)	A_s design (cm²)	A_s adopted (cm²)
421.9	1.625	5.97	6.32 - $8\phi10$
-750	1.625	10.61	A_{s1} = 5.50 - $11\phi8$
			A_{s2} = 5.50 - $11\phi8$

The web reinforcement is provided in the form of a light mesh of orthogonal reinforcement consisting of vertical stirrups and horizontal bars placed near each face and surrounding the extreme vertical bars. The aim of the web reinforcement is mainly to limit the crack widths which may be caused by the principal tensile stresses. REBAP also require designers to comply with a specified minimum amount of flexural steel reinforcement for deep beams to prevent the sudden and brittle failure of a flexural member when the computed flexural strength reinforcement is less than the bending moment that causes cracking. Article 133 of REBAP, state that an orthogonal reinforcement mesh with a minimum area amount defined by $A_{sv,\min} = A_{sh,\min} = 0.0005A_c$ (cm²/m²), where A_c is 0,20 x1.00 m², $A_{sv,min} = A_{sh,min}$ = 1 cm²/face => $\#\phi6//0.25$. In Figure 6, the calculated amount of steel reinforcement and detailing is represented in each beam face.

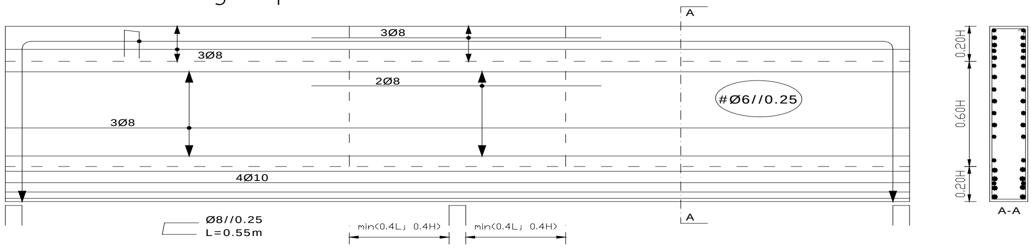


Figure 6. Steel reinforcement detailing per face

— French code provisions

The calculation of the bending steel reinforcement, according to BAEL 91, obey the same procedure as for the Portuguese national code §4.2. Nevertheless, the lever arm z must satisfied Eq. (5) in agreement with annex E5.4.12 of BAEL 91:

$$z = 0.20\,(1.5\,h + l) \text{ se } 1 < l/h < 2.0 \,. \tag{5}$$

The annex E5.4.12 of BAEL 91, state that the steel reinforcement over supports can be interrupted in the same ways as described in the Portuguese national code § 4.2 and should be distributed in a zone limited by the upper face and min{0.10 h; 0.10 l}, Figure 7.

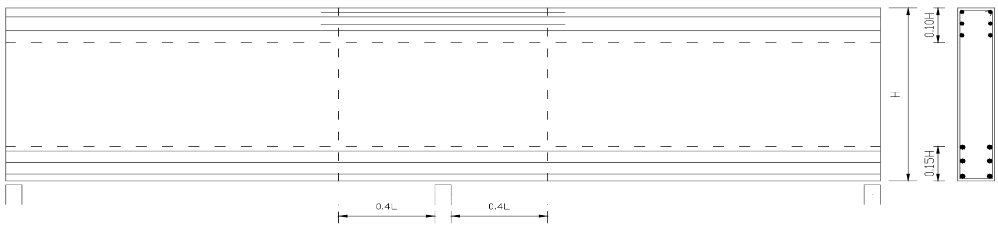


Figure 7. Flexural steel reinforcement detailing

The design is based on Eq. (1) and Eq. (5), and the steel reinforcement amount at mid-span and over the intermediate support are obtained. These amounts are represented in Table 2.

Table 2. Flexural steel reinforcement

M_{sd} (kN.m)	z (m)	A_s design (cm²)	A_s adopted (cm²)
421.9	1.75	5.54	5.74 - $6\phi10 + 2\phi8$
- 750	1.75	9.85	10.04 - $4\phi16 + 4\phi8$

According to the French code, an orthogonal steel reinforcement mesh is compulsatory and should be provided for both deep beam faces. The amount in the vertical direction is given by annex E5.4.21, and must satisfied Eq. (6).

$$\rho_v = \max\left\{\frac{0.8}{f_{yk}};\frac{3}{4}\frac{\tau_{ou}}{f_{syd}}\right\},\ \rho_V = \frac{A_V}{b_O s_V},\ \tau_{ou} = \frac{V_{ou}}{b_o h} \text{ para } l>h, \tag{6}$$

The horizontal amount is defined in annex E.5.4.22 of BAEL 91. Two distinct horizontal nets should be defined (Annex E5.4.221and E5.4.222). The first steel reinforcement net is located between 0.10h e 0.55h, Figure 7, with a horizontal reinforcement amount, ρ_h, given by Eq. (7),

$$\rho_h = \max\left\{0.50\left(0.60 + 15\frac{\tau_{ou}}{f_{c28}}\right)\frac{\tau_{ou}}{f_{syd}};0.50\frac{\tau_{ou}}{f_{syd}};\frac{0.8}{f_{yk}}\right\}, \text{ com } \rho_h = \frac{A_h}{b_o s_h}. \tag{7}$$

Where f_{c28} é o is the concrete characteristic strength, s_h is the horizontal spacement between steel bars. The second steel reinforcement net, Figure 8, is located between 0,55h e 0,90h, with an amount $\rho_h^{'}$ given by Eq. (8). The two nets reinforcement should be extended along the complete adjacent spans.

$$\rho_h^{'} = \max\left\{0.30\left(0.60+15\frac{\tau_{ou}}{f_{c28}}\right)\frac{\tau_{ou}}{f_{syd}};0.30\frac{\tau_{ou}}{f_{syd}};\frac{0.8}{f_{yk}}\right\}. \qquad (8)$$

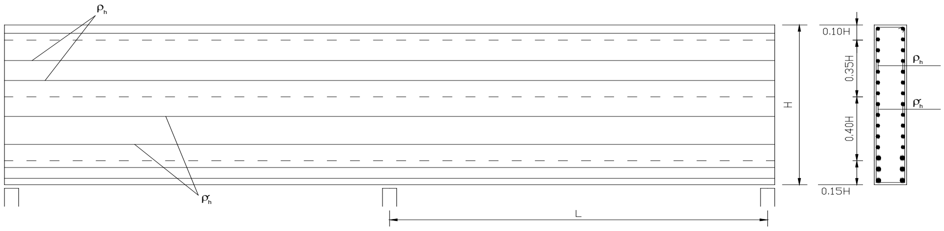


Figure 8. Horizontal reinforcement nets

Table 3 indicates the total steel reinforcement in each net.

Table 3. Vertical and horizontal steel reinforcement amount

V_{ou} (kN)	τ_{ou} (kPa)	ρ_v	ρ_h	$\rho_h^{'}$
450	900	$\phi 8//0.15$	$\phi 8//0.15$	$\phi 8//0.15$

In Figure 8, the BAEL 91 calculated steel reinforcement amount and the corresponding detailing is represented in each beam face.

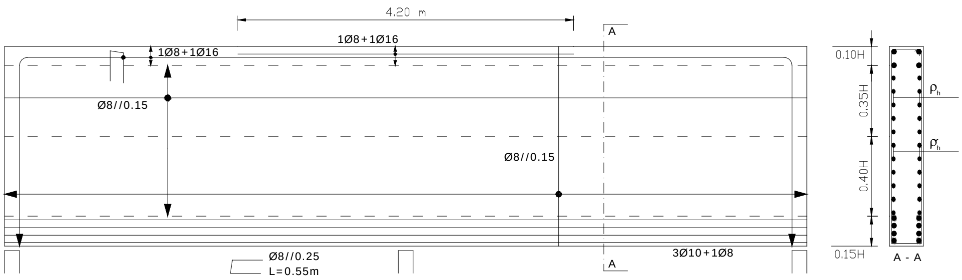


Figure 8. Reinforcement detailing

5. EUROCODE 2

— Eurocode 2 design methods

Eurocode 2 advocate the adoption of one of two design methodologies for deep beams reinforcement and detailing. The first corresponds to a linear elastic analyze by the finite element method [10, 11, 12] and the second methodology is related the strut-and-tie model application, [13, 14].

— Finite element analyse

In order to apply annex F of Eurocode 2, relative to the finite element analyze, the finite element program SAP2000, [15], was used. This computer program allows to obtain the stress field along the deep beam example. For that, the deep beam was discretized in 100 quadrangular 4 nodes elements with 4 integration points per element. Each element is a square 50 cm x 50 cm ($a_x = a_y$) and a thickness equal to 20 cm (e). The elastic longitudinal module is taken equal to 30 GPa and the Poisson coefficient is zero $(\nu = 0)$, [4]. The obtained stress diagrams are represented in Figure 10, Figure 11 and Figure 12.

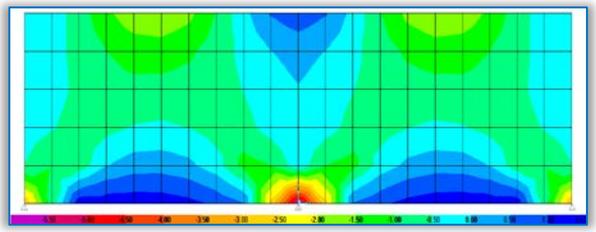


Figura 10. Horizontal stress, σ_x.

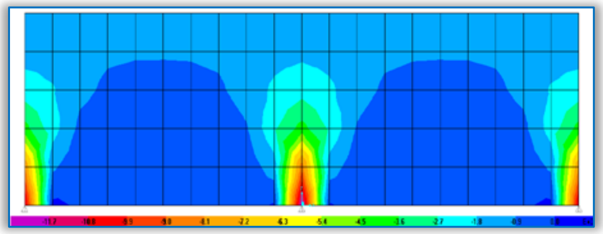


Figure 11. Vertical stress, σ_y.

Figure 10 clearly shows that the higher horizontal steel reinforcement should be disposed along the overall span length at the bottom of the deep beam and also under the support in the top of the deep beam.

Figure 11 indicates that vertical reinforcement is necessary at the support zones. Figure 12 shows that a constant web reinforcement is necessary.

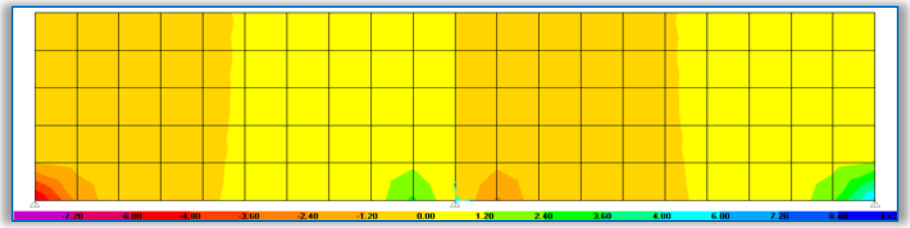


Figure 12. Tangential stress, τ_{xy}.

The Eurocode 2 methodology requires several steps to obtain the reinforcement amount. Beyond that, the concrete compression must be controlled through the stress σ_{cd} defined by Eq. (9).

$$\sigma_{cd} < \nu\, f_{cd}, \tag{9}$$

with $\nu = 0.6\left[1 - \frac{f_{ck}}{250}\right]$.

The diagram represented in Figure 13, define, the necessary steps to obtain the reinforcement amount starting from the stress field knowledge obtained from the finite element analysis.

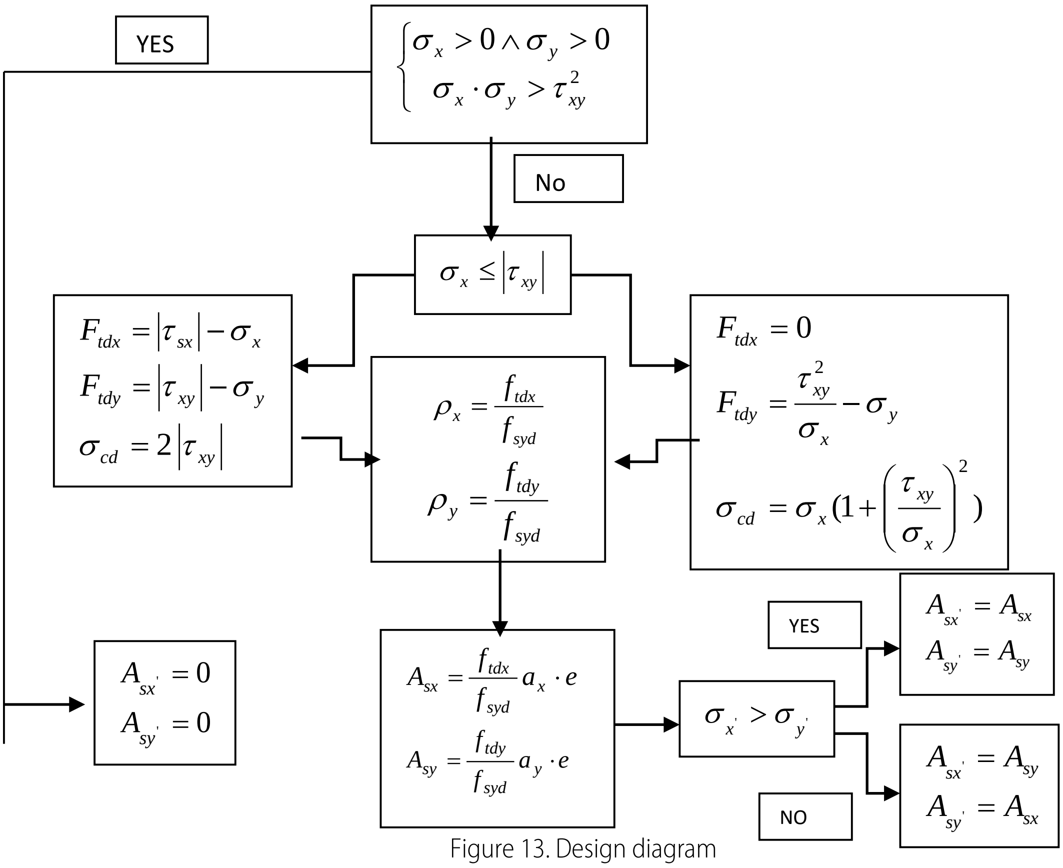


Figure 13. Design diagram

Those steps being followed, allow to obtain for element 6, the reinforcement ration amount per face and per elements in the vertical and horizontal directions and are represented in Table 4.

Table 4. Reinforcement ratio in element 6 Gauss points.

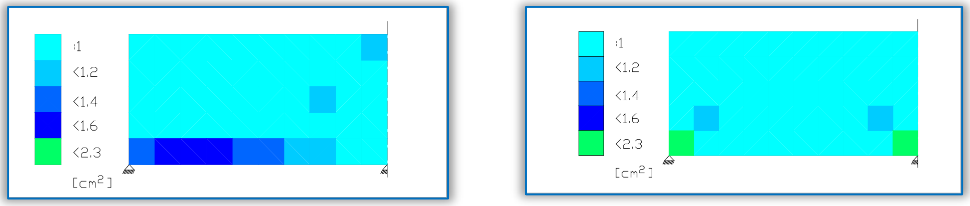
Element 6	Global axis system			Local axis system with $\sigma_x < \sigma_y$				Global axis system: Reinforcement ration in both faces		Global axis system: $\sigma_{cd} < \nu\, f_{cd}$
Gauss point	σ'_x (MPa)	σ'_y (MPa)	τ_{xy} (MPa)	σ_x (MPa)	σ_y (MPa)	f_{tdx} (MPa)	f_{tdy} (MPa)	ρ'_x	ρ'_y	σ_c (MPa)
1	-1,307	-0,186	0,541	-0,186	-1,307	0.727	1.848	0,0042	0,0017	1,082
2	-1,307	0,115	-0,811	0,115	-1,307	0.696	2.118	0,0049	0,0016	1,621
3	1,396	0,115	-0,961	1,396	0,115	0	0.547	0,0000	0,0013	2,057
4	1,396	-0,186	0,391	1,396	-0,186	0	0.296	0,0000	0,0007	1,506

In Table 5 we consider that the reinforcement amount to be placed in each element is the mean value of the reinforcement amount in each Gauss point.

Table 5. Elements 6 and 21, reinforcement amount per face

		Reinforcement ratio in both faces				Steel area (cm²) per face and per element			
		Design reinforcement ratio		Minimum reinforcement ratio					
Element	P.G.	ρ'_x	ρ'_y	$\rho'_{x,\min}$	$\rho'_{y,\min}$	A_x	$A_{x,médio}$	A_y	$A_{y,médio}$
6	1	0,0042	0,0017	0.002	0.002	2.12	1.64	1.00	1.00
	2	0,0049	0,0016	0.002	0.002	2.43		1.00	
	3	0,0000	0,0013	0.002	0.002	1.00		1.00	
	4	0,0000	0,0007	0.002	0.002	1.00		1.00	
21	1	0,0037	0,0006	0.002	0.002	1.85	1.42	1.00	1.00
	2	0,0036	0,0005	0.002	0.002	1.82		1.00	
	3	0,0013	0,0005	0.002	0.002	1.00		1.00	
	4	0,0013	0,0005	0.002	0.002	1.00		1.00	

The biggest compression stress in the Gauss points, for Eq. (9) verification is obtained in elements 45 and 51 located near the middle support and is equal to 17.64 Mpa. In regard to te minimum reinforcement in deep beams, Eurocode 2 recommend the application of a minimum reinforcement ratio in direction x and y equal to $\rho'_{x,\min}, \rho'_{y,\min} = 0{,}001$ per face, with a minimum of 1,50 cm²/m/face and a maximum bar spacement equal to 30 cm. Since the concrete are $A_c = 0.10\,m^2$, $A_{x,\min}, A_{y,min} = 2$cm²/m/face, corresponding to a steel net #ϕ8//0,25. In Figure 14, the computed reinforcement amount in the vertical and horizontal direction and per face are represented.



Horizontal direction Vertical direction

Figure 14. Computed reinforcement amount in both direction and per face

Finally, in Figure 15 the deep beam reinforcement detailing is presented.

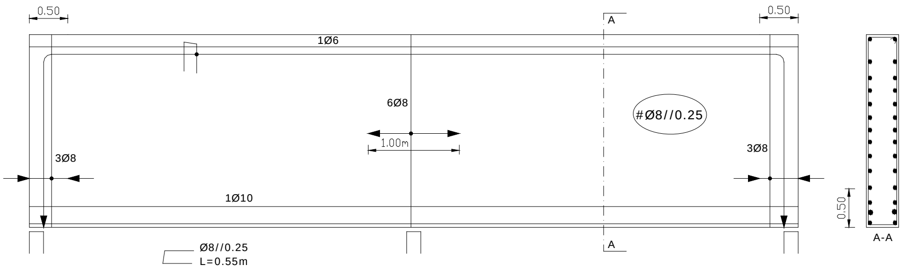


Figure 15. Reinforcement detailing per face

— Strut-and-tie model

A strut-and-tie model consists of 3 main components: struts, ties, and nodes. The struts carry the compressive forces and the ties are the tension members in the model. Nodes, or nodal areas, represent the points where the struts and ties meet. This method only requires that equilibrium and the yield criteria are satisfied and does not require strain compatibility. It is a lower bound plasticity method [16]. Hence, numerous strut-and-tie models can be mapped on to a specific half-joint design, as long as the external applied loads and reaction forces are in equilibrium with the assumed distribution of

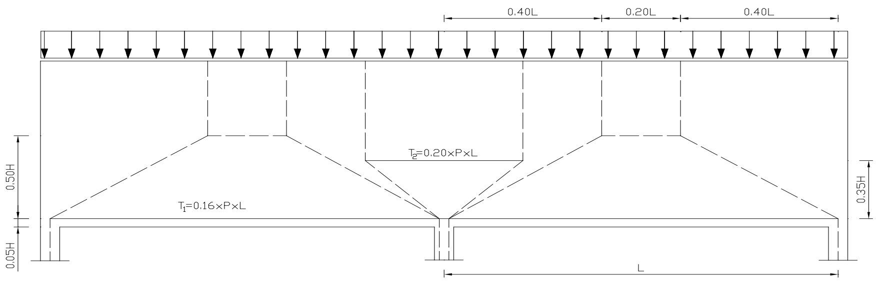


Figure 16. Strut-and-tie adopted model

Table 6. Steel reinforcement amount.

Tie	F_t (kN)	A_s design (cm²)	A_s adopted
T_1	192	4.41	$2\phi 10 + 6\phi 8$
T_2	240	5.52	$12\phi 8$

internal forces and the stresses developed in the struts, ties and nodes are within acceptable limits. Several methods have been developed to identify possible strut-and-tie models, these are all based on elastic solutions, [17]. The tensile forces in the ties are taken up by reinforcement bars. In Figure 16, the strut-and-tie adopted model, corresponding the design example, is represented.

In Table 6 the axial forces on T_1 and T_2 ties are indicated, as the corresponding reinforcement amount. The minimum reinforcement amount for the web net is the same who was already described § 5.2, Finally, Figure 17, is related to the corresponding reinforcement detailing.

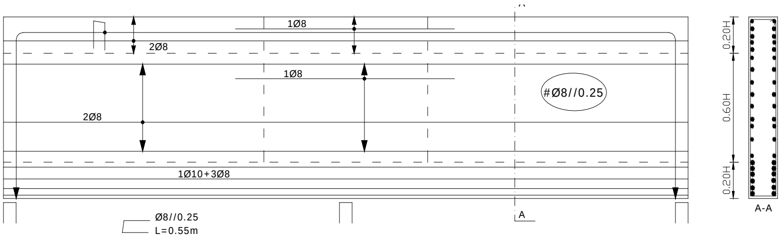


Figure 17. Reinforcement detailing per face

6. RESULTS

— Reinforcement ratios

In tables 7, 8, 9 and 10, related to REBAP, BAEL91 and Eurocode 2, the deep beam reinforcement ratios are presented. According to the results, a based on REBAP provisions, an amount of 42.61 kg/m³ of steel reinforcement in the deep beam design example.

Table 7. REBAP steel reinforcement design

Position	Amount	Length (m)	Weight (kg)	Total weight (kg)	Ratio (kg/m³)
Span	8 ϕ10	10	49.36	49.36	42.61
Support	12 ϕ8	10	47.40	56.09	
	10 ϕ8	2.2	8.69		
Web net	22 ϕ6	10	48.84	94.35	
	82 ϕ6	2.5	45.51		
U	61 ϕ8	0.55	13.25	13.25	

Table 8. BAEL91 steel reinforcement design

Position	Amount	Length (m)	Weight (kg)	Total weight (kg)	Ratio (kg/m³)
Span	6 ϕ10	10	37.02	44.22	56.47
	2 ϕ8	10	7.90		
Support	2 ϕ16	10	31.56	56.04	
	2 ϕ8	10	7.90		
	2 ϕ16	4.2	13.26		
	2 ϕ8	4.2	3.32		
Web net	26 ϕ8	10	102.70	168.86	
	67 ϕ8	2.5	66.16		
U	61 ϕ8	0.55	13.25	13.25	

According to BAEL 91 provisions, a total amount of 56.47 kg/m³ for steel reinforcement is necessary, this result indicates that BAEL 91 provisions requires more reinforcement that REBAP provisions.

Table 9. EC2 Finite element method design

Position	Amount	Length (m)	Weight (kg)	Total weight (kg)	Ratio (kg/m³)
Span	2 ϕ10	10	12.34	12.34	22.79
Support	2ϕ 6	10	4.44	4.44	
Web net	22ϕ8	10	86.90	83.94	
	100ϕ8	2.5	98.75		
U	61 ϕ8	0.55	13.25	13.25	

Table 10. EC2 Strut-and-tie model design.

Position	Amount	Length (m)	Weight (kg)	Total weight (kg)	Ratio (kg/m³)
Span	2 ϕ10	10	12.34	36.04	50.16
	6 ϕ8	10	23.70		
Support	6 ϕ8	10	23.70	33.65	
	6 ϕ8	4.2	9.95		
Web	22ϕ8	10	86.90	167.88	
	82ϕ8	2.5	80.98		
U	61 ϕ8	0.55	13.25	13.25	

The finite element design approach, recommended by Eurocode 2, indicates the need for a total reinforcement amount of 22.79 kg/m^3. This is approximately 46% to 60% lesser than the amount required by the two national codes presented. Finally, the strut-and-tie model application requires a total steel reinforcement amount of 50.16 kg/m^3, this amount is approximately the same as defined by the two national codes studied.

7. CONCLUSIONS

French and Portuguese national code provisions have been used to evaluate the steel reinforcement amount for a deep beam design example. Although based on the same calculation type, the French national code provisions require a higher web reinforcement amount. Nevertheless, the overall results are approximately the same, differing in 25 %.

The two methodologies defined by Eurocode 2, the finite element method and the strut-and-tie method have also been used for the steel reinforcement amount evaluation.

The finite element method gives the lesser steel reinforcement amount when compared to the strut-and-tie method and the national codes provisions. The finite element method application results in a steel reinforcement amount reduction approximately equal to 50% when compared to the national codes and the strut-and-tie method. The strut-and-tie method gives steel reinforcement amounts of the same order than the national French and Portuguese design codes. The results indicate that the finite element method recommended by Eurocode 2 will allow a significant reduction of the steel reinforcement amount in deep beams design.

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ISSN 1584 - 2665 (printed version); ISSN 2601 - 2332 (online); ISSN-L 1584 - 2665