[1.]Prachi JAYASWAL, [2.]Anant PRASAD

HYBRID LUMP-INTEGRAL MODEL FOR THE FREEZING AND MELTING OF A BATH MATERIAL ONTO A HIGH-MELTING TEMPERATURE PLATE ADDITIVE OF NEGLIGIBLE THERMAL RESISTANCE

[1.] National Institute of Technology Jamshedpur, Metallurgical and Materials Engineering, INDIA
[2.] National Institute of Technology Jamshedpur, Mechanical Engineering, INDIA

Abstract: Reduction in the time of unavoidable freezing and melting of the bath material onto an additive that occurs as soon as it is immersed in the hot metal bath during the melt preparation of desired composition for manufacture of steel and cast iron of different grades, is essential in order to decrease their production time, cost and environmental impact for global competitiveness. With this objective, a hybrid non-dimensional lump-integral model is devised for this event of freezing and melting of the bath material around a high melting temperature plate additive of negligible thermal resistance. It exhibits that this event is regulated by non-dimensional independent parameters, the modified Conduction factor,C_{ofm} and the Stefan number, St_b. The series solutions for short times and the numerical solutions for all times are found. It is predicted that the total time for such an event decreases with decrease in C_{ofm}, but for a given C_{ofm}, it remains the same for all St_b investigated. Moreover, there is a startling feature of the total time of freezing and melting, τ_t , equal the modified Conduction factor, C_{ofm} . If the bath is at the freezing temperature of the bath material, only freezing occurs. Here, the maximum frozen layer thickness, ξ_{max} , becomes equal to $\left(1+\frac{1}{St_b}\right)$.
Keywords: melt bath-additive system, freezing and melting, mathematical modeling, additive addition

1. INTRODUCTION

Due to global aggressive market competition for steel and cast iron of different grades, steel makers need to manufacture them at low cost, with reduced energy consumption and high productivity and without degrading their quality and impacting adversely the environment. To produce them, their melts are first prepared by dunking and assimilating the additives in hot metal bath and then treating by passing them through designated metallurgical processes route. The time taken for this is called production time, which needs to be reduced for increasing their productivity but without altering processes involved for global competitiveness. Here, the undesirable freezing and melting of the bath material onto additive soon after its plunging in the bath occurs during the melt preparation for their production. It is due to the high temperature gradient developed on the additive side resulting in requirement of conductive heat far greater than the available bath convective heat. The excess conductive heat is balanced by the latent heat of fusion generated from the freezing of the bath material onto the additive. Elapsing time reduces the additive side temperature gradient and the associated conductive heat till this heat equalises the convective heat of the bath. At this instant, the growth of the frozen layer ceases. After this time, conductive heat becomes less than the bath convective heat permitting the deficient conductive heat to be balanced by less convective heat. The excess convective heat is utilized in melting the frozen layer till it completely melts, exposing the additive at a raised temperature. As this undesirable happening is dependent upon the temperature and thermo-physical properties of the additive-bath system, the bath condition, geometry, shape and size of the additive; control of one or several of them can essentially decrease its time duration resulting in reducing the production time and increasing the productivity.

In view of the above facts, the current study concerns the freezing and melting of the bath material onto a high-melting temperature plate additive of negligible thermal resistance as compared with that of the bath. A hybrid non-dimensional lump-integral model for this event is developed. It exhibits the freezing and melting as functions of non-dimensional independent parameters, the modified Conduction factor, Cofm, related to bath convection and the phase-change parameter of the bath material, the Stefan number, Stb, and gives series solutions for short times and numerical solutions for all times for the freezing and melting and the associated build-up of the temperature in the additive. Their graphical representations indicate the startling features of the total time of the freezing and melting equal to the modified Conduction factor, Cofm, for a certain Stefan number, Stb, and; for a prescribed Cofm, this total time is the same for all Stb considered.

This investigation seldom appears in the literature. However, the freezing and melting of the bath material onto high melting temperature plate[1], cylindrical [2-5] and spherical [6-11] additives was studied when their thermal resistance was comparable with that of the bath. It was also investigated for them [12-14] when the thermal resistance of the frozen layer developed around them was negligible with respect to thermal resistance of these additives. Their solutions were in closed-forms. This situation was further investigated when the additive was a low-melting temperature cylindrical additive [15]. Here also, its solution was in closed-form. The effect of temperature dependent heat capacity on the plate additive was also examined on the freezing and melting of the bath material onto such a plate, when the bath was with [16] and without [17] agitation. They yielded the solutions in closed-forms. The instant interface temperature attained between the additive and the freezing layer at the time of dunking the additive in the bath, called θe at $\tau = 0^+$, influencing the development of the frozen layer was also found for high-melting temperature plate additive [18] and high [19] and low [20] melting temperatures cylindrical additive in closed-form.

2. MATHEMATICAL MODEL

Consider a thin plate additive of semi-thickness 'b', initially at a temperature, T_{ai}, less than its melting temperature, T_{am} .It is dunked in a hot melt bath maintained at a temperature, T_b, greater than the freezing temperature of bath material, T_{bm}. Immediately, on the surface of the plate additive , freezing begins, the additive gets heated and the interface formed between the plate surface and the freezing layer attains an equilibrium temperature, T_e , which lies between the initial temperature of the additive and the freezing temperature of the bath material.

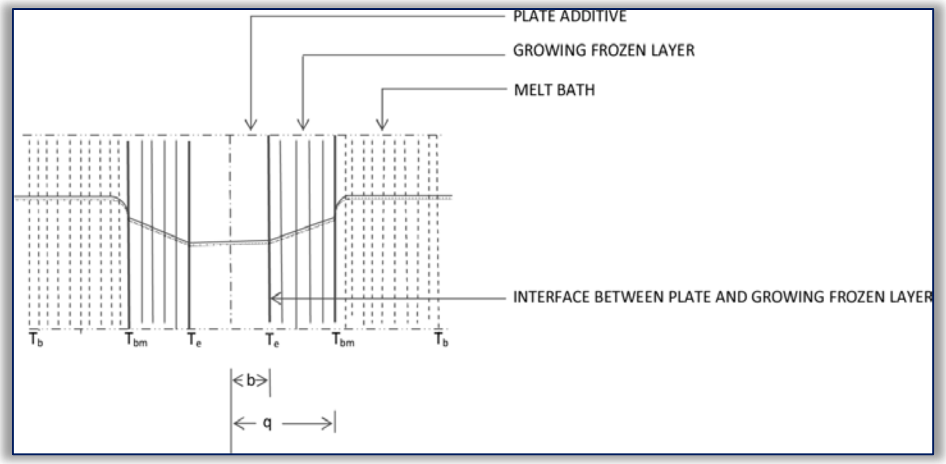

Figure 1: Schematic diagram of freezing and melting of bath material onto a high-melting temperature plate additive of negligible thermal resistance.

Moreover, the plate additive-bath system establishes a temperature field, Figure 1, $T_b>T_{bm}>T_e>T_{ai}$. As the time progresses, the frozen layer grows in thickness to reach a maximum value and then melts completely leaving the additive at an elevated temperature.

The additive is assumed not to react with the bath material and the freezing and melting events along with heating of the additive is regulated by transient one-dimensional heat conduction. The integral form of non-dimensional heat conduction equation controlling the freezing and melting can be cast as

$$\frac{d}{d\tau}\left(\int_{Cr}^{\xi_{bf}} \theta_{bf}\, d\xi_{bf}\right) - \left(\theta_{bf}\Big|_{\xi_{bf}=\xi_{bm}}\right)\frac{d\xi_{bm}}{d\tau} + \left(\theta_{bf}\Big|_{\xi_{bf}=Cr}\right)\frac{dCr}{d\tau} = \frac{\partial\theta_{bf}}{\partial\xi_{bf}}\Big|_{\xi_{bf}=\xi_{bm}} - \frac{\partial\theta_{bf}}{\partial\xi_{bf}}\Big|_{\xi_{bf}=Cr} \tag{1}$$

Its related initial and boundary conditions are

$$\xi_{bf} = Cr, \quad \theta_{bf} = \theta_b\,, \quad \tau = 0 \tag{2}$$

$$\xi_{bf} = Cr, \quad \theta_{bf} = \theta_e\,, \quad \tau > 0 \tag{3}$$

$$\frac{\partial\theta_{bf}}{\partial\xi_{bf}} = Bi_m(\theta_b - 1) + \frac{1}{St_b}\frac{d\xi_{bm}}{d\tau} = \frac{1}{C_{ofm}} + \frac{1}{St_b}\frac{d\xi_{bm}}{d\tau}; \;\; \xi_{bf} = \xi_{bm}, \;\; \theta_{bf} = 1, \quad \tau > 0 \tag{4}$$

The additive plate is of a high-melting temperature material and its thermal resistance is negligible with respect to convective thermal resistance of the bath and the conductive thermal resistance of the growing frozen layer. This feature sets up a uniform temperature in the entire volume of the plate which is equal to the contact interface temperature, θ_e , between the additive and the frozen layer, making the additive act as a lump.

Applying an energy balance to this lump, the heat conducted to the lump from the frozen layer through the contact interface and the increase in the thermal energy of the lump leads to:

$$\frac{d\theta_e}{d\tau} = -\frac{q_n}{B} \tag{5}$$

It is subjected to initial condition

$$\theta_e = 0, \tau = 0 \tag{6}$$

The interface coupling conditions between the additive and the frozen layer can be written as

$$\frac{\partial\theta_{bf}}{\partial\xi_{bf}} = -\frac{q_n}{B}\,, \theta_a = \theta_{bf} = \theta_e\,, \xi_{bf} = Cr\,, \;\; \xi_a = 1; \quad \tau > 0 \tag{7}$$

Note that the equations (1) to (7) recognize as the hybrid lump-integral model of the current problem. These equations are written on the assumptions of the thermo-physical properties of the additive-bath system uniform but different. The additive behaves as a lump whereas the frozen layer acts as an integral system in the direction of its growth. The equation (7) assumes the plate is in perfect contact with the frozen layer, with no interfacial resistance between them. Such assumptions were taken in the recent past investigations [1, 6, 12-13, 19-21] for the freezing and melting of the bath material onto the plate, cylindrical and spherical additives, giving realistic and reliable results. This non-dimensional model indicates the dependence of this problem upon independent non-dimensional parameters, the bath convection, represented by the modified Conduction factor, C_{ofm}; the phase-change parameter of the bath material, St_b; the property-ratio of the additive-bath system, B and the heat capacity ratio, Cr.

3. SOLUTIONS

The hybrid lump-integral model in non-dimensional form just devised for the current problem indicates that it is non-linear due to presence of moving boundary of the frozen layer, equation (4) and coupled due to conjugating conditions, equation (7). They forbid its closed-form solution, applying available exact methods of the literature. In such a situation, alternative semi-analytical methods need to be sought. One of them, known as the integral method, that gave closed-form expression associated with freezing and melting, freezing or melting in the past investigations and reduced similar other problems to initial value problems is employed. Here, the governing equation for the frozen layer has already been expressed in the integral format; equation (1). It is reduced to

$$\frac{d}{d\tau}\left(\int_{Cr}^{\xi_{bf}}\theta_{bf}d\xi_{bf}\right)-\frac{d\xi_{bm}}{d\tau}=\left.\frac{\partial\theta_{bf}}{\partial\xi_{bf}}\right|_{\xi_{bf}=\xi_{bm}}-\left.\frac{\partial\theta_{bf}}{\partial\xi_{bf}}\right|_{\xi_{bf}=Cr};\ Cr\le\xi_{bf}\le\xi_{bm};\tau>0 \quad (8)$$

when equations (3) and (4) are substituted.

To solve this equation, the prior knowledge of temperature field in the frozen layer is required. A linear temperature distribution which satisfies equations (3) and (4) is assumed.

$$\theta_{bf}=\theta_e+(1-\theta_e)\frac{(\xi_{bf}-Cr)}{(\xi_{bm}-Cr)} \quad (9)$$

Such a profile is justified because it yielded accurate results for the phase-change problems [1, 18-19, 22] in previous investigations. Combination of equations (8) and (9) results in

$$\frac{d}{d\tau}\left[\frac{(1+\theta_e)(\xi_{bm}-Cr)}{2}\right]-\frac{d\xi_{bm}}{d\tau}=\left.\frac{\partial\theta_{bf}}{\partial\xi_{bf}}\right|_{\xi_{bf}=\xi_{bm}}-\frac{(1-\theta_e)}{(\xi_{bm}-Cr)} \quad (10)$$

It is changed to

$$\frac{d}{d\tau}\left[\frac{(1+\theta_e)(\xi_{bm}-Cr)}{2}\right]-\frac{d\xi_{bm}}{d\tau}=\frac{1}{C_{ofm}}+\frac{1}{St_b}\frac{d\xi_{bm}}{d\tau}-\frac{(1-\theta_e)}{(\xi_{bm}-Cr)} \quad (11)$$

once, the boundary condition, equation (4) is employed. Substitution of equations (7) and (10), makes equation (5)

$$\frac{d\theta_e}{d\tau}=\frac{(1-\theta_e)}{(\xi_{bm}-Cr)} \quad (12)$$

when equation (9) is used.

The governing equation for the frozen layer, equation (11) and that of the additive, equation (12) are coupled due to the appearance of θe in them. For obtaining solutions, they are arranged in standard format of simultaneous first order ordinary differential equations in time τ. Here, using equation (12), equation (11) can be written as

$$\xi C_{ofm}(1+\theta_e-2St_{bm})\frac{d\xi}{d\tau}=2\xi-(1-\theta_e)(2+\xi)C_{ofm} \quad (13)$$

whereas, equation (12) takes the following format

$$\xi\frac{d\theta_e}{d\tau}=1-\theta_e \quad (14)$$

Here, $\xi=\xi_{bm}-Cr$ and $St_{bm}=\left(1+\frac{1}{St_b}\right)$.

Equations (13) and (14) form an initial value problem with the initial conditions, $\xi_{bm}-Cr=\xi=0$ and $\theta_e=0$ at time τ = 0. Their examination indicate that they do not lead to closed-form solutions nor their solutions are initiated employing the standard fourth order Runge-Kutta method , owing to the appearance of infinity at the initial conditions. To overcome such a difficulty, the series solutions for small times are found. From these, starting values of ξ and θe in the neighborhood of $\tau\to0$ (τ = 10-4) are obtained. Using these, the Runge-Kutta method is now capable of providing numerical values for ξ and θe for all subsequent times.

— Series solutions for small times

In order to find series solutions for small times that lie within the neighborhood of initial time τ=0, (τ=10-4) , the following series solutions are taken:

$$\xi=\sum_{i=1}^{n}a_i\tau^{i/2} \quad (15)$$

$$\theta_e=\sum_{i=1}^{n}b_i\tau^{i/2}\quad i=1,2,3,\ldots,n \quad (16)$$

They fulfill the initial conditions, $\xi=0,\ \theta_e=0$ at time $\tau=0$. To find the higher order coefficients of $\tau^{i/2}$ of equations (15) and (16), they are substituted in equations (13) and (14).

Using MATLAB, the coefficients of higher order of $\tau^{i/2}$ become respectively:

$$a_1 = \pm\sqrt{\frac{St_b}{St_b+2}};\ a_2 = \frac{4[(C_{ofm}-1)St_b - C_{ofm}]}{3C_{ofm}(St_b+2)};\ b_1 = \pm\sqrt{\frac{St_b+2}{St_b}};\ b_2 = \frac{(2-5C_{ofm})St_b - 4C_{ofm}}{6C_{ofm}St_b} \quad (17)$$

Higher order terms of the series above i=2 are not considered due to their negligible effects on ξ and θ_e.

— Numerical solutions for all times

As described, the starting values of ξ and θ_e in the vicinity of $\tau \rightarrow 0$ ($\tau = 10^{-4}$) are estimated from the above equations for small times. These are then applied to Runge-Kutta method to calculate numerical values of ξ and θ_e for all times.

As the time rate of increase in temperature of additive is faster than the time rate of development of the frozen layer with its subsequent melting, the additive attains the melting temperature of the bath material before the frozen layer completely melts. In such a situation, the temperature of the additive and the temperature of the melting front of the remaining frozen layer become at the melting temperature of the bath material. This permits the entire remaining frozen layer and the additive along with the interface between the additive and the frozen layer to be at the melting temperature of the bath material. Due to this, the bath convective heat only melts the remaining frozen layer, which is governed by

$$\left(\frac{-1}{St_b}\right)\frac{d\xi}{d\tau} = Bi_m(\theta_b - 1) = \frac{1}{BC_{of}} = \frac{1}{C_{ofm}} \quad (18)$$

Once θ_e=1 is substituted in equation (13), it readily gives a closed-form solution,

$$\xi = \xi_r - \frac{St_b}{C_{ofm}}(\tau - \tau_r) \quad (19)$$

satisfying the condition $\xi = \xi_r$ at $\tau = \tau_r$ where, ξ_r represents the thickness of the remaining frozen layer at time $\tau = \tau_r$ of the start of the above happening. It provides the total time of freezing and melting, once the frozen layer disappears $(\xi = 0)$.

$$\tau_t = \frac{C_{ofm}}{St_b}\xi_r + \tau_r \quad (20)$$

4. VALIDITY

In absence of freezing and melting of the bath material onto the plate additive, the bath convective heat only heats the additive, transforming the current problem to the transient heat conduction in the additive subjected to convective heat injection. Here, the term related to freezing and melting, $\frac{1}{St_b}\frac{d\xi_{bm}}{d\tau}$ vanishes; $\xi_{bm} = Cr$ and $\frac{(1-\theta_e)}{(\xi_{bm}-Cr)} = \frac{d\theta_e}{d\tau}$ {equation (12)} and the moving front coincides with the surface of the additive. Applying them, equation (11) becomes

$$\frac{d\theta_e}{d\tau} = \frac{1}{C_{ofm}} = Bi_m(\theta_b - 1) \quad (21)$$

It is transformed to

$$\frac{d\theta_e}{d\tau_h} = Bi(\theta_b - \theta_e) \quad (22)$$

Here, $\tau_h = \tau/B$ and $Bi = BBi_m$

Equation (22) provides the closed-form solution satisfying the initial conditions $\theta_e = 0$ at $\tau_h = 0$

$$1 - \frac{\theta_e}{\theta_b} -= e^{-Bi\tau_h} \quad (23)$$

It is exactly the same as reported in the literature [23-25] validating the present problem.

5. SPECIAL CASE: ONLY FREEZING

When the bath temperature is close to the freezing temperature of the bath material $(T_b \rightarrow T_{bm})$, the convective heat of the bath becomes zero {h (T_b-T_{bm}) = 0} and in turn C_{ofm}=∞. Its substitution along with the use of equation (14) makes equation (11)

$$\frac{d}{d\tau}\left[\frac{(1+\theta_e)(\xi)}{2} - (St_{bm})\xi + \theta_e\right] = 0 \quad (24)$$

where, $\xi = \xi_{bm} - Cr$ and $St_{bm} = \left(1 + \frac{1}{St_b}\right)$

It gives a closed-form solution

$$\frac{(1+\theta_e)(\xi)}{2} - (St_{bm})\xi + \theta_e = 0 \quad (25)$$

once, $\xi = 0$ and $\theta_e = 0$ at $\tau = 0$.

It leads to

$$\theta_e = \frac{\xi(2St_{bm}-1)}{(\xi+2)} \quad (26)$$

In case the latent heat of fusion generated due to freezing of the bath material which is conducted to the additive, raises its interface temperature to the freezing temperature of the bath material $(\theta_e = 1)$, no further freezing of the bath material takes place. This results in the development of maximum possible frozen layer thickness, ξ_{max} onto the additive. Applying this, equation (26) gives

$$\xi_{max}(2St_{bm} - 1) = (\xi_{max} + 2) \quad (27)$$

It leads to

$$\xi_{max} = St_{bm} \tag{28}$$

It is noted that if the bath temperature, $T_b > T_{bm}$, the growth of the maximum thickness of the frozen layer is always smaller than that found from equation (28). It is due to the part of the latent heat of fusion supplied to the additive being replaced by the bath convective heat.

Equation (28) can be also derived directly by applying an energy balance between the latent heat of fusion released by freezing of the bath material around the plate additive and the absorption of this heat to raise the temperature of the additive to the freezing of the bath material.

$$2A_s\rho_b(q_{max} - b)L = 2A_s\rho_a bC_{pa}(T_{bm} - T_{ai}) \tag{29}$$

In non-dimensional form, it is exactly the same as that of equation (28).

6. RESULTS AND DISCUSSIONS

A non-dimensional lump-integral model is evolved for the reduction in time of occurrence of unavoidable freezing and melting of the bath material onto a high-melting temperature plate additive of negligible thermal resistance during initial period of the melt preparation of requisite composition by assimilating this plate additive to manufacture steel and cast iron. This reduction decreases the production time and increases the productivity for global competitiveness. The model reveals that this event is dependent upon the bath condition represented by modified conduction factor, C_{ofm}, a product of the property-ratio of the additive-bath system 'B' and the conduction factor 'C_{of}' ($C_{ofm}=BC_{of}$) and; the phase change parameter of the bath material, denoted by the Stefan number, St_b. The latter is the ratio of the sensible heat and latent heat of the bath material. Its high value is representative of low latent heat of fusion, due to which, for the same bath convective heat, the frozen layer developed becomes of larger thickness. A low value of the property-ratio increases the driving bath thermal force to transfer large heat to the additive. The conduction factor, C_{of}, the ratio of the heat conducted to the plate additive, $k_a(T_{bm}-T_{ai})/b$, which arises owing to the temperature difference of the freezing temperature of the bath material and the initial temperature of the additive, and; the convective heat of the bath, $h\,(T_b-T_{bm})$. It ranges between 0 and infinity. Infinity denotes the bath to be at the freezing temperature of the bath material leading to non-availability of the convective heat from the bath. This results in only freezing of the bath material onto the additive. Zero corresponds to the additive heated at the freezing temperature of the bath material before it is immersed in the bath. It causes no conductive heat transfer to the additive, resulting in no frozen layer of the bath material onto it.

Shown in Figure 2 and Figure 4 are behavior of freezing and melting, ξ and the interface temperature θ_e. In each figure, the freezing and melting is parabolic whereas, the temperature θ_e rises quickly to attain the freezing temperature of bath material, $\theta_e=1$ and then remains at this temperature until the frozen layer completely melts. Here, the total time, τ_t, is the time from beginning of the freezing, with the growth of the frozen layer to its maximum value, and its melting, whereas, τ_{max}, is the time of development of the maximum thickness of the frozen layer represented by the time needed to grow this thickness. The difference between the total time and the time of the maximum growth of the frozen layer provides the time of melting, $\tau_m=\tau_t-\tau_{max}$. Moreover, an early part of any figure, representing freezing, is reached quickly the maximum frozen thickness, whereas its melting is slow with its later portion exhibiting a linear behavior with time. The later behavior is the result of the interface temperature, θ_e rapidly rising to the freezing temperature of the bath material before the complete melting of the frozen layer. In this situation, the remaining frozen layer including the interface temperature and the freezing front becomes at the freezing temperature of the bath material. This results in not allowing the convective heat to be absorbed by the frozen layer, rather, the convective heat only melts the frozen layer, which is provided by equation (20). It gives a linear relation between melting of the frozen layer and time.

— Effect of modified conduction factor, C_{ofm}

Displayed in Figure 2 are the time dependent freezing and melting and associated temperature rise for a prescribed value of the phase-change parameter, St_b. The modified conduction factor is assumed to be a parameter. For each of C_{ofm}, the behaviour of freezing and melting, ξ, and the interface temperature, θ_e, are similar to those described in the above section. However, increasing C_{ofm}, increases the total time of freezing and melting, the maximum frozen layer thickness and its time of formation. The interface temperature, θ_e, rises with a faster rate, to attain the freezing temperature, $\theta_e=1$, permitting not to melt completely the developed frozen layer. As stated earlier, the remaining frozen layer becomes at freezing temperature, $\theta_e=1$. This trait is expected due to the heat conducted to the additive is the sum of the convective heat and the latent heat of the fusion generated due to freezing of the bath material onto the additive. In earlier investigations also,[8, 26] , this behavior was found. Increasing C_{ofm}, decreases the convective heat, due to which more latent heat of fusion is required to be released which is affected by the growth of the larger thickness of the frozen layer.

Figure 3 relates the growth of the maximum frozen layer thickness, ξ_{max}, its time of formation, τ_{max}, and the total time of freezing and melting, τ_t, with modified Conduction factor, C_{ofm}, for a prescribed Stefan number, St_b. It is found that for smaller C_{ofm} ($C_{ofm}<100$), ξ_{max} and τ_{max} grow faster and as C_{ofm} increases beyond 100, they increase insignificantly.However their total time of freezing increaes almost linearly with increase in C_{ofm}.

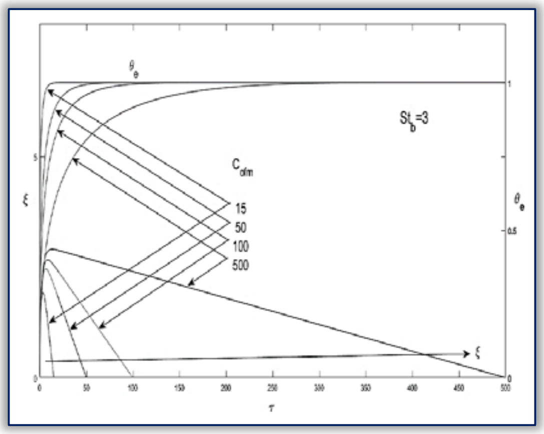

Figure 2 :Time variant freezing and melting,ξ, and the interface temperature, θ_e ,for various values of modified Conduction factor, C_{ofm}, for a given value of the Stefan number, St_b.

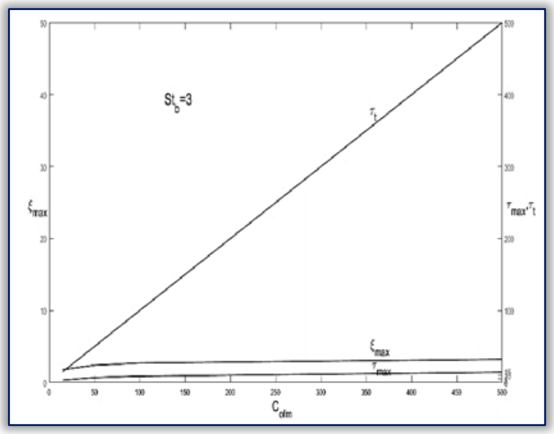

Figure 3 : Modified Conduction factor, C_{ofm}, dependent maximum frozen layer thickness, ξ_{max} , its growth time ,τ_{max} , and total time of freezing with its subsequent melting ,τ_t , for a specified value of Stefan number, St_b.

— Effect of Stefan number, St_b

Indicated in the Figure 4 are the freezing and melting of bath material and the associated interface temperature, θ_e with respect to time for a given modified Conduction factor, for different Stefan number, St_b. It is observed that for a given St_b the freezing and melting assumes a parabolic behaviour. Its apex represents the development of the maximum frozen layer thickness whereas time taken for this development is the time for its growth. Beyond the maximum frozen layer thickness the rest of the parabolic graph represents the melting of the frozen layer developed.This behaviour is same for all the St_b taken in this study, however, increasing St_b increases the size of the parabola, the maximum frozen layer thickness and the time taken to grow this thickness. But the apex of the parabola moves towards the greater time. The Figure 4 also provides a startling exhibit for taking the same total time, equal to the modified Conduction factor (τ_t=C_{ofm}=100), for the freezing and melting irrespective of the change in the Stefan number from 1 to 5.

Variation of maximum frozen layer thickness, ξ_{max}, its development time, τ_{max}, and the total time of freezing and melting, τ_t at a prescribed value of C_{ofm} =100 are displayed in Figure 5 for different Stefan number, St_b. It indicates that although the time taken for the growth of maximum frozen layer thickness is almost linear, the rate of the maximum frozen layer thickness developed becomes larger at a greater time. The total time, however, remains the same with its value equal to the C_{ofm}, for which the Figure 5 is drawn.

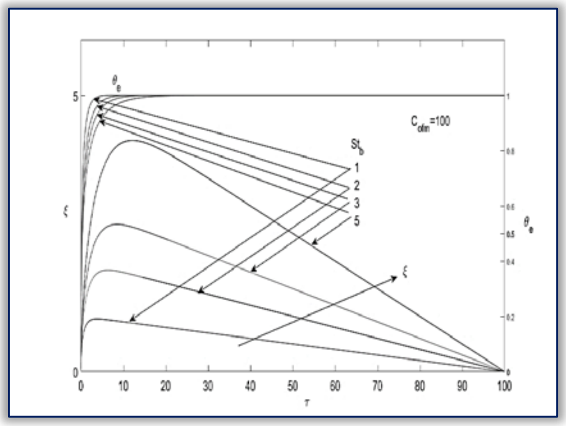

Figure 4: Time dependent freezing and melting,ξ , and the interface temperature, θe , for different Stefan number, St_b for a given value of the modified Conduction factor, C_{ofm} .

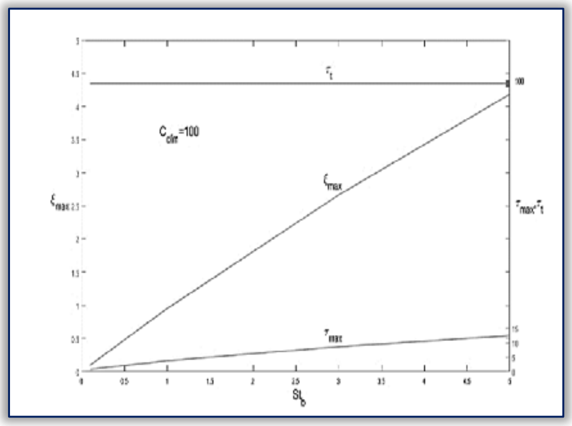

Figure 5: Variation of maximum frozen layer thickness, $\xi_{max,}$ its growth time, τ_{max} , and total time of freezing with subsequent melting, τ_t, with Stefan number, St_b, for a specified value of the modified Conduction factor, C_{ofm} .

7. APPLICATION OF CURRENT MODEL TO MANUFACTURING PROCESSES

The hybrid lump-integral model just presented has yielded closed-form solution for the freezing and melting of the bath material onto high-melting temperature plate additives, eah of which is of negligible thermal resistance. This solution is employed to foretell the time of freezing and melting of the steel bath material [11] onto the plate additives made of different materials, Table 1, used in manufacturing of steel and cast iron of different grades. Here, each of these plate additives is of 1mm semi-thickness.The freezing and melting of these materials plotted in the Figure 6 indicates that Chromium additive takes much less time than that of Molybednum for their same size.

Table 1. Thermo-physical Properties of Bath Material : STEEL BATH [11]

C_{pb}=0.670kJ/kg°C,ρ_b=6850kg/m3,k_b=35,T_{bm}=1531°C, T_b=1600°C, L=271.97kJ/kg, h=3500W/m2K, θ_b=1.03, Stb=3.71

Thermo-physical properties of high melting temperature plate-shaped solid additive of semi-thickness, b=1×10^{-3}m , Tai=25°C					Non-dimensional Parameters		
Additive	T_{am}(°C)	C_P(kJ/kg°C)	ρ(kg/m^3)	k(W/mK)	Bi	B	C_{ofm}
V	1910	0.494	5900	35	0.100	1.579	526.33
Mo	2623	0.251	10200	140.7	0.025	0.446	594.67
Cr	1830	0.448	7150	69.8	0.050	0.718	478.67

8. CONCLUSIONS

A hybrid non-dimensional lump-integral model for the current event, stated earlier, is evolved. It is functions of the modified Conduction factor, C_{ofm} and the Stefan number, St_b. For short times, series solutions for freezing and melting and associated rise in temperature are derived, whereas for all times numerical solutions for them are obtained. The total time of freezing and melting is equal to C_{ofm} (τ_t=C_{ofm}), for a given St_b, whereas, for a given C_{ofm}, this time remains unaltered for all the St_b considered. This predicts that for increasing productivity for a given bath material, C_{ofm} needs to be decreased and in turn, the bath convective heat is to be increased.

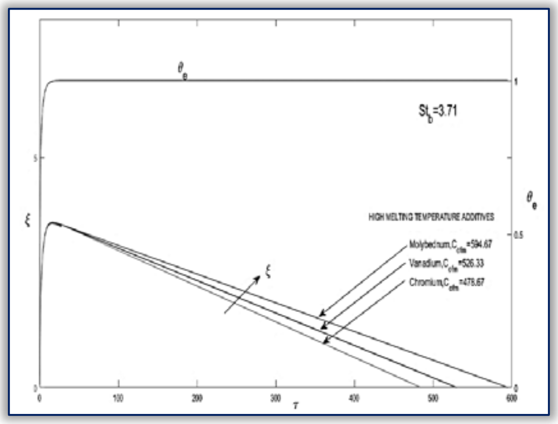

Figure 6 : Time dependent freezing and melting of steel bath material (St_b=3.71) onto the high-melting temperature additives often used in steel manufacturing

Nomenclature & Greek Letters

A_s	Surface area of the additive (m^2)
b	Significant thickness,(m)
B	Property ratio, C_bk_b/C_ak_a
Bi	Biot number, hb/k_b
Bim	Modified Biot number, (hb/k_b)(C_ak_a)/(C_bk_b)
C_a	Heat capacity of the additive, ρ_aC_{Pa}, (Jm^{-3}K^{-1})
C_b	Heat capacity of the bath material, ρ_bC_{Pb}, (Jm^{-3}K^{-1})
Cof	Conduction factor, (k_a/hb)(T_{bm}-T_{ai})/(T_b-T_{bm})
C_{ofm}	Modified conduction factor, BC_{of}
C_{pa}	Specific heat capacity of the additive, (Jkg^{-1}K^{-1})
C_{pb}	Specific heat capacity of the bath material, (Jkg^{-1}K^{-1})
Cr	Heat capacity ratio of the additive-bath system, C_b/C_a
H	Convective heat transfer coefficient of the bath, (Js^{-1}m^{-2}K^{-1})
k_a	Thermal conductivity of the additive, (Js^{-1}m^{-1}K^{-1})
k_b	Thermal conductivity of the bath material , (Js^{-1}m^{-1}K^{-1})
L	Latent heat of fusion of the bath material (Jkg^{-1})
q	Moving frozen layer front measured from the central axis of the additive at any time,(m)
q_a	Heat flux injection to the additive (Js^{-1}m^{-2})
q_n	Dimensionless growing frozen layer front at any time t, q_ab/(k_a(T_{bm}-T_{ai}))
St_b	Stefan number of the bath material, C_{Pb}(T_{bm}-T_{ai})/L
t	Time measured from the initiation of the events , (s)
T_a	Temperature of the additive,(K)
T_{ai}	Initial temperature of the additive,(K)
T_{am}	Melting temperature of the additive,(K)
T_b	Temperature of the bath, (K)
T_{bf}	Temperature of the frozen layer at any time, (K)
T_{bm}	Freezing temperature of the bath material,(K)
T_e	Thermal equilibrium temperature at the interface,(K)
α_a	Thermal diffusivity of the additive, k_a/C_a, (m^{2}s^{-1})
ξ_{bf}	Dimensionless moving frozen layer front measured from the central axis of the additive at any time, (C_b/C_a)(q/b)
ξ	Dimensionless moving frozen layer front measured from interface at any time, $\xi_{bf} - \mathrm{Cr}$
ρ_a	Density of the additive material, (kgm^{-3})
ρ_b	Density of the bath material, (kgm^{-3})
θ_a	Dimensionless temperature of the additive, (T_a-T_{ai})/(T_{bm}-T_{ai})
θ_{ai}	Dimensionless initial temperature of the additive, (T_{ai}-T_{ai})/(T_{bm}-T_{ai})
θ_{am}	Dimensionless melting temperature of the additive, (T_{am}-T_{ai})/(T_{bm}-T_{ai})
θ_b	Dimensionless temperature of the bath, (T_b-T_{ai})/(T_{bm}-T_{ai})
θ_{bf}	Dimensionless temperature of the frozen layer at any time,(T_{bf}-T_{ai})/(T_{bm}-T_{ai})
θ_{bm}	Dimensionless freezing temperature of the bath material,(T_{bm}-T_{ai})/(T_{bm}-T_{ai})
θ_e	Dimensionless thermal equilibrium temperature at the interface,(T_e-T_{ai})/(T_{bm}-T_{ai})
τ	Diensionless time , $B\alpha_a t/b^2$

Acknowledgement
The first author thanks Prof. R. P. Singh, Head of Dept. of Met. & Mat. Eng., National Inst. of Tech. Jamshedpur, India, for allowing her to work on a project under the guidance of senior author of this paper.

References:

[1] R.P. SINGH AND A. PRASAD,'Integral model based freezing and melting of a melt material onto solid additive.', Mathl. Comput. Modeling, 2003, vol. 37,pp. 849-62.
[2] U.C.SINGH,'Ph.D. thesis 2014, Birla Institute of Technology, Mesra, Ranchi,India'.
[3] S.SANYAL, S.CHANDRA, S.KUMAR AND G.G.ROY,'An improved model of cored wire injection in steel melt', ISIJ, International, 2004,vol. 44, pp 1157-65.
[4] J.LI, G. BROOKS AND N.PROVATAS,'Kinetics of scrap melt in liquid steel',Metall.Trans.B, 2005,vol.36B, pp 293-02.
[5] L. PANDELAERS, F. VERHAEGHE, D. BARRIER, P. GARDIN, P. WOLLANTS AND B. BLANAIN,'Theoretical evolution of influence of convective heat transfer and original sample size on shell melting time during Titanium dissolution in secondary steel making',Ironmak. Steelmak., 2010, vol. 37, pp. 516-21.
[6] Q. JIAO AND N.J. THEMELIS, 'Mathematical modelingg of heat transfer during the melting of solid particles in liquid slag or melt bath.',Canadian Metall, Quarterly, 1993, vol. 32, No. 1 , pp. 75-83
[7] E. ROHMEN, T. BERGSTRON AND T.A.EANGH,'Thermal behavior of spherical addition to molten metals',INFACON, Trondheim, Norway, 1995, pp. 683-95
[8] R. KUMAR, S.CHANDRA AND A. CHATERJEE,'Kinetics of ferroalloy dissolution in hot metal and steel', Tata search, 1997, pp. 79-85.
[9] S. TANIGUCHI, M. OHMI, S. ISHIURA AND S. YAMAUCHI,'A cold model study of gas injection upon the melting rate of solid sphere in a liquid bath', Transactions ISIJ, 1983, vol. 23, pp. 565-70.
[10] S. TANIGUCHI, M. OHMI AND S. ISHIURA,'A hot model study of gas injection upon the melting rate of solid sphere in a liquid bath', Transactions ISIJ, 1983, vol. 23, pp. 571-75.
[11] B.K.LI, X.F. MA, X.R. ZHANG AND J.C.HE,'Mathematical model for melting processes of solid particles in melt bath.',Acta Metallurgica Sinica, 1999, vol. 12, No. 3, pp. 259-66.
[12] U.C. SINGH, A. PRASAD AND A. KUMAR,'A lump integral model for freezing and melting of a bath material onto a plate shaped solid additive in agitated bath',Acta Metallurgica Slovaca, 2013, vol. 19, No. 1, pp. 60-72.
[13] U.C. SINGH, A. PRASAD AND A. KUMAR, 'Freezing and melting of a bath material onto cylindrical solid additive in an agitatd bath', J. Min. Metall, Sect B-Metall, 2012, vol. 48(1) B, pp. 11-23.
[14] L. ZHANG,'Modeling on melting of sponge iron particles in iron bath.Steel Res., 1996,vol. 66(11), pp. 466-72
[15] S. PRASAD, A. PRASAD AND A. KUMAR, 'Study of Axi-symmetric freezing and melting of an agitated bath material around a low melting temperature cylindrical solid additive', Int. J. of Eng. ANNALS of Faculty Engineering Hunedoara,2018, Fascicule 2, XVI, pp. 53-66
[16] R.P. SINGH AND A. PRASAD, 'Freezing and melting of a bath material onto a negligible thermal resistance plate-additive having its heat capacity temperature dependent', 7th European Oxygen Steelmaking Conference, Trinec, Czech Republic, Sept 09-11, 2014

ISSN 1584 - 2665 (printed version); ISSN 2601 - 2332 (online); ISSN-L 1584 - 2665