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ESTIMATION OF WATER TABLE FLUCTUATIONS IN SLOPING AQUIFER SUBJECTED TO TRANSIENT RECHARGE AND SEEPAGE FROM ADJACENT STREAM

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Abstract: Approximation of water table fluctuations in response to stream-stage variations and downward recharge is a challenging task owing to continuously varying hydrological conditions of the surroundings. In this work, a stream-aquifer model is developed and solved analytically to predict the spatio-temporal variations of water table in an unconfined sloping aquifer subjected to seepage from a time dependent stream level and transient downward recharge. To simulate the subsurface seepage problem, Boussinesq equation with moving boundary conditions is used and solved analytically to obtain closed form expressions for water head and flow rate. The results obtained in this study are illustrated with an example indicating significant built up of the water table in the middle part of the aquifer as a consequence of continuous recharge.
Keywords: Sloping aquifers, Streams, Laplace transform, Boussinesq equation

1. INTRODUCTION

Estimation of transient water interactions between streams and aquifers has gained tremendous importance in past few decades for its central role in catchment hydrology. Estimation is also useful in designing controlled replenishment activity, and analyses aquifer's responses to withdrawal and recharge activity. Mathematical models are one of the important tools that helped understand complex interplay between dynamic boundary conditions, aquifer parameters and spatio-temporal variations in the water table.

Various analytical and numerical studies have been carried out to estimate water exchange between stream-aquifer systems under diverse boundary conditions. Theis (1941) was among the first few researchers who studied unsteady behavior of surface-ground interaction due to pumping effect. The significant developments related to fluctuations of water table due to constant or time varying recharge were mainly happened in the time period between 1960 and 2000. Modeling techniques for subsurface seepage flow in confined as well as unconfined aquifers was introduced and analyzed by Polubarinova-Kochina (1962), Hantush (1965), Marino (1973), Lockington (1997), Hunt (1999) etc. Linearization of nonlinear partial differential equation for one dimensional seepage flow was presented by Marino (1973) in his mathematical models. Two diverse approximation techniques were studied by previous researchers for approximation of groundwater flow in unconfined aquifer: the first approach is based on Dupuit assumptions in which the streamlines are considered to be horizontal. The second approach is based on Dupuit–Forchheimer assumption, and accordingly, stream lines are nearly horizontal, and according to Dupuit–Forchheimer assumption streamlines are considered to be nearly parallel to the horizontal base. Both the aspects have been widely used in the past by the researchers such as Wooding and Chapman (1966); Childs (1971), Verhoest and Troach (2000), Verhoest et al. (2002), Upadhyaya and Chauhan (2001), Butler and Zlotnik (2001), Moutsopolous (2010) etc. to establish noteworthy results in groundwater hydrology. The majority of the existing investigations are focused on approximation of water head and flow rate in a horizontal aquifer when the adjacent stream-stage vary linearly. Some researchers have considered a sloping porous formation which is in contact with a uniformly rising water bodies. Many of these studies used Boussinesq's equation- a non-linear parabolic advection-diffusion partial differential equation- for approximation of surface-groundwater interaction. In these studies, both one-dimensional and two-dimensional domain has been considered. These studies reveal that the variation in the phreatic surface is mainly dependent upon sloping angle, adjacent stream variation rate and the rate of replenishment. Most of the existing models have inherent restrictive assumption that either the aquifer base is completely horizontal or the stream stage variation is linear/uniform. As a matter of fact, the bases of unconfined aquifers in sedimentary basins are often sloping, and the stream variations are non-uniform. Thus, applying the existing results on all variety of cases may underestimate or overestimate the actual scenario. Some researchers such as Bansal and Das (2011), Teloglou and Bansal (2012), Bansal (2012), Zlotnik and Kacimov (2017) have studied the variation of water table in presence of vertical stream bank with horizontal and sloping base. In most of these studies, the partial differential equation is solved by techniques like Laplace transform and Fourier transform. Numerical techniques like predictor corrector method, finite difference method, Adomian decomposition, perturbation method are also used to developed numerical models. The analysis presented by all these researchers indicates significant variations in groundwater table in variation of stream.

In this paper, a mathematical model is developed for estimation of water table fluctuations in response to combined effects of bed slope and non-uniform recharge and seepage. The geological system consists of an unconfined aquifer resting on a downward sloping and perfectly impervious bed. The water table variations are caused by seepage from an adjacent stream whose water level varies from a known initial value to a final value by a non-uniform rate. Moreover, the aquifer domain also receives a downward percolation at a time varying rate. Dupuit–Forchheimer assumption is used to formulate the subsurface seepage flow as a nonlinear Boussinesq equation. Laplace transform technique is used for solving the linearized version of the Boussinesq equation. Closed form analytical expressions are obtained to represent the water head in the aquifer. Flow rate at the stream–aquifer interface is also calculated. The results obtained in this study are demonstrated with an illustrative example. Sensitivity of various parameter is analyzed and plotted.

2. MATHEMATICAL FORMULATION AND ANALYTICAL SOLUTION

An unconfined downward sloping aquifer is resting on an impervious bed with downward slope tan β. The aquifer is hydrologically connected with a water body at one end in which a constant water level $h_0$is maintained throughout. The other end of the aquifer is connected with a stream whose level is rising from initial level h_L to a final level $h_0$by a known function of time t. Furthermore, the entire domain of the aquifer receives a downward transient recharge. The stream is assumed to penetrate full depth of the aquifer.

Following Chapman (1980), the equation of the groundwater flow in an unconfined sloping aquifer is

$$K\cos^2\beta\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)-K\sin\beta\cos\beta\frac{\partial h}{\partial x}+N(t)=S\frac{\partial h}{\partial t} \tag{1}$$

where h is the height of water table in the aquifer, K is the hydraulic conductivity of the porous medium, S is the specific yield of the aquifer and x is the horizontal x–axis. $N(t)$ is the transient downward recharge per unit of the aquifer, given by the following exponential function of time t

$$N(t)=N_0+N_1e^{-\lambda t}. \tag{2}$$

where λ is a positive constant characterizing the rate at which recharge decreases from an initial value N_0+N_1 to a final value N_0.

The initial water table in the aquifer is linear joining both saturated ends of the aquifer. If L denotes the length of the domain then initial conditions can be specified as

$$h(x,t=0)=h_0-(h_0-h_L)\frac{x}{L} \tag{3}$$

While a constant water head h_0 is maintained at the left end of the aquifer; the aquifer is in contact with a stream of varying water level at the right end right. The water in the stream rises from an initial level h_L to its final level h_0 by an exponential function of time. These conditions can be summarized as

$$\begin{aligned}&h(x=0,t)=h_0\\&h(x=L,t)=h_0+(h_L-h_0)\ e^{-\gamma t}\end{aligned} \tag{4a-4b}$$

where γ is a positive constant characterizing the rise rate. Equation (1) is a nonlinear parabolic partial differential equation whose analytical solution cannot be obtained. In order to find an approximate analytical solution, equation (1) is linearized as follows:

$$\frac{\partial^2 h}{\partial x^2}-\frac{\tan\beta}{h}\frac{\partial h}{\partial x}+\frac{N(t)}{K\hbar\cos^2\beta}=\frac{S}{K\hbar\cos^2\beta}\frac{\partial h}{\partial t} \tag{5}$$

where $\hbar$ = $(h_i + h_t)/2$ is the average saturated depth of the aquifer (Marino 1973). Now define the following dimensionless variables

$$H=\frac{h-h_0}{h_L-h_0}\ ;\ \tau=\frac{(K\hbar\cos^2\beta)t}{SL^2}\ ;\ X=\frac{x}{L}\ ;\ \alpha=\frac{L\tan\beta}{2\hbar} \tag{6a - 6d}$$

Using them in equation (5), one obtains

$$\frac{\partial^2 H}{\partial X^2}-2\alpha\frac{\partial H}{\partial X}+\left(N_0'+N_1'e^{-\lambda_1\tau}\right)=\frac{\partial H}{\partial\tau} \tag{7}$$

where

$$N_0'=\frac{L^2N_0}{K\hbar(h_L-h_0)\cos^2\beta}\ ;\ \lambda_1=\frac{(SL^2)\lambda}{K\hbar\cos^2\beta}\ ;\ N_1'=\frac{L^2N_1}{K\hbar(h_L-h_0)\cos^2\beta} \tag{8 a -8c}$$

The initial and boundary conditions are reduced to

$$H(X,\tau=0)=X$$
$$H(X=0,\tau)=0 \tag{9a-9c}$$
$$H(X=1,\tau)=e^{-\gamma t}$$

where

$$\gamma_1=\frac{SL^2}{K\hbar\cos^2\beta}\tau \tag{10}$$

Analytical Solution is obtained by using Laplace Transform method defined as

$$L\{H(X,\tau)\tau\to p\}=\theta(X,p)=\int_0^\infty e^{-p\tau}H(X,\tau)d\tau \tag{11}$$

Applying Laplace transform to both sides of equation (7) and on the initial & boundary condition given above and using it, we will get

$$\frac{d^2\theta}{dX^2}-2\alpha\frac{d\theta}{dX}+\left(\frac{N_0'}{p}+\frac{N_1'}{p+\lambda_1}\right)=p\theta-X \tag{12}$$

Equation (12) can be solved by any standard method. Its solution is

$$\begin{aligned}\theta(X,p)=&\left(\frac{2\alpha-N_0'}{p^2}\right)\left\{e^{\alpha X}\frac{\sinh\left\{(1-X)\left(\sqrt{\alpha^2+p}\right)\right\}}{\sinh\left(\sqrt{\alpha^2+p}\right)}+e^{-\alpha(1-X)}\frac{\sinh\left(\sqrt{\alpha^2+p}\right)X}{\sinh\left(\sqrt{\alpha^2+p}\right)}-1\right\}\\&+\frac{N_1'}{p(p+\lambda_1)}\left\{-e^{\alpha X}\frac{\sinh\left\{(1-X)\left(\sqrt{\alpha^2+p}\right)\right\}}{\sinh\left(\sqrt{\alpha^2+p}\right)}-e^{-\alpha(1-X)}\frac{\sinh\left(\sqrt{\alpha^2+p}\right)X}{\sinh\left(\sqrt{\alpha^2+p}\right)}+1\right\}\\&+e^{-\alpha(1-X)}\left\{\frac{1}{(p+\gamma_1)}\frac{\sinh\left(\sqrt{\alpha^2+p}\right)X}{\sinh\left(\sqrt{\alpha^2+p}\right)}\right\}-e^{-\alpha(1-X)}\left\{\frac{1}{p}\frac{\sinh\left(\sqrt{\alpha^2+p}\right)X}{\sinh\left(\sqrt{\alpha^2+p}\right)}\right\}+\frac{X}{p}\end{aligned} \tag{13}$$

Taking Laplace Inverse Transform on equation (16) and using the Calculus of residues method it gives water head as

$$\begin{aligned}H(X,\tau)=&X-\left(2\alpha-N_0'\right)\sum_{n=1}^{\infty}\frac{2n\pi\left\{1-(-1)^n e^{-\alpha}\right\}e^{\alpha X}\sin n\pi X}{\left(\alpha^2+n^2\pi^2\right)^2}\left\{1-e^{-\left(\alpha^2+n^2\pi^2\right)\tau}\right\}\\&+\frac{N_1'}{\lambda_1}\left(1-e^{-\lambda_1\tau}\right)-\frac{N_1'}{\lambda_1}\left(1-e^{-\lambda_1\tau}\right)e^{\alpha X}\left[\frac{\sinh\left\{(1-X)\sqrt{\alpha^2-\lambda_1}\right\}+e^{-\alpha}\sinh\left(X\sqrt{\alpha^2-\lambda_1}\right)}{\sinh\left(\sqrt{\alpha^2-\lambda_1}\right)}\right]\\&+N_1'\sum_{n=1}^{\infty}\frac{2n\pi\left\{1-(-1)^n e^{-\alpha}\right\}e^{\alpha X}\sin n\pi X}{\left(\alpha^2+n^2\pi^2\right)\left(\alpha^2+n^2\pi^2-\lambda_1\right)}\left\{1-e^{-\left(\alpha^2+n^2\pi^2\right)\tau}\right\}-e^{-\alpha(1-X)}\frac{\sinh(\alpha X)}{\sinh(\alpha)}\\&+e^{-\alpha(1-X)}\frac{\sinh\left(\sqrt{\alpha^2-\gamma_1}\right)X}{\sinh\left(\sqrt{\alpha^2-\gamma_1}\right)}e^{-\gamma_1\tau}-e^{-\alpha(1-X)}\sum_{n=1}^{\infty}\frac{2n\pi(-1)^n\sin n\pi X\, e^{-\left(\alpha^2+n^2\pi^2\right)\tau}}{\left(\alpha^2+n^2\pi^2\right)}\\&+e^{-\alpha(1-X)}\sum_{n=1}^{\infty}\frac{2n\pi(-1)^n\sin n\pi X}{\left(\alpha^2+n^2\pi^2-\gamma_1\right)}e^{-\left(\alpha^2+n^2\pi^2\right)\tau}\end{aligned} \tag{14}$$

Equation (14) describes the water head profile in an unconfined sloping aquifer under the combined influence of time dependent recharge and seepage. The analytical expressions clearly underline the dependence of water head on the sloping angle β and rise parameters λ and γ.

3. DETERMINATION OF FLOW RATE AT STREAM-AQUIFER INTERFACE

Flow rate in an unconfined sloping aquifer is defined as

$$q=-K\,h\left(\frac{\partial h}{\partial x}-\tan\beta\right)\cos^2\beta \tag{15}$$

Define a dimensionless flow Rate as

$$Q=\frac{qL}{K\left(h_0-h_L\right)^2} \tag{16}$$

Invoking the dimensionless variable, we get

$$Q=-\left[H+\frac{h_0}{(h_L-h_0)}\right]\left[\left(\frac{\partial H}{\partial X}\right)-\frac{L\tan\beta}{(h_L-h_0)}\right]\cos^2\beta \tag{17}$$

The flow rate at the stream–aquifer interface, i.e. at $X=1$ is given by

$$(\mathrm{Q})_{X=1}=-\left[H(X=1,\tau)+\frac{h_0}{(h_L-h_0)}\right]\left[\left(\frac{\partial H}{\partial X}\right)_{X=1}-\frac{L\tan\beta}{(h_L-h_0)}\right]\cos^2\beta \tag{18}$$

Invoking equation (14) in (18), the analytical expression for the flow rate at the stream aquifer interface is given by

$$(\mathrm{Q})_{X=1}=-\left[e^{-\gamma_1\tau}+\frac{h_0}{(h_L-h_0)}\right]\left[\begin{array}{l}1+\left(2\alpha-N_0'\right)\sum\limits_{n=1}^{\infty}\dfrac{2n^2\pi^2\left\{1-(-1)^n e^{\alpha}\right\}}{\left(\alpha^2+n^2\pi^2\right)^2}\left\{1-e^{-\left(\alpha^2+n^2\pi^2\right)\tau}\right\}\\ -\dfrac{N_1'\alpha}{\lambda_1}\left(1-e^{-\lambda_1\tau}\right)-\dfrac{N_1'}{\lambda_1}\left(1-e^{-\lambda_1\tau}\right)\left(\sqrt{\alpha^2-\lambda_1}\right)\left[\dfrac{\cosh X\left(\sqrt{\alpha^2-\lambda_1}\right)-e^{\alpha}}{\sinh\left(\sqrt{\alpha^2-\lambda_1}\right)}\right]\\ +N_1'\sum\limits_{n=1}^{\infty}\dfrac{2n^2\pi^2\left\{(-1)^n e^{\alpha}-1\right\}}{\left(\alpha^2+n^2\pi^2\right)\left(\alpha^2+n^2\pi^2-\lambda_1\right)}\left\{1-e^{-\left(\alpha^2+n^2\pi^2\right)\tau}\right\}-\dfrac{e^{\alpha}\alpha}{\sinh(\alpha)}+\alpha e^{-\gamma_1\tau}\\ +\dfrac{\left(\sqrt{\alpha^2-\gamma_1}\right)\cosh\left(\sqrt{\alpha^2-\gamma_1}\right)}{\sinh\left(\sqrt{\alpha^2-\gamma_1}\right)}e^{-\gamma_1\tau}+\sum\limits_{n=1}^{\infty}\dfrac{2n^2\pi^2 e^{-(\alpha^2+n^2\pi^2)\tau}}{\left(\alpha^2+n^2\pi^2-\gamma_1\right)}\\ -\sum\limits_{n=1}^{\infty}\dfrac{2n^2\pi^2 e^{-\left(\alpha^2+n^2\pi^2\right)\tau}}{\left(\alpha^2+n^2\pi^2\right)}-\dfrac{L\tan\beta}{(h_L-h_0)}\end{array}\right]\cos^2\beta \tag{19}$$

4. DISCUSSION OF RESULTS

Analytical solutions developed in this study are implemented in FORTRAN environment with a set of hypothetical aquifer parameters as listed in Table 1. Several numerical tests were conducted to assess the convergence of infinite series involved in equations (14) and (19). It was observed that these series are fast converging and merely first 70 terms are sufficient to approximate the whole series.

Table 1: Values of aquifer parameters

Parameter	Value
K	2.5 m/h
S	0.2
L	100 m
h_0	5 m
h_L	3 m
N_0	4 mm/h
N_1	2 mm/h
λ	0.2 day^{-1}
β	3 deg
γ	0.2 day^{-1}

Development of dynamic water table in a 3 deg downward sloping aquifer is plotted at $t=0$, 8, 16, 24, 48 and 120 hrs. The same is presented in Figure 1. Due to continuous downward recharge and seepage from adjacent water bodies, the phreatic surface in the aquifer grows with time and evolves in the form of a mound. During early phases, the mound remains confined in the vicinity of left interface of stream and aquifer, but gradually drifts towards right side where the aquifer meets the rising stream. Numerical experiments with large value of time reveal that the center of the mound eventually settles at the midpoint of the aquifer.

The growth of water table depends on the complex interplay between recharge rate, stream rise rate and the bed slope of the aquifer. In Figure 2, the water table development in a horizontal aquifer is presented for the same set of aquifer parameters.

Comparison of the two water table graphs reveals that the growth of water table is significantly high in the sloping aquifer, primarily due to the bed slope that lets more inflow of water from the left end water body.

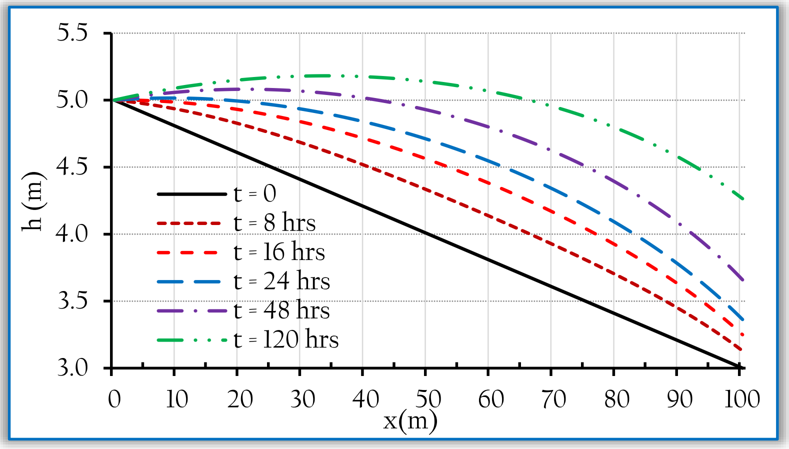

Figure 1: Development of water table in 3 deg downward sloping aquifer

Stream water rise rate γ plays important role in transient water head values. In Figure 3, the water head height at the midpoint of the aquifer plotted against time t for various values of γ. It can be observed from this figure that when an aquifer is connected with a rapidly rising stream (for instance $\gamma = 4$ day^{-1}), the water table in it at any specific instant of time attains higher values than the case when the stream rise rate is slower (γ = 0.2 and 2 day^{-1}). In fact, a faster rising stream creates higher hydraulic gradient at the right end of the aquifer, letting more water to flow into the aquifer from the right end. As a result, the transient water head attains higher values. However, numerical experiments for large value reveal that the steady state values of water head is independent of γ.
A similar simulation of water head for various values of sloping angle β is presented in Figure 4 in which negative value of sloping angle signifies an upward sloping impervious base.
When the aquifer's base is down sloping, a natural hydraulic gradient is created towards the positive direction of x–axis, which causes more water to flow into the aquifer from the left end water body. As a result, hydraulic head increases with down sloping angle. On the contrary, when the aquifer is upward sloping, the inflow from left end decreases, causing lower level of water head in the aquifer.
Downward percolation rate has significant impact on the transient as well as the final level of the water table water. Previous studies reveal that every cycle of downward recharge follows a typical pattern including a rising limb and a recession curve. Interestingly, the recession limb can be approximated an exponentially decreasing function of time. In the current example, the recharge rate is considered to decrease to N_0 from initial value $N_0 + N_1$by an exponent law given by $N(t) = N_0 + N_1 e^{-\lambda t}$ in which the parameter λ determines the rate at which the final value from its initial value. In Figure 5, the water head at the midpoint of the aquifer is plotted against time t for various values of λ. Taking the observational point at x = 50 m, tt is worth noting that at any specific

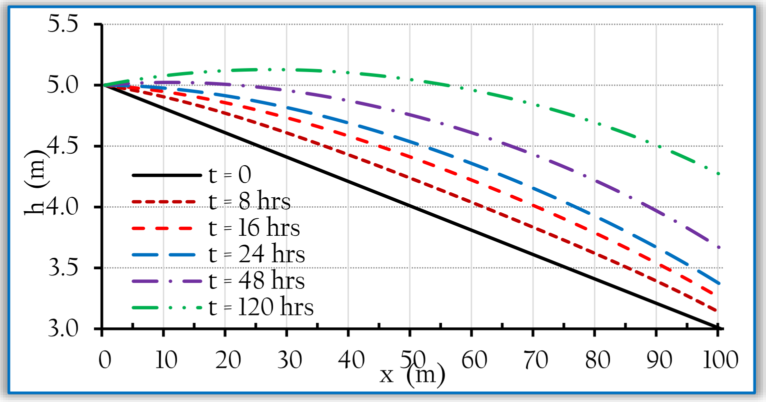

Figure 2: Water table profiles in a horizontal aquifer

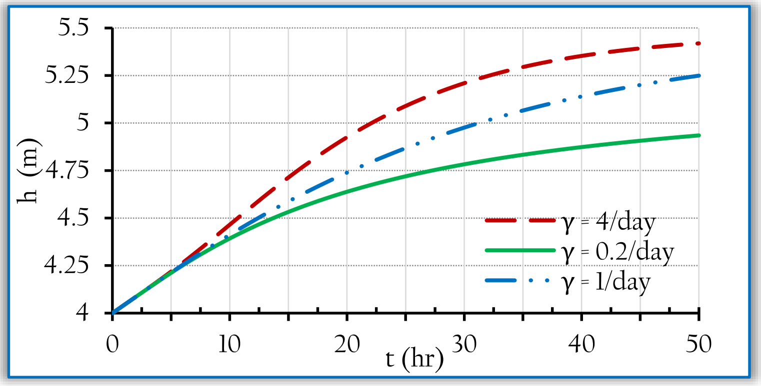

Figure 3: Water head height at the midpoint of the aquifer for various values of γ

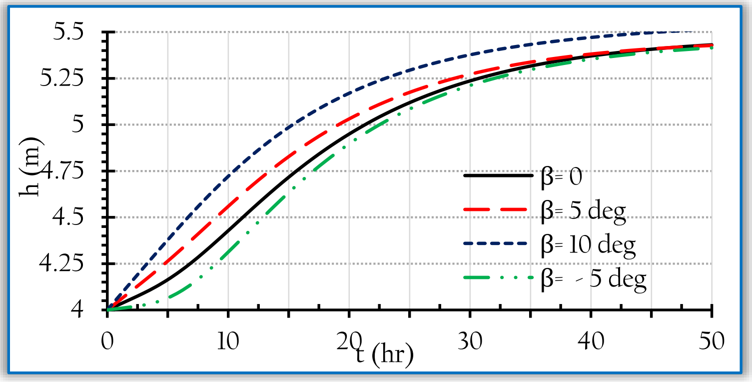

Figure 4: Water head height at the midpoint of the aquifer for various values of β

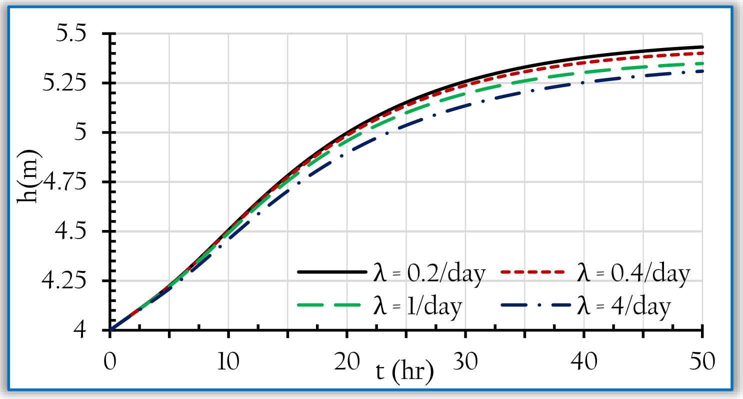

Figure 5: Water head height at the midpoint of the aquifer for various values of λ

instant of time *t*,the transient water head height decreases as the parameter λ increases.

Flow rate hydrograph at the interface of the aquifer and the rising stream, i.e. at *x* = 100 m are presented in Figure 6.

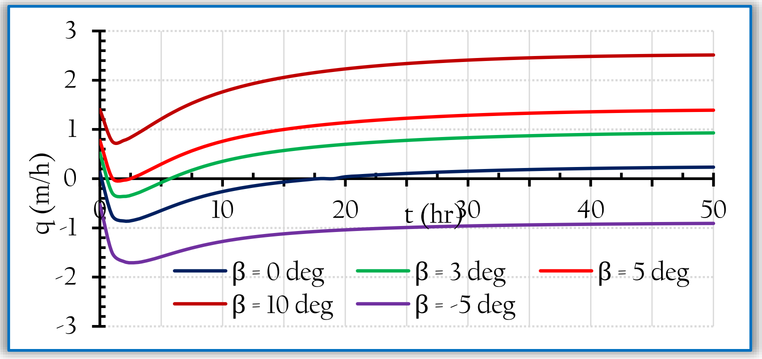

Figure 6: Flow rate hydrographs for various values of sloping angle β

Typically, the flow rate decreases at the initial instants of time. Once a minimum value is attained, it increases thereafter and reaches a steady-state value. It is noteworthy that a negative value of flow rate at the stream aquifer interface characterizes an inflow of water into the aquifer from the rising stream. No inflow is observed in the aquifer for a substantially large value of sloping angle, viz. β= 5 and 10 deg. However, for small down sloping (β = 0 and 3 deg) or up sloping aquifers (β = − 5 deg), a inflow is observed during stream-stage variations. Duration and magnitude of this inflow depends on the sloping angle as well as the stream rise rate. A fast rising stream creates an inward hydraulic gradient, and thus facilitates flow of water in the aquifer.

5. CONCLUSIONS

Stream-aquifer models are often developed to get an insight into the dynamic behavior of water table in unconfined aquifers. In the present study, closed form analytical expressions are derived and implemented for simulation of hydraulic head distribution in a stream-aquifer system under the influence of stream-stage variations and downward percolation. The aquifer is underlain by a sloping impervious bed and the variations in both stage and recharge rate are time dependent. In an illustrative example, it is demonstrated that the water table in the aquifer evolves as a mound due to continuous recharge whose height doesn't depends the recharge rate alone. It is revealed in this study that the interplay between various aquifer parameters is deterministic of spatio-temporal variations in the water head.

References

[1] Bansal, R.K.: Groundwater fluctuations in sloping aquifers induced by time-varying replenishment and seepage from a uniformly rising stream. Transport in Porous Media 92(2): 817-836, 2012
[2] Bansal, R.K. and Das, S.K.: Response of an unconfined sloping aquifer to constant recharge and seepage from the stream of varying water level. Water Resources Management 25, 893–911, 2011
[3] Chlids, E.C.: Drainage of Ground water resting on a sloping bed. Water Resource Research 7(5), 1971
[4] Hantush, M.S.: Wells near streams with semi-pervious beds. J Geophysical Research 70(12): 2829–2838, 1965
[5] Hunt, B.: Unsteady stream depletion from ground water pumping. Ground Water 37, 98–102, 1999
[6] Butler, J.J., Zlotnik, V.A. and Tsou M.A.: Drawdown and stream depletion produced by pumping in the vicinity of a partially penetrating stream. Ground Water 39(5), 651-9, 2001
[7] Lockington, D.A.: Response of unconfined aquifer to sudden change in boundary head. J Irrigation Drainage Engineering 123(1), 24–27, 1997
[8] Marino M.: Water-table fluctuation in semi pervious stream–unconfined aquifer systems. J Hydrology 19, 43–52, 1973
[9] Moutsopoulos, K.N.: The analytical solution of the Boussinesq equation for flow induced by a step change of the water table elevation revisited. Transport in Porous Media 85, 919–940, 2010
[10] Polubarinova-Kochina P.Y.: Theory of Groundwater Movement (trans: deWiest, J.M.R.) Princeton University Press, Princeton, 1962
[11] Rai, S.N. and Manglik, A.: Modelling of water table variation in response to time-varying recharge from multiple basins using linearised Boussinesq equation. J. Hydrology 220, 141–148, 1999
[12] Teloglou, I.S. and Bansal, R.K.: Transient solution of stream–unconfined aquifer interaction due to time varying stream head and in the presence of leakage. J. Hydrology 428-429, 68–79, 2012
[13] Upadhyaya, A. and Chauhan H.S.: Interaction of stream and sloping aquifer receiving constant recharge. J. Irrigation and Drainage Engineering 127(5), 295–301, 2001
[14] Verhoest, N.E.C., Pauwels, V.R.N., Troch, P.A. and Troch, F.P.D.: Analytical solution for transient water table heights and outflow from ditch-drained terrains. J. Irrigation and Drainage Engineering 128(6), 358–364, 2002
[15] Verhoest, N.E.C. and Troch, P.A.: Some analytical solution of the linearized Boussinesq equation with recharge for sloping aquifer. Water Resources Research 36(3), 793–800, 2000
[16] Wooding, R.A. and Chapman, T.G.: Groundwater flow over a sloping aquifer impermeable layer, 1, application of Dupuit–Forchheimer assumption. J. Geophysical Research 71(12), 2895–2902, 1966
[17] Zlotnik, V.A. and Tartavoski, D.M.: Stream depletion by groundwater pumping in leaky aquifer. Journal of Hydrologic Engineering, 13(2), 2008
[18] Zlotnik, V.A. and Kacimov, A.: Estimating groundwater mounding in sloping aquifers for Managed Aquifer Recharge. Groundwater, 55(6), 2017