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INTEGRAL BASED MODEL FOR DEVELOPMENT OF INSTANT INTERFACE TEMPERATURE AT TIME $\tau = 0^+$ OF A HIGH MELTING TEMPERATURE SPHERICAL SOLID ADDITIVE-BATH SYSTEM

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Abstract: The current investigation relates to the development of integral form of model to predict the temperature at the interface formed between the high melting temperature spherical solid additive and the freezing layer of the bath material around it as soon as the additive is dunked in the bath. It shows that this temperature is function of property-ratio B of the additive-bath system and the phase-change parameter the Stefan number S_t of the freezing bath material and leads to a closed-form solution for this θ_e which gets raised from the initial temperature of the additive to the freezing temperature of the bath material once B $(0 \le B \le \infty)$ increases or S_t $(\infty \ge S_t \ge 0)$ decreases. For $S_t \to 0, 0 \le B \le \infty$ it takes an expression $\theta_e = \frac{\sqrt{2B}}{\sqrt{3}+\sqrt{2B}}$.
Keywords: instant interface temperature, mathematical model, solid additive melt bath system

1. INTRODUCTION

Attainment of thermal equilibrium temperature at the interface formed between the additive and the freezing of the hot melt bath material onto it just after dunking it in the bath plays an important role in determining the duration of unavoidable freezing and melting of the bath material around the additive and its subsequent dissolution or melting and assimilation in the bath to prepare the bath of requisite composition for production of steel and cast iron of different grades. Such a melt then undergoes various metallurgical treatments before it is cast. This temperature is known as interface temperature θ_e at time $\tau = 0^+$.

In view of this, knowledge of this temperature is essential but as this phenomenon is regulated by the condition and temperature of the bath, geometry and shape of the additive, its temperature before its immersion, and thermo physical properties of the additive-bath system these make the prediction of θ_e at time $\tau = 0^+$is extremely difficult unless a suitable mathematical model is devised. Nevertheless, in the previous studies such a temperature in closed-form was obtained when two semi-infinite plates at different temperatures[1,2] came in contact. It was also found when the phase of either plate or both plates [3,4,5] changed after they were brought in contact. Moreover, this temperature at the interface formed soon after immersing high melting temperature plate [6] and cylindrical additive [7] and low melting temperature cylindrical additive [8] was predicted. It was shown that the higher interface temperature attained immediately after dunking the plate [8] and cylindrical additive [9] of high melting temperature and cylindrical additive of low melting temperature permits the reduction in the time of freezing and melting onto them. In turn this diminishes the production time and increases the productivity for making the product globally competitive. Incase of spherical additive, the investigation of such a temperature was not reported in the literature although dissolution of this spherical additive including the freezing and melting of the bath material onto it was investigated by several authors.[14-19]

The intent of the present study is to evolve a suitable nondimensional mathematical model for attainment of the interface temperature at the contact interface developed between the high melting temperature spherical solid additive and the freezing of the bath material around it just after its immersion in the hot melt bath.

2. MATHEMATICAL MODEL

In order to find instant temperature at the interface formed between the additive immersed in the melt bath and the freezing layer of the bath material onto it, a spherical solid additive is considered at its initial temperature T_{ai} less than its melting temperature T_{af}. It is dunked in the hot melt bath maintained at a temperature T_b greater than the freezing temperature of the bath material T_{bf}. Immediately the interface between the additive and the freezing layer onto the additive arrives at a temperature T_e that lies between the initial temperature of the additive, T_{ai} and the freezing temperature, T_{bf} of the bath material and develops a temperature field in the additive bath system represented by $T_b > T_{af} > T_{bf} > T_e > T_{ai}$. Moreover, heating of the additive and freezing of the bath material onto the additive initiate. With passing of the time, the interface temperature T_e rises, the frozen layer grows in thickness, and the heat penetration depth in the additive increases. This event is assumed to be regulated by transient heat conduction and the temperature field established in the additive and the frozen layer is symmetric about the center of the sphere.

The dimensionless differential equation for the heat conduction governing the temperature field in the heated region of the additive can be cast as:

$$\frac{1}{B}\cdot\frac{1}{\xi_a^2}\cdot\frac{\partial}{\partial\xi_a}\left(\xi_a^2\frac{\partial\theta_a}{\partial\xi_a}\right)=\frac{\partial\theta_a}{\partial\tau}\qquad 0\le\xi_a\le\eta,\quad \tau=0 \qquad [1]$$

Its initial and boundary conditions are:

$$\theta_a=0,\quad 0\le\xi_a\le\eta,\quad \tau=0 \qquad [2]$$

$$\theta_a=\theta_e,\quad \xi_a=1,\quad \tau>0 \qquad 3]$$

$$\frac{\partial\theta_a}{\partial\xi_a}=0,\quad \theta_a=0,\quad \xi_a=\eta,\quad \tau>0 \qquad [4]$$

The dimensionless heat conduction equation in differential form for the frozen layer can be expressed as:

$$\frac{1}{\xi_m^2}\cdot\frac{\partial}{\partial\xi_m}\left(\xi_m^2\frac{\partial\theta_m}{\partial\xi_m}\right)=\frac{\partial\theta_m}{\partial\tau}\qquad C_r\le\xi_m\le\xi_f,\quad \tau>0 \qquad [5]$$

Its associated initial and boundary conditions are:

$$\theta_m=\theta_b,\quad \xi_m=\xi_f=C_r,\quad \tau=0 \qquad [6]$$

$$\theta_m=\theta_e,\quad \xi_m=C_r,\quad \tau>0 \qquad [7]$$

$$\frac{\partial\theta_m}{\partial\xi_m}=B_{im}(\theta_b-1)+\frac{1}{S_t}\frac{d\xi_f}{d\tau},\quad \theta_m=1,\xi_m=\xi_f,\tau>0 \qquad [8]$$

The conditions that couples at the interface between the additive and the freezing layer of the bath material onto the additive can be written as,

$$\theta_a=\theta_m=\theta_e,\quad \xi_m=C_r,\xi_a=1,\tau>0 \qquad [9]$$

$$\frac{\partial\theta_a}{\partial\xi_a}=B\frac{\partial\theta_m}{\partial\xi_m},\quad \xi_m=C_r,\xi_a=1,\tau>0 \qquad [10]$$

Equations (1) to (10) forms the mathematical model of the current problem and are based on the assumptions of perfect contact at the interface between the additive and the freezing layer despite, the fact that there always occurs an interfacial resistance between the additive and the frozen layer due to imperfect contact. The values of such a resistance range from 1.9×10^{-4} m^2 sK/J to 2.1×10^{-4} m^2 sK/J[10] .Their effect is negligible with respect to thermal resistance offered by the additive to heat transfer to the frozen layer permitting to assume a perfect contact. Moreover, the additive and the frozen layer have uniform but different thermophysical properties. Equation [4] is derived from the energy balance applied to the interface between the frozen layer and the bath.

3. SOLUTION

The above model consisting of equations (1) through (10) exhibits non-linearity owing to the presence of phase-change moving boundary, equation (4) and coupled due to equations (9) and (10). The non-linearity prohibits its closed-form solution when exact analyses present in the literature are applied. In such a situation, semi-analytical techniques become important. One such methods known as integral method that yielded closed-form solutions for solidification and melting, and cooling and heating problems in the previous investigations [5,6,7] is employed in the current problem. It requires the transformation of the differential form of equations to the integral form. Equation (1) related to heating of the additive is converted to the following integral form:

$$\frac{d}{d\tau}\int_\eta^1\xi_a^2\theta_a d\xi_a-[\xi_a\theta_a]_{\xi_a=1}\frac{d1}{d\tau}+[\xi_a\theta_a]_{\xi_a=\eta}\frac{d\eta}{d\tau}=\frac{1}{B}\left[\xi_a^2\frac{\partial\theta_a}{\partial\xi_a}|_{\xi_a=1}-\xi_a^2\frac{\partial\theta_a}{\partial\xi_a}|_{\xi_a=\eta}\right] \qquad [11]$$

$$\eta\le\xi_a\le1,\quad \tau>0$$

whereas that of Equation (5) for the growth of the frozen layer becomes,

$$\frac{d}{d\tau}\int_{C_r}^{\xi_f}\xi_m^2\theta_m d\xi_m-[\xi_m^2\theta_m]_{\xi_m=\xi_f}\frac{d\xi_f}{d\tau}+[\xi_m^2\theta_m]_{\xi_m=C_r}\frac{dC_r}{d\tau}$$

$$=\xi_m^2\frac{\partial\theta_m}{\partial\xi_m}|_{\xi_m=\xi_f}-\xi_m^2\frac{\partial\theta_m}{\partial\xi_m}|_{\xi_m=C_r},C_r\le\xi_m\le\xi_f,\tau>0 \qquad [12]$$

Applying boundary conditions equations (3) and (4), equation (11) is reduced to,

$$\frac{d}{d\tau}\int_\eta^1\xi_a^2\theta_a d\xi_a=\frac{1}{B}\left[\xi_a^2\frac{\partial\theta_a}{\partial\xi_a}|_{\xi_a=1}\right],\quad \eta\le\xi_a\le1,\tau>0 \qquad [13]$$

On the other hand, equation (12) becomes,

$$\frac{d}{d\tau}\int_{C_r}^{\xi_f}\xi_m^2\theta_m d\xi_m-[\xi_m^2\theta_m]_{\xi_m=\xi_f}\frac{d\xi_f}{d\tau}=\xi_m^2\left[B_{im}(\theta_b-1)+\frac{1}{S_t}\frac{d\xi_f}{d\tau}\right]\xi_m^2\frac{\partial\theta_m}{\partial\xi_m}|_{\xi_m=C_r},$$

$$C_r\le\xi_m\le\xi_f,\tau>0 \qquad [14]$$

once, equations (7) and (8) are applied.

Note that the integral equations (13) and (14) can provide solutions in closed-form or reduce to initial value problems only when the temperature distribution in the frozen layer and the heated region of the additive are prescribed. In the frozen layer a first degree polynomial that satisfies equations (7) and (8),

$$\theta_m = 1 + (\theta_e - 1)\left\{1 - \frac{\xi_m - C_r}{\xi_f - C_r}\right\} \qquad [15]$$

And in the heated region of the additive, a third degree polynomial,

$$\theta_a = \theta_e\left[1 - \frac{1-\xi_a}{1-\eta}\right]^3 \qquad [16]$$

fulfilling the boundary conditions (3) and (4) are assumed.

The choice of such profiles for analogous problems of freezing and melting of the bath material and associated heating of the plate [2,12] and cylindrical shape additive [9,13] and freezing of the bath material onto them yielded accurate results in the previous investigations. Substitution of Equation (16) in integral equation(13) gives

$$\frac{d}{d\tau}\left[\theta_e\left\{\frac{1-\eta}{4} - \frac{(1-\eta)^2}{10} + \frac{(1-\eta)^3}{60}\right\}\right] = \frac{3\theta_e}{B(1-\eta)} \qquad [17]$$

whereas, application of equation (15) to equation (14) leads to

$$\frac{d}{d\tau}\left[\frac{1}{3}(\xi_f^3 - C_r^3) + (\theta_e - 1)\left\{\frac{C_r^2}{2}(\xi_f - C_r) + \frac{C_r}{3}(\xi_f - C_r)^2 + \frac{1}{12}(\xi_f - C_r)^3\right\} - \frac{1}{3}\xi_f^3\right]$$

$$= B_{im}\xi_f^2(\theta_b - 1) + \frac{1}{3S_t}\frac{d\xi_f^3}{d\tau} + \frac{C_r^2(\theta_e - 1)}{\xi_f - C_r} \qquad [18]$$

Note that equations (17) and (18) comprise of three unknown variables ξ_f, η, and θ_e. For their unique solutions one more equation is needed which is provided by the coupling condition, equation (10). It takes the form,

$$\frac{3\theta_e}{B(1-\eta)} = -\frac{\theta_e - 1}{\xi_f - C_r} \qquad [19]$$

when equations (15) and (16) are applied. Equations (17) to (19) readily lead to solutions for all times.

— **Solutions for time $\tau = 0^+$**

Equations (17) to (19) gives solutions for all times. To find solutions for time $\tau = 0^+$, order of magnitude analysis is applied. It compares various terms of equations (17) to (19) with respect to each other and drops those having insignificant order of magnitude. Here at this time the steep temperature gradient on the additive side of the contact interface occurs. It requires conductive heat transfer much greater than the convective heat supplied from the bath. Consequently, the excess heat conducted to the additive is provided by generation of latent heat of fusion owing to freezing of the bath material onto the additive. In such a case the order of magnitude of the convective heat supplied from the bath is much less than the latent heat of fusion allowing the dropping of the convective heat term $B_{im}(\theta_b - 1)$ from equation (18). Also, the rate of growth of the frozen layer $\frac{d\xi_f}{d\tau}$ and the rate of increase in the heat penetration depth $\frac{d(1-\eta)}{d\tau}$ become high as compared with the rate of change of equilibrium temperature $\frac{d\theta_e}{d\tau}$ leading to the order of magnitude of $\frac{d\theta_e}{d\tau}$ much less than those of $\frac{d\xi_f}{d\tau}$ and $\frac{d(1-\eta)}{d\tau}$. They permit deleting the terms associated with $\frac{d\theta_e}{d\tau}$ from equations (17) and (18). Consequently, these two equations are reduced, respectively to,

$$\theta_e\left\{-\frac{1}{4} + \frac{1-\eta}{5} - \frac{(1-\eta)^2}{20}\right\}\frac{d\eta}{d\tau} = \frac{3\theta_e}{B(1-\eta)} \qquad [20]$$

$$\left[(\theta_e - 1)\left\{\frac{C_r^2}{2} + \frac{2}{3}C_r(\xi_f - C_r) + \frac{1}{4}(\xi_f - C_r)^2\right\} - \frac{\xi_f^2}{S_t}\right]\frac{d\xi_f}{d\tau} = \frac{C_r^2(\theta_e - 1)}{\xi_f - C_r} \qquad [21]$$

Equation (20) is simplified to give the closed-form solution,

$$\frac{\tau}{B} = \frac{(1-\eta)^2}{24} - \frac{(1-\eta)^3}{45} + \frac{(1-\eta)^4}{240} \qquad [22]$$

It satisfies the initial conditions, η =1, τ =0. In contrast with the plate additive that provides the closed-form solution for dependent variables, the heat penetration depth as a function of an independent variable time, τ, the spherical additive represented by spherical coordinates gives closed-form solutions in equation (22) as an inverse function of $\tau = \{f(1-\eta)\}$ .To obtain the solution for the frozen layer thickness, ξ_f very near to $\tau = 0^+$, the series solution of the inverse function

$$\xi_f = \sum_{i=0}^{\eta} a_i(1-\eta)^i \qquad i = 1,2,3, \ldots. \qquad [23]$$

is employed for all times in view of equation (22). As for time $\tau = 0^+$, second and higher orders terms become progressively smaller due to which they are neglected from equation (23), giving

$$\xi_f - C_r = a_1(1-\eta) \qquad [24]$$

Equation (24) satisfies equation (6) and denotes an appropriate solution for the frozen layer thickness ξ_f. Its coefficient a_1 can readily be derived applying equation (20) related to heat penetration depth in the additive, equation (19) for the coupling condition and equation (21).
Applying equation (24), equation (19) becomes,

$$\theta_e = \frac{B}{3a_1 + B} \quad [25]$$

whereas, equation (21) related to frozen layer is converted to

$$-\frac{3a_1}{B+3a_1}\left\{\frac{C_r^2}{2} + \frac{2}{3}C_r a_1(1-\eta) + \frac{a_1^2}{4}(1-\eta)^2\right\}\left\{-a_1\frac{d\eta}{d\tau}\right\} - \frac{\{C_r + a_1(1-\eta)\}^2}{S_t}\left\{-a_1\frac{d\eta}{d\tau}\right\} = -\frac{3C_r^2}{(B+3a_1)(1-\eta)} \quad [26]$$

The above equation is transformed to the following form

$$\frac{1}{3C_r^2}\left\{\frac{A_0^*}{2}(1-\eta)^2 + \frac{A_1^*}{3}(1-\eta)^3 + \frac{A_2^*}{4}(1-\eta)^4\right\} = \tau \quad [27]$$

where, $A_0^* = \frac{3a_1^2C_r^2}{2} + \frac{a_1C_r^2(B+3a_1)}{S_t}$, $A_1^* = \frac{(2a_1^3C_r + 2a_1C_r)a_1(B+3a_1)}{S_t}$, $A_2^* = \frac{3a_1^4}{4} + \frac{a_1^3(B+3a_1)}{S_t}$
Equating equation (22) with (27) leads to the following equation

$$\left(\frac{B}{24} - \frac{A_0^*}{6C_r^2}\right)(1-\eta)^2 - \left(\frac{B}{45} - \frac{A_1^*}{9C_r^2}\right)(1-\eta)^3 + \left(\frac{B}{240} - \frac{A_2^*}{12C_r^2}\right)(1-\eta)^4 = 0 \quad [28]$$

Equation (28) results in providing coefficient of each of $(1-\eta)^2$, $(1-\eta)^3$, and $(1-\eta)^4$ zero giving,

$$\frac{B}{24} - \frac{A_0^*}{6C_r^2} = 0 \quad [29]$$

$$\frac{B}{45} - \frac{A_1^*}{9C_r^2} = 0 \quad [30]$$

$$\frac{B}{240} - \frac{A_2^*}{12C_r^2} = 0 \quad [31]$$

As in the neighborhood of $\tau = 0^+$, the heat penetration depth $(1-\eta)$ is small, increasing its order, its value becomes progressively smaller. Consequently, the terms containing $(1-\eta)^3$ and $(1-\eta)^4$in equation (28) become insignificant .They are, therefore, deleted from equation (28) leaving equation (29). It gives

$$A_0^* = \frac{BC_r^2}{4} \quad [32]$$

Employing the expression for A_0^* from equation (27), equation (32) leads to

$$a_1^2(6S_t + 12) + 4Ba_1 - BS_t = 0 \quad [33]$$

Its solution becomes

$$a_1 = \frac{-2B \pm \sqrt{4B^2 + BS_t(6S_t + 12)}}{(6S_t + 12)} \quad [34]$$

Since the frozen layer developed remains always positive, only the positive values of a_1 from equation (34) are taken for assigned values of S_t and B.

4. SPECIAL CASES AND VALIDITY

— CASE 1: $\mathbf{B \to 0}$, $\mathbf{0 \le S_t \le \infty}$

Using $B \to 0$ in equation (25) gives $\theta_e = 0$. From physical point of view, it is realistic since $B \to 0$ denotes the additive of infinite heat capacity. Due to this, the temperature of the additive does not rise beyond its initial temperature even though a large amount of heat is conducted to the additive provided by the convective heat from the bath and the latent heat of fusion generated due to freezing of the bath material onto the additive. As a result, the additive behaves as a thermal reservoir. The conductivity of the bath $k_m \to 0$ also gives $B \to 0$. It makes the frozen layer of the bath material of almost infinite thermal resistance. This property does not allow a large quantity of heat available from the bath including the heat generated by the freezing of the bath material to be conducted through the frozen layer to the additive. Consequently, the temperature at the interface does not rise above the initial temperature of the additive. The same result was reported for the plate, low and high melting temperature cylindrical additives in previous studies [2,9,10].

— CASE 2: $\mathbf{B \to \infty}$, $\mathbf{0 \le S_t \le \infty}$

When it is applied to equation (25), it gives $\theta_e = 1$. It is correct as $B \to \infty$ is indicative of an additive of negligible heat capacity. It permits the instant rise of the interface temperature to the freezing temperature of the bath material, despite a small amount of heat is conducted to the additive, Such heat is provided by the convective heat of the bath and the latent heat of fusion generated by the freezing of the bath material. $B \to \infty$ also signifies the thermal conductivity of the additive of negligible value, $k_a \to 0$. In this situation, the additive behaves almost as a perfect insulator with infinite thermal resistance. Here owing to this property, even a small amount of heat conducted to the additive, which is supplied by the bath and the freezing of the bath material increases the interface temperature to the freezing temperature of the bath material. It is the same which was obtained earlier for the high melting temperature plate and cylinder, and low melting temperature cylindrical additives [2,9,10].

— CASE 3: $\mathbf{S_t \to 0, \ \ 0 \le B \le \infty}$

$S_t \to 0$ is indicative of the bath material of infinite latent heat of fusion liberating a large amount of heat which when conducted to the additive raises the interface temperature to the freezing temperature of the bath material, $\theta_e = 1$. It is corroborated from the present findings of $\theta_e = 1$ when $S_t \to 0$ is substituted in equation (34), which in turn, is employed in equation (25). It is noted that the same results was derived for high melting temperature plate and cylindrical additives and low melting temperature cylinder[2,9,10].

— CASE 4: $\mathbf{S_t \to \infty, \ \ 0 \le B \le \infty}$

Its application to equation (33) , gives $a_1 = \sqrt{\frac{B}{6}}$ for positive value of the frozen layer thickness. The corresponding expression for θ_e from equation (25) becomes,

$$\theta_e = \frac{\sqrt{6B}}{3 + \sqrt{6B}} \qquad [35]$$

It is exactly the same as obtained for the high melting temperature cylindrical additive [9]. It is realistic because $S_t \to \infty$ denotes the bath material of negligible latent heat of fusion due to which a large thickness of the bath material freezes onto the additive, simulating the additive at an initial temperature brought in contact with the frozen layer at the freezing temperature of the bath material giving equation (35). Prasad and Nandi [1], in case of hot plate was brought with a cold plate, and Singh and Prasad [2], in case of freezing of the bath material onto the additive obtained the interface temperature θ_e as

$$\theta_e = \frac{\sqrt{24B}}{3 + \sqrt{24B}} \qquad [36]$$

The above expression possibly differs due to geometry of the additive.

5. RESULTS AND DISCUSSIONS

The current investigation devises a mathematical model of integral format for instant interface temperature at the contact surface formed as soon as the spherical solid additive immersed in the hot melt bath. The model gives a closed-form expression for this temperature. It is functions of the phase-change parameter of the frozen of the bath material, Stefan number S_t and the property-ratio B of the additive-bath system. The B is the ratio of the effusivity of the bath material to that of the additive. Physically, B behaves as a thermal force. Its low value is indicative of low driving thermal force developed in the bath for transferring less heat to the additive resulting in development of lower temperature at the interface (Figure 2). The Stefan number S_t is the ratio between the sensible heat and latent heat of fusion of the bath material. Its large value denotes the bath material of small latent heat of fusion owing to which large thickness of the frozen layer develop onto the additive for the same convective heat available from the bath. The values of B and S_t for different additives-bath systems are presented in Table 1. For extreme values of B, $(0 \le B \le \infty)$, and S_t $(0 \le S_t \le \infty)$, the expression for θ_e in equation (25) becomes those of the high melting temperature cylindrical additive [9].

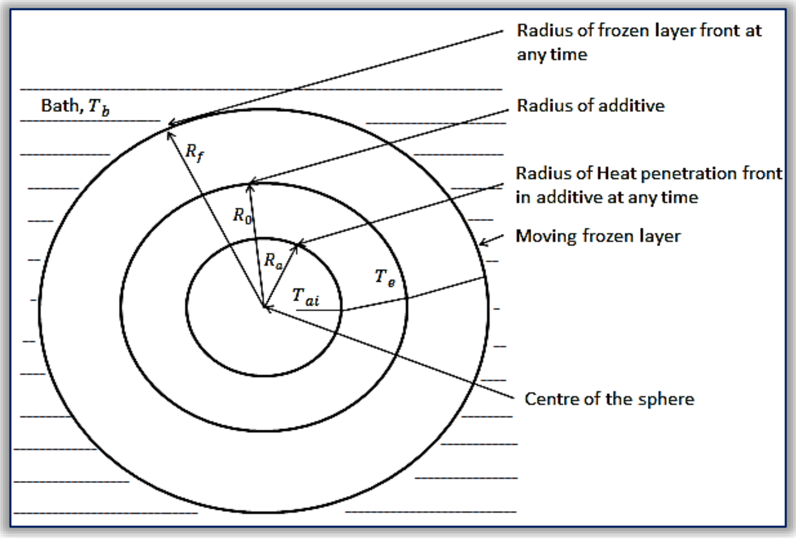

Figure 1—Schematic of freezing of the bath material onto the spherical shaped additive immersed in bath for time $\tau = 0^+$

Table 1: Thermophysical properties of additive bath system with their non-dimensional parameters

Solid additive	ρ kg m^{-3}	Melting temperature K	C_p J kg^{-1}°C^{-1}	k W m^{-1}K^{-1}	αx10^6 m^2 s^{-1}	B
Mo	10240	2883	251	138	53.7	0.453
Ti	4500	1953	611	22.0	8.0	2.655
V	6100	2192	502	31.4	10.3	1.670

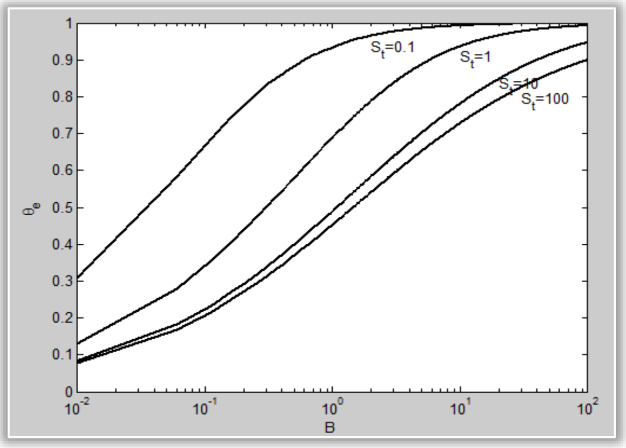

Figure 2—Property ratio B dependent instant interface temperature θ_e with Stefan number S_t as a parameter

Shown in figure 2 is the behavior of instant interface temperature θ_e with the property-ratio B for different values of S_t. For each S_t, it denotes that increasing B from 0 to ∞ increases θ_e swiftly from initial temperature of the additive to the freezing temperature of the bath material. It is realistic since $B \rightarrow \infty$ reduces the thermal resistance of the frozen layer to such a value that there is no drop in the temperature occurring in the frozen layer. It results in the temperature at the interface same as that of the moving front. When S_t decreases, the θ_e increases at any B and assumes asymptotically the freezing temperature for large B. Moreover, at low values of B, θ_e moves towards the freezing temperature once S_t assumes a small value. From physical point of view, this behavior is correct because small S_t represents the bath material of large latent heat of fusion resulting in the growth of thin frozen layer that offers negligible thermal resistance to the heat flow from the bath to the additive. Such a situation allows the interface to attain the freezing temperature. Also, these values are lower than those found for the plate additive in the previous investigation [2] because during the same time the rate of development of the frozen layer is slower than that obtained for the plate additive. The heat liberated from this frozen layer is less. It gives the total heat available from the freezing and the convective heat from the bath conducted to the additive smaller than that transferred to the plate additive leading to attainment of lower temperature at the interface between the spherical additive and the frozen layer.

The Stefan number S_t variant instant interface temperature θ_e is depicted in figure 3 for different property-ratio B. It states that θ_e reduces as S_t increases for each B but as B reduces from 100 to 0.1, the rapidity of reduction in θ_e increases for $S_t \leq 0.1$ whereas for $S_t > 10$, an asymptotic decrease in θ_e takes place in the entire range of S_t ($10^{-2} \leq S_t \leq 10^2$), θ_e increases with increase in B. These predictions appear to be true because for a particular B a small S_t denoting a bath material of large latent heat of fusion provides growth of a smaller thickness of a frozen layer that gives less thermal resistance and absorbs a small quantity of sensible heat. Consequently, more heat is available that is transferred to the additive resulting in larger θ_e (figure 3). On the other hand, decreasing B for a prescribed S_t, the thermal resistance for the frozen layer increases permitting less amount of heat to be transferred to the additive. It results in smaller θ_e. Comparing these values with those of the plate additive, these values are lower than those of the plate additive studied earlier [2].

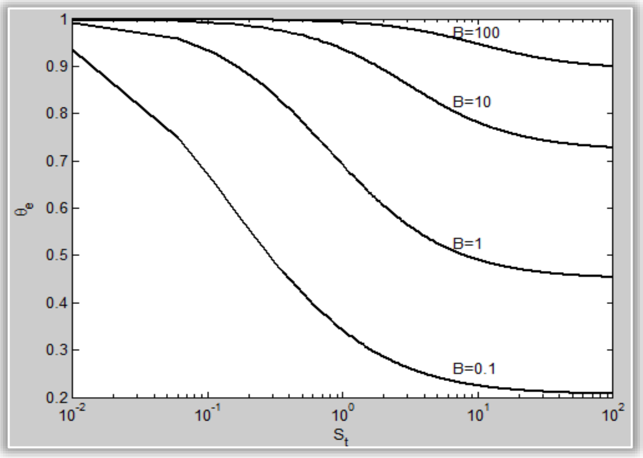

Figure 3—Stefan number S_t variant instant interface temperature θ_e with the property ratio B as a parameter

Thermophysical properties of the steel bath: $C_p = 0.670$ kJ kg^{-1} °C^{-1}, $\rho = 6850$ kg m^{-3}, $T_{bm} = 1531$°C, $T_b = 1600$°C, L=271.97 kJ/kg, $\theta_b = 1.03$, $S_{tb} = 3.71$

6. CONCLUSIONS

The integral model developed above for the interface temperature attained soon after dunking the additive in the bath gives closed-form solution for this temperature. It exhibits θ_e dependence on the property ratio B and the Stefan number S_t. For any combination of B and S_t, the values of θ_e are exactly the same as those of high melting temperature cylindrical additive but lower than those of the high melting temperature plate additive. θ_e reaches the initial temperature of the additive when $B \to 0,\ \ 0 \le S_t \le \infty$, whereas it attains the freezing temperature of the bath material once $B \to \infty,\ \ 0 \le S_t \le \infty$ or $S_t \to 0,\ \ 0 \le B \le \infty$. Incase $S_t \to \infty,\ \ 0 \le B \le \infty$, it assumes $\frac{\sqrt{6B}}{3+\sqrt{6B}}$.

NOMENCLATURE:

B property-ratio (=$K_m \rho_m C_{pm}/K_a \rho_a C_{pa}$)
B_i Biot number (=hR_0/K_a)
B_{im} modified Biot number (=(hR_0/K_a)*($K_a C_a/K_m C_m$))
C Heat capacity (=ρC_p), J m^{-3} K^{-1}
C_p specific heat, J kg^{-1} K^{-1}
C_r heat capacity-ratio (=C_f/C_a)
h heat transfer coefficient, W m^{-2} K^{-1}
k thermal conductivity, W m^{-1} K^{-1}
L latent heat of fusion, J kg^{-1}
S_t Stefan number (=$C_m(T_f - T_i)/\rho_m L_m$)
t time, s
T temperature, K
T_f freezing or melting temperature of the bath material, K
R radius, m
R_f radius of frozen layer front at any time, m
R_a radius of heat penetration front at any time, m
R_0 radius of the spherical shaped additive, m

GREEK LETTERS

α thermal diffusivity (m^2 s^{-1})
ξ_a nondimensional radius within heat penetration (R_a/R_0)
ξ_f nondimensional radius of the frozen layer front ($C_m R_a/C_a R_0$)
η nondimensional radius of the heat penetration front in the additive at any time (R_a/R_0)
ρ density (kg m^{-3})
θ nondimensional temperature ($(T - T_i)/(T_f - T_i)$)
τ nondimensional time [$(K_m C_m/C_a^2 R_0^2)t$]

SUBSCRIPTS

a spherical additive
b bulk condition of the bath
e at the interface between the additive and the frozen layer at time, $\tau = 0^+$
m frozen bath material

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