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OPTIMIZATION AND PREDICTION OF COMPRESSIVE STRENGTH OF CONCRETE USING IBEARUGBULEM'S APPROACH

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Abstract: In this paper, a mathematical model is developed to predict and optimize the compressive strength of normal cement concrete using Ibearugbulem's optimization approach. The response function expressed as a multivariable function of the proportions of the component materials, which are water, ordinary Portland cement, sand and gravel is based on the establishment of spatial domain for each concrete mixture variable. The response function was developed within the defined spatial domain and was optimized using a variational approach. A total of twenty mix ratios were used to demonstrate the applicability of the present mathematical model. The first ten mix ratios were used to formulate the model while the remaining ten mix ratios were used as check points for the model validation. The formulated model was tested for reliability at 95% level of confidence using F-statistic and was found to be adequate. A visual basic program, based on the formulated model was written to speed up the process of selecting the proportions of the component materials corresponding to a desired compressive strength value and vice versa. The optimum value of compressive strength predictable by the model is 45.55N/mm^2 corresponding to mix ratios of 0.45:1.00:1.02:1.45 (water: cement: sand: granite).
Keywords: mathematic model; compressive strength; Ibearugbulem's optimization approach, variational approach; multivariable function

1. INTRODUCTION

The rate of structural failure and collapse of buildings in Nigeria in recent times is worrisome. These unfortunate developments have led to loss of many lives and damage of properties worth millions of Naira. The structural failure and the eventual collapse of most buildings may be attributed to improper concrete mixture proportioning (Chendo and Obi, 2015; Oloyede et al., 2010). Proper concrete mixture proportioning will go a long way to enhancing building performance in service and will also make concrete production cost effective. The increase in price of cement in recent times in Nigeria amid the present economic situation is a source of concern (THISDAY Newspaper, October 2016). This development has denied most citizens of Nigerian, the opportunity to afford their own shelter. There are existing regression models that can be used to model mix components interaction within the mixture matrix or spatial domain (Obam and Osadebe, 2006; Anyaogu and Ezeh, 2013; Onwuka and Sule, 2017; Ndububa and Osadebe, 2007). However, they cannot (most times) fit a set of mixes that had hitherto been carried out. They demand to have a predetermined set of mixes before the regression model can be formulated.

To overcome this challenge, Ibearugbulem et al. (2013) introduced a new regression approach that is capable of modeling a set of mixes that had already been carried out without having a predetermined number of mixes. The original work on the Ibearugbulem model is quite demanding to understand and to interpret the way the CC matrix was obtained is another daunting task. This shortcoming posed serious challenge to using this approach. Another issue with the approach is that regression space was open bounded making it an unconstrained optimization problem. This made it difficult to optimize the regression. In this paper, a mathematical model is developed to predict and optimize the compressive strength of normal cement concrete using Ibearugbulem's new regression function. A domain was provided for each mix component in the mixture. By so doing, the entire mixture space domain was defined. Within this defined spatial domain for the mixture matrix, the regression was optimized using the extremizing principles of calculus of variation. A computer program coded in Visual Basic language based on the formulated model was written to predict various mix ratios corresponding to the desired compressive strength value and vice versa. The optimization process is simple and straightforward.

2. MATERIALS AND METHODS

The cement used as binder for this study is ordinary Portland cement with properties conforming to BS EN 197, part 1, 2000. The water used was fresh, odourless and free from any kind of organic matters. The fine aggregate was granulated river sand and was obtained from Otamiri River in Imo State. It was washed and sundried to bone-dry state before it was used for concrete production. The grading and properties of fine aggregates were determined according to the requirements of BS 882, part 2, 1992. The granitic gravel used as coarse aggregate was obtained from Crushed Rock Industry in Port Harcourt, Rivers State. It was properly washed and sundried to bone-dry state. Mixing of the materials was done manually using spade and hand trowel. Three cubes were cast and cured for 28 days. After curing the cubes, they were crushed using a Universal Compression Machine in their saturated surface dry (SSD) state. Twenty mixes were used, which gave a total of 60 cubes. The mix ratios are as shown on Table 1. The first ten mix ratios were used to formulate the model while the remaining ten mix ratios were used as check points to validate the model.

Table 1: Mix ratios and their corresponding compressive strength values

Mix number	Water	Cement	Sand	Gravel	Stress	x1	x2	x3	x4
N1	0.6	1	2	4	28.59	0.0789	0.1316	0.2632	0.5263
N2	0.55	1	2	3	31.63	0.0840	0.1527	0.3053	0.4580
N3	0.5	1	1.5	1.5	37.78	0.1111	0.2222	0.3333	0.3333
N4	0.45	1	1	1.5	44.96	0.1139	0.2532	0.2532	0.3797
N5	0.58	1	2	3.5	30.67	0.0819	0.1412	0.2825	0.4944
N6	0.55	1	1.75	2.75	31.93	0.0909	0.1653	0.2893	0.4545
N7	0.5	1	1.5	2.75	31.56	0.0870	0.1739	0.2609	0.4783
N8	0.53	1	1.75	2.25	33.26	0.0958	0.1808	0.3165	0.4069
N9	0.48	1	1.5	2.25	33.41	0.0918	0.1912	0.2868	0.4302
N10	0.45	1	1.25	1.5	40.52	0.1071	0.2381	0.2976	0.3571
c1	0.58	1	2	3.75	29.19	0.0791	0.1364	0.2729	0.5116
c2	0.47	1	1.38	1.88	37.41	0.0994	0.2114	0.2918	0.3975
c3	0.53	1	1.63	2.13	34.96	0.1002	0.1890	0.3081	0.4026
c4	0.47	1	1.25	1.88	39.85	0.1022	0.2174	0.2717	0.4087
c5	0.56	1	1.88	3.13	30.00	0.0852	0.1522	0.2861	0.4764
c6	0.51	1	1.63	2.5	32.81	0.0904	0.1773	0.2890	0.4433
c7	0.5	1	1.5	2.13	34.15	0.0975	0.1949	0.2924	0.4152
c8	0.56	1	1.88	2.88	33.48	0.0886	0.1582	0.2975	0.4557
c9	0.48	1	1.38	2.13	34.37	0.0962	0.2004	0.2766	0.4269
c10	0.57	1	1.88	3.13	32.00	0.0866	0.1520	0.2857	0.4757

— Derivation of fundamental equation of the mathematical model

The mix component is four. It comprises of water, cement, sand and gravel (granite). The mix quantity (x_i) of each material in a particular mix ratio is determined by dividing the individual ratios (s_i) by the sum of the ratios (S). That is:

$$x_i = \frac{s_i}{S} \quad (1)$$

$$S = s_1 + s_2 + s_3 + s_4 \quad (2)$$

In this work, the spatial domain in which the model is restricted to are mix ratio domains given as:

$$s_{1\,min} \le s_1 \le s_{1\,max} \quad (3)$$

$$s_2 = 1 \quad (4)$$

$$s_{3\,min} \le s_1 \le s_{3\,max} \quad (5)$$

$$s_{4\,min} \le s_1 \le s_{4\,max} \quad (6)$$

From equation (1),

$$s_i = x_i \cdot S \quad (7)$$

Substituting equation (7) into equation (2) gives the sum of all the mix quantities to be unity as:

$$x_1 + x_2 + x_3 + x_4 = 1 \quad (8)$$

The response function to be adopted herein is a quadratic function of the component proportions given as:

$$\begin{aligned} y = {} & a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_1^2 \; a_6x_2^2 + a_7x_3^2 + a_8x_4^2 \; a_9x_1x_2 \\ & + a_{10}x_1x_3 + a_{11}x_1x_4 + a_{12}x_2x_3 + a_{13}x_2x_4 + a_{14}x_3x_4 \end{aligned} \quad (9a)$$

That is:

$$y = [x_i][a_i] \quad (9b)$$

Equation (9b) was used to obtain the array response equation for the set of mix ratios used in the formulation as:

$$[y^k] = [x_i^k][a_i] \quad (9c)$$

Where k denotes the mix number (or observation point number); $[a_i]$ is the coefficient vector, and $[x_i]$ is the shape function vector. They are:

$$[a_i] = [a_1 \; a_2 \; a_3 \; a_4 \; a_5 \; a_6 \; a_7 \; a_8 \; a_9 \; a_{10}]^T \quad (10)$$

$$[x_i] = [x_1 \; x_2 \; x_3 \; x_4 \; x_1^2 \; x_2^2 \; x_3^2 \; x_4^2 \; x_1x_2 \; x_1x_3 \; x_1x_4 \; x_2x_3 \; x_2x_4 \; x_3x_4]^T \quad (11)$$

Pre-multiplying both sides of equation (9c) with a weighting function (transpose of the shape function) for the set of mixes for the formulation gives the weighted response equation (WRE) as:

$$\left[x_i^{\ k}\right]^T\left[y^k\right]=\left[x_i^{\ k}\right]^T\cdot\left[x_i^{\ k}\right]\left[a_i\right] \qquad (12a)$$

This multiplication did not change the generality of the regression function as the weighting function can easily cancel out from both the left and right hand sides of equation (12a). It is clear from here that the approach used in the original work of Ibearugbulem model (Ibearugbulem et al., 2013) is weighted response approach (WRA).

The weighted response equation (equation (12a)) can be rewritten as:

$$[F]=[CC]\cdot[a_i] \qquad (12b)$$

Where the weighted response vector, F and CC matrix are defined as:

$$F=\left[x_i^{\ k}\right]^T\cdot\left[y^k\right] \qquad (13)$$

$$[CC]=\left[x_i^{\ k}\right]^T\cdot\left[x_i^{\ k}\right] \qquad (14)$$

In simpler words, [CC] is the matrix whose arbitrary element CC_{ij} is obtained by array multiplication of transpose of Column "i" with Column "j" of the shape function vector.

— Algorithm for optimization of the model

To optimize the response function (equation (9)), iteration principle was employed. Since there are four variables, three iterating factors ($e_1 = 0.001$, $e_2 = 0.001$ and $e_3 = 0.001$) were used. The constraints are as set in equation (2) to equation (7). Start the iteration with the first quantities, x_{1min}, x_{2min}, x_{3min} and x_{4min}. Substitute these quantities into equation (1) to get the first set of mix ratios,$^1[s_1, s_2, s_3$ and $s_4]$. Note: $^n[\ ]$ denotes n^{th} set.

Substitute the first quantities, x_{1min}, x_{2min}, x_{3min} and x_{4min} (That is: $^1[x_1, x_2, x_3$ and $x_4]$) into the response function. The first response is taken as y_m (optimum response). Now, add the iterating factors (e_1, e_2, and e_3) to the first set of quantities, that is, $x_{1min} + e_1$, $x_{2min} + e_2$ and $x_{3min} + e_3$ respectively, to obtain the second set of quantities, $^2[x_1, x_2$ and $x_3]$. Subtract their sum from unity (that is 1) to obtain $^2[x_4]$. Divide $^2[x_1, x_2, x_3$ and $x_4]$ by $^2[x_2]$ to get $2[s_1, s_2, s_3$ and $s_4]$.

These mix ratios, $^2[s_1, s_2, s_3$ and $s_4]$ must be subjected the constraints of equation (3) to equation (6). If they pass the tests then substitute them into the response function. The second response is compared with the first one. If it is more than the first one then it replaces it, if not the first one is retained as y_m. This procedure is continued within loop until all the possible combinations of the quantities have been used.

— Fitting the model with the mixes used herein

Table 1 contains the values of quantities of mix components, x_i. Ensure to normalize and approximate x_i at four decimal places such that condition of Equation 8 will not be violated. The summation of x_i in each mix ratio on Table 1, was ensured to be equal to unity (in accordance with equation (8)). The values of x_i on Table 1 were used to determine the shape function and weighted response. The transpose of the response of the first ten mix ratios is taken directly from Table 1 and is given as:

$$\left[y^k\right]=\left[28.59\ \ 31.63\ \ 37.78\ \ 44.96\ \ 30.67\ \ 51.93\ \ 31.56\ \ 33.26\ \ 33.41\ \ 40.52\right]$$

The shape function for the first ten mixes (mix numbers N1 to N10) is taken from Table 1 and substituted into equations (1) and (2). The transpose of the shape function is:

$$\left[x^k\right]^T=$$

$$\begin{vmatrix}
0.0789 & 0.0840 & 0.1111 & 0.1139 & 0.0819 & 0.0909 & 0.0870 & 0.0958 & 0.0918 & 0.1071 \\
0.1316 & 0.1527 & 0.2222 & 0.2532 & 0.1412 & 0.1653 & 0.1739 & 0.1808 & 0.1912 & 0.2381 \\
0.2632 & 0.3053 & 0.3333 & 0.2532 & 0.2825 & 0.2893 & 0.2609 & 0.3165 & 0.2868 & 0.2976 \\
0.5263 & 0.4580 & 0.3333 & 0.3797 & 0.4944 & 0.4545 & 0.4783 & 0.4069 & 0.4302 & 0.3571 \\
0.0062 & 0.0071 & 0.0123 & 0.0130 & 0.0067 & 0.0083 & 0.0076 & 0.0092 & 0.0084 & 0.0115 \\
0.0173 & 0.0233 & 0.0494 & 0.0641 & 0.0199 & 0.0273 & 0.0302 & 0.0327 & 0.0366 & 0.0567 \\
0.0693 & 0.0932 & 0.1111 & 0.0641 & 0.0798 & 0.0837 & 0.0681 & 0.1001 & 0.0823 & 0.0886 \\
0.2770 & 0.2098 & 0.1111 & 0.1442 & 0.2444 & 0.2066 & 0.2287 & 0.1655 & 0.1851 & 0.1276 \\
0.0104 & 0.0128 & 0.0247 & 0.0288 & 0.0116 & 0.0150 & 0.0151 & 0.0173 & 0.0175 & 0.0255 \\
0.0208 & 0.0256 & 0.0370 & 0.0288 & 0.0231 & 0.0263 & 0.0227 & 0.0303 & 0.0263 & 0.0319 \\
0.0416 & 0.0385 & 0.0370 & 0.0433 & 0.0405 & 0.0413 & 0.0416 & 0.0390 & 0.0395 & 0.0383 \\
0.0346 & 0.0466 & 0.0741 & 0.0641 & 0.0399 & 0.0478 & 0.0454 & 0.0572 & 0.0548 & 0.0709 \\
0.0693 & 0.0699 & 0.0741 & 0.0961 & 0.0698 & 0.0751 & 0.0832 & 0.0736 & 0.0823 & 0.0850 \\
0.1385 & 0.1399 & 0.1111 & 0.0961 & 0.1396 & 0.1315 & 0.1248 & 0.1288 & 0.1234 & 0.1063
\end{vmatrix}$$

The shape function and its transpose were substituted into equation (14) to obtain CC matrix. This CC matrix as obtained was copied from Microsoft Excel worksheet and pasted on Microsoft word page to discharge inherent formulas and approximate the values to enable it have acceptable inverse. In the same manner, the transpose of the shape function and the response vector from the first ten mixes were Substituted into equation (13) to obtain the weighted response vector. The CC matrix and the weighted response vector are respectively presented as:

$$\text{CC matrix} = \begin{vmatrix} 0.0902 & 0.1789 & 0.273 & 0.4005 & 0.0088 & 0.0354 & 0.0796 & 0.1735 & 0.0176 & 0.0262 \\ 0.1789 & 0.3576 & 0.5354 & 0.7784 & 0.0176 & 0.072 & 0.1561 & 0.3338 & 0.0354 & 0.0519 \\ 0.273 & 0.5354 & 0.8402 & 1.2399 & 0.0262 & 0.1035 & 0.2461 & 0.5424 & 0.0519 & 0.0796 \\ 0.4005 & 0.7784 & 1.2399 & 1.9 & 0.0377 & 0.1466 & 0.3583 & 0.8503 & 0.074 & 0.1153 \\ 0.0088 & 0.0176 & 0.0262 & 0.0377 & 0.0009 & 0.0036 & 0.0077 & 0.0161 & 0.0018 & 0.0026 \\ 0.0354 & 0.072 & 0.1035 & 0.1466 & 0.0036 & 0.0151 & 0.0302 & 0.0613 & 0.0073 & 0.0103 \\ 0.0796 & 0.1561 & 0.2461 & 0.3583 & 0.0077 & 0.0302 & 0.0726 & 0.1558 & 0.0152 & 0.0234 \\ 0.1735 & 0.3338 & 0.5424 & 0.8503 & 0.0161 & 0.0613 & 0.1558 & 0.3865 & 0.0312 & 0.0496 \\ 0.0176 & 0.0354 & 0.0519 & 0.074 & 0.0018 & 0.0073 & 0.0152 & 0.0312 & 0.0036 & 0.0051 \\ 0.0262 & 0.0519 & 0.0796 & 0.1153 & 0.0026 & 0.0103 & 0.0234 & 0.0496 & 0.0051 & 0.0077 \end{vmatrix}$$

$$[F] = \begin{vmatrix} 32.9879 \\ 65.5172 \\ 99.4572 \\ 146.3477 \\ 3.2116 \\ 13.0244 \\ 28.9432 \\ 63.4054 \\ 6.4404 \\ 9.5465 \\ 13.7894 \\ 18.9335 \\ 27.1190 \\ 42.0340 \end{vmatrix}$$

Substituting the CC matrix and the weighted response vector obtained hitherto into equation (12b) and solving the equation gave the coefficient vector of the model as:

$$\begin{aligned}[a_i] = [&-3441.961158\ -1506.084391\ 1684.488411\ -535.4056549\ 4657.104546 \\ &3742.870874\ -2344.587029\ 544.8332977\ 601.2803984\ 8481.879105 \\ &1033.220638\ -2477.768341\ 2889.420326\ -613.8609848]^T\end{aligned} \tag{15}$$

3. RESULTS

Substituting the model coefficients into equation (9a) gives the response function for the mix ratios used herein as:

$$\begin{aligned} y = &-3441.961158x_1 - 1506.084391x_2 + 1684.488411x_3 - 535.4056549x_4 + 4657.104546x_1^2 \\ &+ 3742.870874x_2^2 - 2344.587029x_3^2 + 544.8332977x_4^2 + 601.2803984x_1x_2 + 8481.879105x_1x_3 \\ &+ 1033.220638x_1x_4 - 2477.768341x_2x_3 + 2889.420326x_2x_4 - 613.8609848x_3x_4 \end{aligned} \tag{16}$$

— Visual Basic program for prediction and optimization of the developed model

The visual basic program in accordance to the algorithm of section 2.2 and equation (16) was invoked to select the best mix ratios corresponding to a particular desired compressive strength value and vice versa.

— Test of adequacy of the model

The predicted compressive strength values for the control mixes as obtained from the program are presented on Table 2. They were compared with the results from the laboratory (as shown on Table 2) using F-statistics test at 95% level of confidence.

Table 2: F-statistic test of compressive strength model based on Ibearugbulem's new regression function

Control point	y_o	y_p	$y_o - \bar{y}_o$	$y_p - \bar{y}_p$	$(y_o - \bar{y}_o)^2$	$(y_p - \bar{y}_p)^2$
C1	29.19	29.95	-4.632	-3.401	21.455	11.567
C2	37.41	36.16	3.588	2.809	12.874	7.890
C3	34.96	34.46	1.138	1.109	1.295	1.230
C4	39.85	37.38	6.028	4.029	36.337	16.233
C5	30	31.07	-3.822	-2.281	14.608	5.203
C6	32.81	32.39	-1.012	-0.961	1.024	0.924
C7	34.15	34.18	0.328	0.829	0.108	0.687
C8	33.48	31.97	-0.342	-1.381	0.117	1.907
C9	34.37	34.84	0.548	1.489	0.300	2.217
C10	32	31.11	-1.822	-2.241	3.320	5.022
	33.822	33.351			91.437	52.880

y_o , y_p = Observed and predicted value of compressive strength respectively

$$\bar{y}_o = \frac{\Sigma y_o}{n}; \quad \bar{y}_p = \frac{\Sigma y_p}{n}$$

From Table 2,

$$S_o^{\,2} = \frac{91.437}{9} = 10.1597067$$

$$S_p^{\,2} = \frac{52.880}{9} = 5.87556556$$

The F-statistic is given by:

$$F = \frac{10.1597067}{5.87556556} = 1.72915$$

From standard statistical table, $F_{0.95} = (9,\ 9) = 3.179$.

The calculated value of F (1.72915) is less than the F-value (3.179) obtained from standard statistical table. The model is therefore adequate for the prediction and optimization of compressive strength of normal weight concrete.

4. DISCUSSION OF RESULTS AND CONCLUSION

A mathematical model has been developed to predict and optimize the compressive strength of normal weight concrete based on Ibearugbulem's new regression function. The formulated model was tested for reliability at 95% confidence level and was found to be adequate. A short Visual Basic program was written to estimate the optimum compressive strength value and optimum mix ratios through an iterative technique. This program predicts the mix ratios when the compressive strength is known and, is also capable of predicting the compressive strength when the mix ratios are known. The optimum value of compressive strength predictable by the model is 45.55N/mm^2 corresponding to mix ratios of 0.45:1.00:1.02:1.45 (water: cement: sand: granite).

The Visual Basic program is user-friendly and can predict with reasonable accuracy, the optimum value of compressive strength and the corresponding mix ratios.

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