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ANALYSIS OF MICROPOLAR FLUID FLOW THROUGH A POROUS CHANNEL DRIVEN BY SUCTION/INJECTION WITH HIGH MASS TRANSFER

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Abstract: In this paper, the flow of a micropolar fluid conveyed through porous channel driven by suction or injection with high mass transfer is analyzed using the regular perturbation method to solve the coupled nonlinear ordinary equations arising from the mechanics of fluid. The developed analytical solutions are used to investigate the effects of flow and rotation parameters such as Reynolds number and micro rotation parameters. Obtained analytical results when compared to results of the other methods in the existing works in literature are in good agreements. The results obtained from this paper can be used to further the study of the behavior of micropolar fluids in applications including as lubricants, blood flow porous media, micro channels and flow in capillaries.
Keywords: Micropolar fluids; Porous media, Regular perturbation method; mass transfer

1. INTRODUCTION

The theory of micropolar fluid was established by Eringen [1] in his bid to model the behavior of non-Newtonian flow whose micro constituents rotate during fluid flow. Moreover, in his work, he developed the constitutive relation to include more material parameters and micro rotation vectors making the usual equations for Newtonian flow non-linear. Also, in the study of micropolar fluids, Idris [2] studied the effect of non-uniform temperature gradient on micropolar fluids under convective heat transfer while Yuan [3] investigated the behavior of micropolar fluids under laminar flow condition within a porous channel. Kelson [4-5] presented the effect of surface conditions on micropolar fluid flow over a stretching sheet with strong suction and injection. The flow of viscous fluid was studied by Zaturska et al. [6] along a porous wall during suction. Power law variations were adopted by Cheng [7] to study micropolar fluid from a vertical truncated cone under natural convection. Joneidi et al. [8] applied the differential transformation method (DTM) to heat transfer problems of nonlinear equations while Hassan [9] adopted the DTM in solving Eigen value problems. Magyari and Keller [10] studied boundary layer flows induced by permeable walls using exacts solutions. Natural convective flow over horizontal plate was investigated by Murthy and Singh [11] presenting the thermal effects with surface mass flux on convection.

The relevance and importance of pertubation solutions to provide approximate analytical solutions have been proven beyond reasonable doubt in literatures. However, owing to the problem of weak nonlinearites and small parameters which are sometimes artificial makes it necessarily to develop other analytical methods of solutions to overcome these limits [12-26]. Consequently, the use of other approximate analytical methods such as differential transform method (DTM), homotopy analysis method (HAM), optimal homotopy asymptotic method (OHAM), variational iteration method (VIM), Adomian decomposition method (ADM) and some other approximation methods have been developed. Methods such as DTM, HAM and ADM however require the need to find an initial condition that will satisfy the boundary condition which theories have not been rigorously proven for all cases. Making it necessary to use computational tools resulting to higher computational cost to provide problem solutions. Also, OHAM requires determining constants using auxilliary fuctions which may be too rigorous to determine for some nonlinear problems. Since, the solutions reported for the other relatively sophisticated methods to nonlinear problems have good accuracy, but they are more complicated for applications than perturbation methods. Therefore, over the years, the relative simplicity and high accuracy especially in the limit of small parameter have made perturbation method an interesting tool among the most frequently used approximate analytical methods [27-30]. Therefore, in this paper, the flow and rotation of micropolar fluids transported through porous channels with high mass transfer is studied using the regular perturbation method. The effect of material and microrotation constituent on the flow process is investigated.

2. MODEL DEVELOPMENT AND ANALYTICAL SOLUTION

Consider the laminar, incompressible and isothermal flow of a micropolar fluid through a channel with porous walls where fluid undergoes suction or injection with speed q. The channel wall are parallel to the x axis as described using Cartesian co-ordinate with width of distance 2h and located at a reference y = ±h. The formulation of the model development of the micropolarfluid is developed with respect to the above conditions following the assumptions that the fluid is incompressible, flow is steady and laminar. Also radiation heat transfer is negligible.

Following the assumptions, the governing equations of the channel flow are given as

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \quad (1)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+(\mu+\kappa)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\kappa\frac{\partial N}{\partial y} \quad (2)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+(\mu+\kappa)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\kappa\frac{\partial N}{\partial x} \quad (3)$$

$$\rho\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=-\frac{\kappa}{j}(\mu+\kappa)\left(2N+\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)+\frac{\mu_s}{j}\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}\right) \quad (4)$$

The governing equations expressed in Eqs. (1)-(4) include micro rotation or angular velocity and material parameters which direction is in the xy- plane consistent with other micropolar fluid studies. In this study, material parameters are taken as independent and constant.

$$u(x,\pm h)=0, v(x,\pm h)=\pm q,$$
$$N(x,\pm \mathrm{h})=-\mathrm{s}\left.\frac{\partial u}{\partial y}\right|_{x=\pm h} \quad (5)$$

Fluid flow is assumed symmetric about y=0

$$\frac{\partial u}{\partial y}(x,0)=v(x,0)=0 \quad (6)$$

The value of s depicts various flow situation of the micropolar fluid. When s=0 the microelement close to the porous wall surface are unable to rotate while when s=0.5 the microrotation is same as the fluid vorticity at the boundary. Similarly fluid injected or removed from the stream is depicted by the value of q. Given that suction is the condition when q>0 and injection is the situation when q<0. The governing equation is therefore simplified by including micropolar effects by assuming stream functions and micropolar to the Berman's similarity solution [26]:

$$\psi=-qxF(\eta) \quad (7)$$

$$N=\frac{qx}{h^2}G(\eta) \quad (8)$$

where

$$\eta=\frac{y}{h}, u=\frac{\partial\psi}{\partial y}=-\frac{qx}{h}F^{'}(\eta), v=-\frac{\partial\psi}{\partial x}=qF(\eta) \quad (9)$$

Dimensionless micropolar parameters and non-zero cross flow Reynolds number are introduced as

$$N_1=\frac{\kappa}{\mu}, N_2=\frac{v_s}{\mu h^2}, N_3=\frac{j}{h^2}, \mathrm{Re}=\frac{\rho q}{\mu}h \quad (10)$$

With the aid of Eqs. (7)-(10) the Eqs (1)-(4) may be reduced to ordinary nonlinear differential equations as stated below:

$$(1+N_1)\frac{d^4F}{d\eta^4}-N_1\frac{d^2G}{d\eta^2}-\mathrm{Re}\left(F\frac{d^3F}{d\eta^3}-\frac{dF}{d\eta}\frac{d^2F}{d\eta^2}\right)=0 \quad (11)$$

$$N_2\frac{d^2G}{d\eta^2}+N_1\left(\frac{d^2F}{d\eta^2}-2G\right)-N_3\,\mathrm{Re}\left(F\frac{dG}{d\eta}-G\frac{dF}{d\eta}\right)=0 \quad (12)$$

With the appropriate boundary conditions defined as

$$F(\pm 1)=1, \mathrm{F}^{'}(\pm 1)=0, G(\pm 1)=sF^{''}(\pm 1) \quad (13)$$

Symmetry of fluid flow through the porous channel is assumed therefore boundary condition takes the form

$$F(0)=F^{''}(0)=F^{'}(1)=0, \mathrm{F}(1)=1, \mathrm{G}(1)=\mathrm{sF}^{''}(1) \quad (14)$$

The regular pertubation method which is an analytical scheme for providing approximate solutions to the ordinary differential equations,is adopted in generating solutions to the coupled ordinary nonlinear differential equation .The flow and rotation series solution where ε is the small pertubation parameter, may be presented in the following form.

$$F=F_0+\varepsilon F_1+\varepsilon^2F_2+O(\varepsilon^3) \quad (15)$$

$$G=G_0+\varepsilon G_1+\varepsilon^2G_2+O(\varepsilon^3) \quad (16)$$

Substituting Eqs.(15) and (16) into (11) and selecting at the the terms of the same orders, yields

$$\varepsilon^0 : \frac{d^4 F_0}{d\eta^4} \tag{17}$$

$$\varepsilon^1 : \quad \frac{d^4 F_1}{d\eta^4} + N_1 \frac{d^4 F_0}{d\eta^4} + \mathrm{Re}\frac{dF_0}{d\eta}\frac{d^2 F_0}{d\eta^2} - \frac{d^3 F_0}{d\eta^3} F_0 - N_1 \frac{d^2 G_0}{d\eta^2} \tag{18}$$

$$\varepsilon^2 : \quad \frac{d^4 F_2}{d\eta^4} + \mathrm{Re}\frac{dF_0}{d\eta}\frac{d^2 F_1}{d\eta^2} + \frac{d^2 F_0}{d\eta^2}\frac{dF_1}{d\eta} - \frac{d^3 F_0}{d\eta^3} F_1 - \frac{d^3 F_1}{d\eta^3} F_0 + N_1 \frac{d^4 F_1}{d\eta^4} - N_1 \frac{d^2 G_1}{d\eta^2} \tag{19}$$

Substituting Eqs.(15) and (16) into (12) and selecting at the various orders yields

$$\varepsilon^0 : \frac{d^2 G_0}{d\eta^2} \tag{20}$$

$$\varepsilon^1 : \quad \frac{d^2 G_1}{d\eta^2} + N_1 \frac{d^2 F_0}{d\eta^2} - 2\frac{G_0}{N_2}\eta + N_3 \mathrm{Re}\frac{dF_0}{d\eta} G_0 - F_0 \frac{dG_0}{d\eta}\frac{1}{N_2} \tag{21}$$

$$\varepsilon^2 : \quad \frac{d^2 G_2}{d\eta^2} + N_1 \frac{d^2 F_1}{d\eta^2} - 2\frac{G_1}{N_2} + N_3 \mathrm{Re}\, G_1 \frac{dF_0}{d\eta} + G_0 \frac{dF_1}{d\eta} - F_0 \frac{dG_1}{d\eta} - F_1 \frac{dG_0}{d\eta}\frac{1}{N_2} \tag{22}$$

The boundary conditions for the leading order equation is given as

$$F_0(0) = F_0''(0) = F_0'(1) = 0, \mathrm{F}_0(1) = 1, \mathrm{G}_0(0) = G_0(1) = 0 \tag{23}$$

With the aid of the boundary conditions Eq. (23) it could be expressed easily that Eq. (17) and (20) can be shown as

$$F_0 = -\frac{\left(\eta\left(\eta^2 - 3\right)\right)}{2} \tag{24}$$

Eqs. (24),(27)and (30) are substituted back into the series solution Eq. (15) The flow profile solution is expressed in its final form as

$$G_0 = 0 \tag{25}$$

$$F_1(0) = F_1''(0) = F_1'(1) = 0, \mathrm{F}_1(1) = 1, \mathrm{G}_1(0) = G_1(1) = 0 \tag{26}$$

With the aid of the boundary conditions in Eq. (26), the solutions of Eqs. (18) and (21) can be shown as

$$F_1 = -\frac{\left(\mathrm{Re}\,\eta\left(\eta^2 - 1\right)^2\left(\eta^2 + 2\right)\right)}{280} \tag{27}$$

$$G_1 = \frac{N_1 \eta(\eta^2 - 1)}{2N_2} \tag{28}$$

The boundary conditions for the second order equation are given as

$$F_2(0) = F_2''(0) = F_2'(1) = 0, \quad \mathrm{F}_2(1) = 1, \mathrm{G}_2(0) = G_2(1) = 0 \tag{29}$$

Using the boundary conditions in Eq. (29), it can be easily shown that the solutions of Eq. (19) and (22) are

$$F_2 = \frac{\left(\mathrm{Re}\,\eta(\eta^2 - 1)^2 \begin{pmatrix} 9240N_1 - 703\mathrm{Re} + 4620N_1\eta^2 \\ -530\mathrm{Re}\,\eta^2 - 357\mathrm{Re}\,\eta^4 + 14\mathrm{Re}\,\eta^6 \end{pmatrix}\right)}{1293600} + \frac{N_1(\eta^2 - 1)}{40N_2} \tag{30}$$

$$G_2 = \frac{N_1 \eta(\eta^2 - 1)\begin{pmatrix} 3N_1\eta^2 - 7N_1 + 3N_3 \\ +\mathrm{Re} + 3N_3\mathrm{Re}\,\eta^2 \end{pmatrix}}{60N_2^2} + \frac{N_1 \mathrm{Re}(\eta^2 - 1)^2(\eta^2 + 2)}{280N_2} \tag{31}$$

Table: Comparison of Numerical and regular perturbation solution when $N_1=N_2=1$,,$N_3=0.1$ and Re=-1.

η	G(η)			
	NM [13]	Present work	NM [13]	Present work
0	0.0000	0.0000	0.0000	0.0000
0.05	0.0752	0.0749	-0.0202	-0.0214
0.1	0.1500	0.1495	-0.0401	-0.0424
0.15	0.2240	0.2232	-0.0595	-0.0629
0.2	0.2969	0.2959	-0.0780	-0.0824
0.25	0.3683	0.3671	-0.0954	-0.1006
0.3	0.4378	0.4365	-0.1113	-0.1172
0.35	0.5051	0.5035	-0.1256	-0.1319
0.4	0.5696	0.5680	-0.1378	-0.1445
0.45	0.6311	0.6295	-0.1477	-0.1544
0.5	0.6892	0.6876	-0.1550	-0.1615
0.55	0.7435	0.7420	-0.1592	-0.1654
0.6	0.7937	0.7922	-0.1601	-0.1658
0.65	0.8392	0.8379	-0.1572	-0.1623
0.7	0.8798	0.8787	-0.1503	-0.1545
0.75	0.9152	0.9143	-0.1388	-0.1423
0.8	0.9448	0.9442	-0.1225	-0.1251
0.85	0.9685	0.9681	-0.1009	-0.1027
0.9	0.9858	0.9856	-0.0736	-0.0746
0.95	0.9964	0.9963	-0.0401	-0.0405
1.00	1.0000	1.0000	0.0000	0.0000

Substituting Eqs. (24),(27) and (30) into the series solution Eq. (15) The rotation profile solution is give be expressed in its final form as

$$F=-\frac{\left(\eta\left(\eta^2-3\right)\right)}{2}-\frac{\left(\mathrm{Re}\,\eta\left(\eta^2-1\right)^2\left(\eta^2+2\right)\right)}{280}+\frac{N_1(\eta^2-1)}{40N_2}+\frac{\left\{(\mathrm{Re}\,\eta(\eta^2-1)^2\begin{pmatrix}9240N_1-703\,\mathrm{Re}+4620N_1\eta^2\\ -530\,\mathrm{Re}\,\eta^2-357\,\mathrm{Re}\,\eta^4+14\,\mathrm{Re}\,\eta^6\end{pmatrix}\right\}}{1293600} \quad (32)$$

Also, after substituting Eqs. (25),(28) and (31) into the series solution Eq. (16). The rotation profile solution is give be expressed in its final form as

$$G=\frac{N_1\eta(\eta^2-1)}{2N_2}+\frac{N_1\eta(\eta^2-1)\begin{pmatrix}3N_1\eta^2-7N_1+3N_3\\ +\mathrm{Re}+3N_3\,\mathrm{Re}\,\eta^2\end{pmatrix}}{60N_2^2}+\frac{N_1\,\mathrm{Re}(\eta^2-1)^2(\eta^2+2)}{280N_2} \quad (33)$$

3. RESULTS AND DISCUSSION

The result obtained from the analytical solutions is discussed here, where effect of parameters on flow and rotation is reported graphically. The effect of micropolar fluid parameters at various values on the velocity and rotation profile is presented.

The Figure 1 shows the effect of the Reynolds number (Re) on velocity profile. It can be depicted that the velocity distribution decreases as Re increases when fluid is undergoing suction and during injection the velocity profile increases for increasing values of Re.

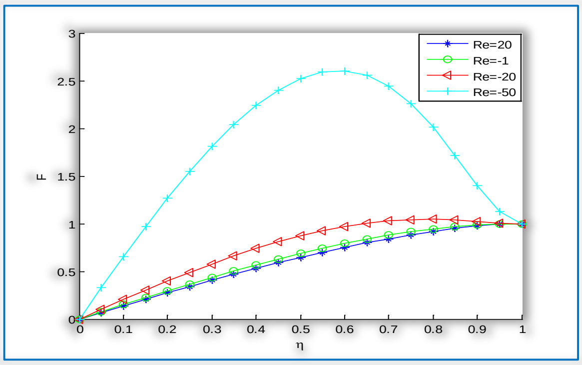

Figure 1. Effect of Reynold's number (Re) on velocity profile when $N_1=N_2=1$ and $N_3=0.01$.

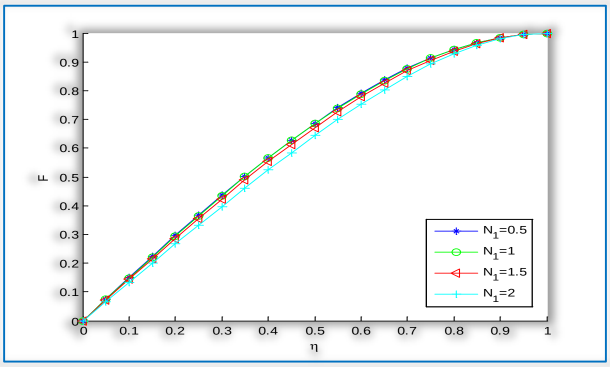

Figure 2. Effect of micro rotation parameter, N_1 on velocity profile when $-Re=N_2=1$ and $N_3=0.01$.

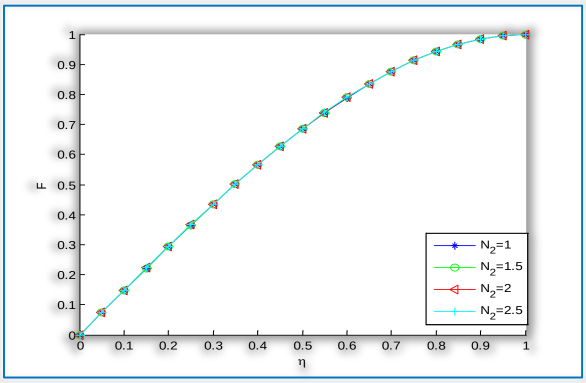

Figure 3. Effect of micro rotation parameter, N_2 on velocity profile when $-Re=N_1=1$ and $N_3=0.01$.

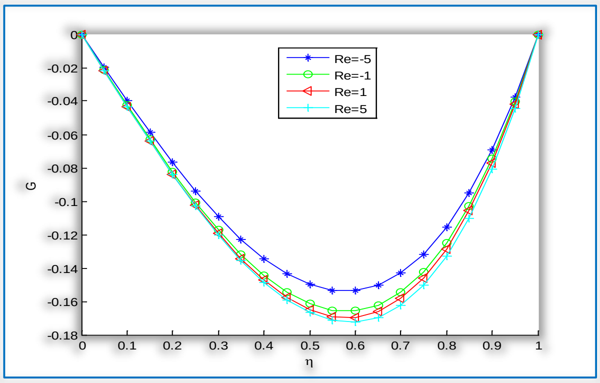

Figure 4. Effect of Reynolds number, Re on rotation profile when $N_1=N_2=1$ and $N_3=0.01$.

Figure 2 shows the effect of microrotation parameter (N_1). From the figure, increasing values of N_1 parameter the velocity profile decreases slightly which is as a result of an increase in rate of shear at the wall causing a decrease in boundary layer thickness. Effect of the microrotation parameter (N_2) on the velocity profile is depicted in Figure 3. The result shows a slight increase in velocity distribution at increasing values of N_2 parameter due to increase in momentum boundary layer thickness near the porous wall.

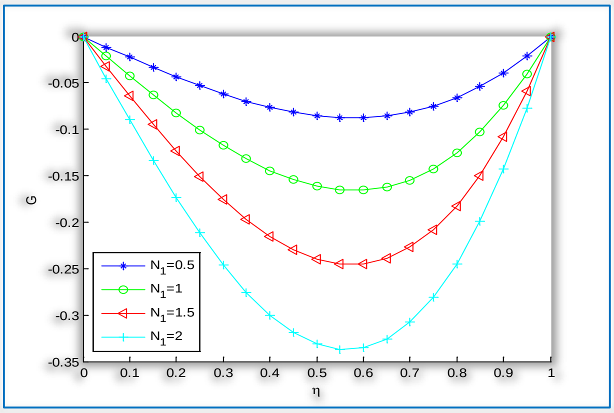

Figure 5. Effect of micro rotation parameter, N_1 on rotation profile when –Re =N_2=1 and N_3=0.01.

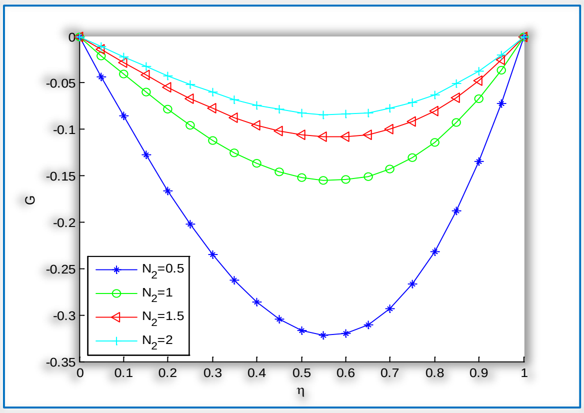

Figure 6. Effect of micro rotation parameter, N_2 on rotation profile when –Re =N_1=1 and N_3=0.01.

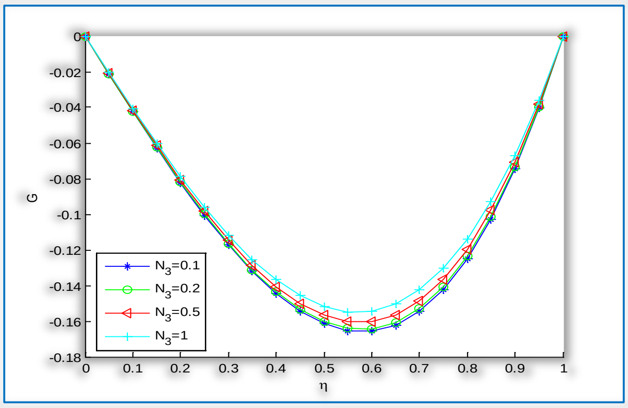

Figure 7. Effect of micro rotation parameter, N_3 on rotation profile when –Re =N_1 =N_2=1

Figure 4 shows the effect of Reynolds number (Re) on rotation profile. It could be seen from the figure thatat increasing values of Re rotation distribution decreases up till point η = 0.6 (not accurately determined) for suction, thereafter rotation distribution increases for increasing values of Re during injection. This can be physically explained that at increasing Re minimum point of micropolar fluid rotation is still retained at the origin. As micro rotation parameter N_1 increases for suction flow the rotation profile decreases till η=0.56 (not accurately determined) then the reverse is the case for injection as depicted in Figure 5 illustrating there is an increase from suction to injection. During suction flow at increasing values of micro rotation parameter N_2 it is shown from the Figure 6 that rotation profile increases for suction thereafter reduces during injection. Also the effect of microrotation parameter N_3 on rotation profile is seen in Figure 7. As it is observed, increasing values of N_3 parameter shows an increasing rotation distribution for suction till point η=0.6 (not accurately determined). Thereafter rotation distribution decreases for injection flow.

4. CONCLUSION

In this work, the flow of a micropolar fluid conveyed through porous channel driven by suction or injection with high mass transfer has been analyzed using the regular perturbation method. The developed analytical solutions are used to investigate the effects of flow and rotation parameters such as Reynolds number and micro rotation parameters. The results obtained can be used to advance the study of micropolar fluid in processes such as blood flow, turbulent shear flow, micro channel and porous channel.

Nomenclature

F	dimensionless streamfunction	s	microrotation boundary condition
G	dimensionless microrotation	u,v	Cartesian velocity components (ms^{-1})
H	width of channel (m)	x,y	Cartesian coordinate parallel and normal to channel (m)
j	micro-inertia density	η	dimensionless normal distance
N	microrotation/angular velocity (S^{-1})	μ	dynamic viscosity ($kgm^{-1}s^{-1}$)
$N_{1,2,3}$	dimensionless parameter	κ	coupling coefficient ($kgm^{-1}s^{-1}$)
p	embedding parameter	ρ	fluid density (kgm^{-3})
q	mass transfer parameter(ms^{-1})	ψ	stream function (m^2s^{-1})
Re	Reynolds number	υ_s	microrotation / spin gradient viscosity (m kg s-)

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