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MATHEMATICAL MODELING OF WORKERS ACTIVITY DURING PLANT SHUTDOWN-STARTUP ACTIVITIES WITHIN THE FRAMEWORK OF COOPERATIVE GAME THEORY

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Abstract: In every kind of industry to have esteemed production in limited time and cost marks its capital gain. It may be any section of the industry, such as technical, mechanical, human resource or sales, all are co-related with the end products a factory produces to be marketed off in the world spectrum. This paper reflects a short and applicable method to overcome the difficulties arising in the all-around production of goods in industries when time and capital are limited. In simple terms, this model formatted on games theory (*n*- person cooperative games). Lastly, for more illustration, a case study is posted to explain the issues with examples for better understanding of Shapely value and the concept of the nucleolus. All departments of a production house can benefit equally.
Keywords: Game theory, project management, employee, cost allocation

1. INTRODUCTION

Cooperative game theory assumes that groups of players, called coalitions, are the primary units of decision-making, and may enforce cooperative behaviour. Consequently, cooperative games can be seen as a competition between coalitions of players, rather than between individual players. The basic assumption in cooperative game theory is that the grand coalition that is the group consisting of all players will form. One of the main research questions in cooperative game theory is how to allocate in some fairway the payoff of the grand coalition among the players. The answer to this question is related to a solution concept which, roughly speaking, is a vector that represents the allocation to each player. Different solution concepts based on different notions of fairness have been proposed in the cooperative game theory literature. At present, the division has fully believed in non-cooperative aspects and to format dynamic surroundings to dissolve any problems arising to diminish the productivity of any kind of industry. Time cost trade-off is totally effective in reducing the cost of production, labour charges and cost of material whereas time management aspect helps in producing the products of desirable quality in less time. It is generally believed that less the value of the raw materials and other initial expenses, time is taken to finish off the project will be higher. They were putting practical use of Critical Path Method (CPM) as it is widely known, minimising the cost for non-essential activities eventfully, which will lead the projects to finish in the proposed time.

Today heuristics commonly termed as mathematical programming is widely worked out for time-cost trade-off analysis which proves inefficient to bigger scale CPM networks. Few programs like analogous to natural selection, genetics in reproduction and genetic algorithm have proved to dissolve a large number of problems arising in the scientific and mechanical arena. Feng [2] formatted an algorithm revolving around the principles of genetic algorithms (GA) for designing the methodology. In other programs, Feng [3] has done initiative representation process essential for verifying stochastic effects. Even then, a common strategy algorithm will be quite useful to give the right direction to possess favourable solutions. Ho [7] researched the effects of bid compensation for further improving the rightful bid compensation process to verify the comparative relation between competing bidders and project filers. The model evolved during this research revolves around bid compensation aided with equilibrium compensation, quantitative formula and qualitative usage. Ho [8] analysed theoretic foundations for the effective applicable common and private partnership procurements and management strategies for decision. It can be rightly stated that the study is an innovative step to design a framework and systematic workout to estimate the working motions. Recently, Ho [9] verified by scanning the primary characteristic pattern of the participants if get indulged in opposite unexpected goals in unfavourable conditions. The study states that when the end result is more profitable than initiative capital loss, the players are mentally unprepared to meet the challenges, which may create quite potential problems to achieve the desired goal in a fixed time. Medda [10] put forward a structure to aid formwork subcontractors to hire daily wages open workers in place of workers related to the Union. They can earn considerable gain as the labour cost will be less and can surely complete the proposed works in the given time. This is sure to help in increasing their capacity to complete the project at less cost, hence gain more contracts eventually. Payoff process is well materialised for single contractors as well as a group of contractors. The profit can be equally distributed among the collaborated contractors by utilising the Shapely Value and nucleolus. Shen [14] elongate the process in which an innovative approach to performing build-operate-transfer (BOT) concession model (BOTCeM), helping to recognise the concession period with the aid of bargaining game theory. It is done by considering the bargaining procedure of two or more parties such as the subcontractor, investor or the party representing any Government department. Perng [11] had a similar opinion that a Formwork subcontractor can gain more capital by appointing regular workers than union workers. More profit can be gained if worked collaborating with other subcontractors. Shapely value and nucleolus help in a greater way in allocating the gained capital to the listed subcontractors in the collaborated group. Asgari [1] portrayed a thought to save time and cost by trading time in between two or more sequential projects without any conditional clauses placed written while signing the contract with general contractors. All the above studies reflect that varied kind of beneficial approach has been done to satisfactorily solve the issues related to construction. It has been studied that multiple analysers used hybrid and linear programming methods to solve the issues which were successful to a limited extent. Hegazy [6] believed in utilising genetic algorithm while few others believed in combining both integer programming as well as a genetic algorithm. They fully concentrated on ways to reduce cost and overlooked the application to decrease the time period.

This paper is mainly concerned to aid the people who manage the planning department in every construction projects such as sub-contractors, permanent employees and workers based on daily wages. The subject portrays the overall view to reducing the time during shut-down–start-up functions. The primary reason behind this consideration is that the planning department has control over all the activities involved in the project. As time is one of the vital factors to complete the project in the given period, hence the case is explained with a clear description of an example.

2. COOPERATIVE GAME THEORY AND SOLUTION CONCEPTS

A game theory [5] is considered by mentioning every value of the cooperative game for each coalition. Basically, the coalition game includes of a finite set of players N, called the grand coalition, and a characteristic function $v:2^N \to R$ from the set of all possible coalitions of players to a set of payments that satisfies $v(\varphi)=0$. The procedure illustrates the amount of collective payoff; a set of players can possess by agreeing to form a coalition. For further easy understanding, the set may even be termed as value game or a profit game. The players are given an independent choice to select which coalition team they want to be a member. The factors considered while finalising the team are the method applied to distribute the payoff among the coalition members. In accordance with that, a coalition game can even be described as a characteristic cost function $c:2^N \to R$ satisfying $c(\varphi)=0$. In this setting, players must accomplish some task, and the characteristic function c represents the cost of a set of players accomplishing the task together. A game of this kind is known as a cost game [5]. For allocating profits, some conceptual solutions refer to a 'range' of values that fulfil certain conditions like 'The Core' and some conceptual solutions suggest a unique point-like 'The Shapley value' and 'The Nucleolus'.

The core [4]: Let v be a game. The core of v is the set of payoff vectors

$$C(v)=\left\{x \in R^N:\ \sum_{i\in N} x_i = v(N);\ \sum_{i\in S} x_i \ge v(s),\ \forall\ S\subseteq N\right\}. \tag{1}$$

In simple terms, the core is the set of imputations under which no coalition has a value greater than the sum of its members' payoffs. Therefore, no coalition has the incentive to leave the grand coalition and receive a greater payoff.

The Shapley value [13]: It is a beneficial mode to share the capital profit to every player in the coalition in accordance to the distribution agreement. It reflects a "fair" sharing as it induces specific features listed below: In accordance with the formula of Shapely value the capital gain a player *i* gets given a coalitional game (v, N) is

$$\phi_i(v)=\sum_{S\subseteq N\setminus\{i\}} \frac{|S|!\ (n-|S|-1)!}{n!}\,[v(S\subseteq\{i\})-v(S)] \tag{2}$$

where *n* is the total number of players and the sum extends over all subsets *S* of *N* not containing player *i*. The formula can be interpreted as follows: imagine the coalition being formed one actor at a time, with each actor demanding their contribution $v(S\subseteq\{i\})-v(S)$ as fair compensation, and then for each actor take the average of this contribution over the possible different permutations in which the coalition can be formed.

The nucleolus [12]: Let $v:2^N \to R$ be a game, and let $x \in R^N$ be a payoff vector. The excess of *x* for a coalition $S \subseteq N$ is the quantity

$$v(S)-\sum_{i\in S} x_i \tag{3}$$

which is, the gain that players in coalition *S* can obtain if they withdraw from the grand coalition *N* under payoff *x* and instead take the payoff $v(S)$. Now let $\theta(x)\in R^{2^N}$ be the vector of excesses of *x*, arranged in nonincreasing order. In other words, $\theta_i(x) \geq \theta_j(x), \ \forall i < j$. Notice that x is in the core of v if and only if it is a pre-imputation and $\theta_1(x) \leq 0$. To define the nucleolus, we consider the lexicographic ordering of vectors in R^{2^N}: For two payoff vectors x, y, we say $\theta(x)$ is lexicographically smaller than $\theta(y)$ if for some index k, we have $\theta_i(x) = \theta_j(y), \ \forall i < k$ and $\theta_k(x) = \theta_k(y)$. The nucleolus of v is the lexicographically minimal imputation, based on this ordering.

3. OVERVIEW OF THE PROPOSED APPROACH AND MODEL

Conventional time-cost trade-off procedure can be only applicable while considering certain aspects like time factor and cost option included in the activities of the project. It has to be considered that they are visualised as deterministic, but in a real sense, they seem to be ambiguous. Thus, the factor needs to be noted before applying the policy of time – cost trade-off while reducing the period and initial cost requirement during the process of any project. Hence, it is beneficial to use simulation techniques for verifying stochastic effects. It is even seen that a general strategy or you can say the algorithm is needed to gain appropriate solutions. This paper reflects an exclusive elongated approach to applying the combined efforts of simulation techniques as well as genetic algorithms to have esteem profit by solving time cost trade-off unexpected solutions. The analysis report shows that genetic algorithm along with simulation techniques is capable of giving efficient and practical modes to possess appropriate project schedules while considering the uncertain risks involved in term of period and cost of shutting down and start-up activities. This innovative approach gives practical solutions for planning engineers working in polyester manufacturing companies. It provides an excellent platform to minimise the time and cost factor. Other connected scenarios can be verified to decide to have the needed time and cost while applying shut-down – start-up functions.

Cost, duration and quality are the main features deciding the success of any projects. The efficiency of the projects totally depends on these three factors. Hence, they place an essential challenge. The working capability of workmen signifies the profit in the projects. Of course, the planning engineer efficient collaboration with his subordinates and labour force is quite essential to raise the profit, but even a skilled, experienced engineer has too many negative factors like price fluctuations while buying raw material, unfavourable weather, breakdown of equipment in the middle of the process, shortage of working hands, not having right kind of tools, inefficient machines which are enough to create nuisance while the project is in progress. Succumbing to these negative elements is sure to increase the completion period and capital to complete the project. The trading scheme proposed in these papers is sure to aid in eliminating the drawbacks of the above-mentioned problems by providing appropriate solutions by applying certain time efficiency functions like successful negations can be conducted between the management team, employees and the working labours, before the shut-down activities starts-up. The negotiations process help in formatting a new understanding between the management and working forces enabling to complete the production in time and in the proposed budget. Thus, it helps in allowing for maximising the final profit for the coalition team to distribute the gain capital according to the earlier fixed terms.

4. THE MATHEMATICAL MODEL

Basically, the intention behind presenting the model is to feature the relation between time and cost in esteeming the gain in any kind of projects. it will help in the smooth functioning of shut-down start-up activities in production departments. It is designed to increase the efficiency of work and as well as totally aid in reducing the initial cost and lessening the time taken to complete varied projects. The work can be distributed to weekly, monthly and yearly basis. The paper is formulated considering the working abilities all kind of employees, the subcontractors and the planning engineers. Scientifically it can be termed as efficiency is the ratio of the work done by a workgroup to the work that can be done potentially by the same group. In simple terms, it can be explained as the efficiency is always valued between ratio 0 and 1. It becomes essential to note that unexpected cost may arise while the projects attend its completing period. Labour cost is the payoff function of workmen, including both permanent company employee and contractual worker.

From the total workmen to be employed, consider R_{ij}^r is the number of workmen type j required for activity i on time t, R_{ij}^n is the number of workmen type j required for activity i on time t available at normal condition, M_{ij} is the daily cost to company (CTC) of workmen type j required for activity i, P_{ij} is the daily cost to company (CTC) of workmen type j required for activity i above normal condition and t_i the time-activity i. Thus,

$$C_{d_{ij}} = R_{ij}^r \times M_{ij} \times t_i + P_{ij} \times t_i \times (R_{ij}^r - R_{ij}^n) \tag{4}$$

where the premium is added when the number of workmen of type j required for activity *i* exceeds of availability, so that P_{ij} is given by

$$P_{ij} = \begin{cases} V_{ij} - M_{ij} & \text{for } R_{ij}^{r} > R_{ij}^{n} \\ 0 & \text{for } R_{ij}^{r} \le R_{ij}^{n} \end{cases} \tag{5}$$

where: v_{ij} is the cost rates of workmen type *j* at activity *i* above normal shutdown-startup availability limit.

The proposed daily average cost is calculated:

$$c' = C_{d_{ij}}(T)/T \tag{6}$$

$$c(j) = c'/R(j) \tag{7}$$

where: c' denotes the daily average cost to the company for workmen; $C(T)$ is the cost of shutdown-startup activities in time T; T is the time of the project (day) for workmen; $c(j)$ is the daily real cost for workmen; j is the index of the day

$R(j)$ represents the efficiency in the day of *j*.

Now the real cost of a project can be calculated considering time-cost and time-efficiency functions by integrating daily real costs [1]:

$$C_R(T) = \sum_{S}^{F} c(j) = C(T)/T \sum_{S}^{F} 1/R_i(j) \tag{8}$$

$$T = F - S \tag{9}$$

where: $C_R(T)$ denotes the real cost of all activities for shutdown-startup activities; *S* is the start date of shut-down activities; *F* represents the finish date of start-up activities.

In coalition status, subsequent workmen can trade time and decrease their total real cost. The model allows the subsequent planning engineers to choose the best start and finish date, ignoring shutdown-startup constraints. Decision variable of the model is started and finish time of each shutdown-startup activities in the coalition [1].

$$C_R(S) = \text{Min} \sum_{i \in S} \left\{ C_i(T_i)\,(1/T_i) \sum_{S_i}^{F_i} 1/R_i(j) \right\} \tag{10}$$

$$T_i = F_i - S_i \quad \forall i \in S \tag{11}$$

$$F_i \le S_{i+1} \quad \forall i \in S \tag{12}$$

To solve simple problems, it is possible to use the total search method while in complicated problems, the metaheuristic methods may be used. After time trading between shutdown-startup activities, it is necessary to assign new shutdown-startup activities to have to accept all risks of shutdown-startup activities.

5. ILLUSTRATIVE EXAMPLE

The example is presented to describe in detail the procedure and the capability of the performance of the given proposal. Time cost and Time-efficiency functions for activities are represented in table 1, 2, 3 & 4.

The model is solved in two cases:

1- Ignoring time-efficiency function

2- Considering the time-efficiency function

Using equations (10), (11) and (12), total real costs are calculated for all coalitions formed by departmental workmen {1}, {2} and {3} that are represented in table 5 and 6.

Figure 1: workmen position in shutdown-startup activities

where T_N = normal time for shutdown-startup sub-activities, C_N = cost in normal time without considering efficiency, C_{NR} = real cost in normal time.

Table 1: Time-cost function for Production & Technical department {1}

Time	60	61	62	63	66	67	68	74	77	78	84	87
Cost	364350	363300	359100	341250	339150	329700	320250	313950	312900	307650	301350	300300

Table 2: Time-cost function for Mechanical department {2}

Time	100	101	102	100	104	105	106	107	108	109	110	111
Cost	279972	269472	268947	268422	252672	252147	251622	249417	246267	231042	223167	222642
	112	114	115	116	118	119	120	121	122	124	125	126
	222117	221067	220542	220017	219387	218862	218337	218022	217497	216447	215922	215397
	128	131	132	133	134	137	138	139	140	142	143	145
	214872	214557	214137	213822	213297	213171	213087	212457	212037	211827	211617	211197
	148	151	154	156	158	159	161	169				
	210567	210147	210021	209895	209790	209727	209622	209454				

Table 3: Time-cost function for Electrical department {3}

Time	60	61	62	63	65	66	67	68	71	73	74	77
Cost	279972	269472	268947	268422	252672	252147	251622	249417	246267	231042	223167	222642
	78	80	81	83	84	87	90	92	94	102	105	
	214872	214557	214137	213822	213297	213171	213087	212457	212037	211827	211617	

Table 4: Time-efficiency function for departments in each period of project duration

Date (days)	0-10	11-20	21-30	31-50	51-70	71-100	101-195	195-254	255-275
Prod&Tech.dept.	0.5	0.5	0.75	1	1	1	1	-	-
Mech.dept.	-	-	-	-	0.5	0.75	1	1	-
Elect.dept.	-	-	-	-	-	-	1	1	0.5

Table 5: Cost and duration for each coalition not considering the time-efficiency function

Coalition	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
Cost	444522	236695	280741	523992	555450	425922	739242
Start-Finish	0-67	67-195	195-273	0-84, 84-195	0-67, 195-275	67-182, 82-275	0-78, 78-189, 189-275
Duration	67	128	78	84,111	67,80	115,93	78,111,86

Table 6: Cost and duration for each coalition considering the time-efficiency function

Coalition	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
Cost	444522	236695	280741	583388	725262	478099	859146
Start-Finish	0-67	67-195	195-273	17-85, 85-195	0-67, 195-273	67-177, 177-260	19-87,87-197, 197-275
Duration	67	128	78	68,110	67,78	110,83	68,110,78

For coalition {1,2,3} and in the case of ignoring time-efficiency function, the total real cost is decreased from 770322$ to 739242$ (Table 5). This saving can be allocated to the workmen in the coalition according to one of the available approaches (i.e. the shapely value, the nucleolus, etc.). Considering the time-efficiency, it is evident that the entire shutdown-startup time is not necessarily utilized. For example, Prod&Tech. dept. {1} Initiates his shutdown-startup activities on day 19 wherein previous days thus had anticipated very low efficiency. Using various solution concepts in cooperative game theory, the total real cost of the grand coalition, {1,2,3}, could be allocated between its players. The results are indicated in tables 7 and 8.

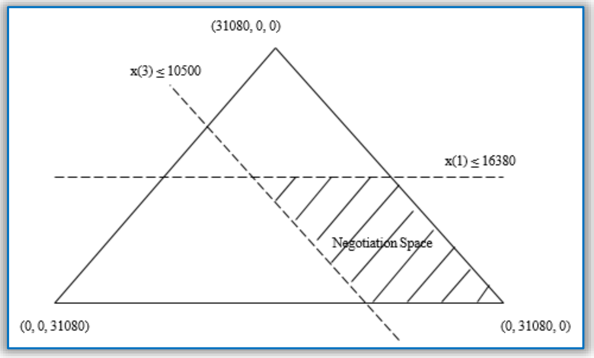

Figure 2: Space of the core not considering the time-efficiency function

Table 7. Cost allocation not considering the time-efficiency function

Solution concept	Prod&Tech. dept.	Mech. dept.	Elect. dept.
Cost allocated by The Shapley value	320811	198633	219801
Benefit by The Shapley value	8889	16239	5949
Benefit (%) by The Shapley value	2.70%	7.56%	2.64%
Cost allocated by The Nucleolus	324450	194292	220500
Benefit by The Nucleolus	5250	20580	5250
Benefit (%) by The Nucleolus	1.59%	9.58%	2.33%

Table 8. Cost allocation considering the time-efficiency function

Solution concept	Prod&Tech.dept.	Mech.dept.	Elect.dept.
Cost allocated by The Shapley value	407060	179565	272523
Benefit by The Shapley value	37464	57133	8217
Benefit (%) by The Shapley value	8.43%	24.14%	2.93%
Cost allocated by The Nucleolus	442031	138867	278250
Benefit by The Nucleolus	2493	97829	2493
Benefit (%) by The Nucleolus	0.56%	41.33%	0.89%

Because of Mech.dept. {2} key role in all beneficial coalitions, he should earn more benefit than others. Therefore all solution concepts have assigned the most shares to Mech.dept. {2}. Additionally, this game for Prod&Tech.dept. {1} is more beneficial than {3} because coalition {1, 2} is more beneficial than coalition {1, 3}.

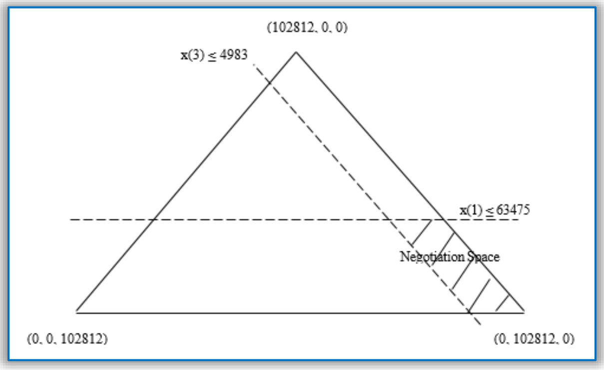

Figure 3: Space of the core in case of considering the time-efficiency function

6. SUMMARY, CONCLUSIONS AND ADDITIONAL WORK

Most of the shut-down start-up activities include many sequential activities which are implanted by different departments, but for the sake of brevity, we have considered only three major departments. Initial allocated time cannot be taken as constant as we need to consider the working ability of hired labour. Results of the working research conducted show that the presented model proves one of the best suitable methods to increase the profit of any production house by cutting down the initial cost and keeping the time limit in check. It works quite well for a team of sub-contractors as well as quite beneficial for individual leading projects. To fruitfully finish the significant activities of any department like shut-down–start-up activities can be quickly done by implying the above-illustrated methods.

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