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# IDENTIFICATION OF THE OPTIMAL LOCATION OF ELASTOPLASTIC DAMPING JOINTS IN SQUARE-PLAN FOLDED LOW SHELLS UNDER DYNAMIC ACTION

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**Abstract:** This work is devoted to identifying the optimal location of elastoplastic damping joints incorporated into folded low shells under dynamic action. To do this, the system of governing motion equations of folded low shells is established using the Dirac δ-functions and Heaviside unit function. The solution is obtained by Bubnov-Galerkin method for determining the minimum frequencies in free vibrations of the folded low shells according to the wave number and fold edge number. Based on the obtained minimum frequencies and the associated vibratory forms, the optimal locations of the elastoplastic damping joints of 12m x12m square plane folded low shells are determined depending on the wave number and fold edge number.
**Keywords:** Folded low shell, wave number, fold edge number

## 1. INTRODUCTION

The damping of structural vibrations is a problem present in all areas of construction. Damping solutions range from passive devices (e.g., introduction of friction or integration of viscoelastic materials) [1] to active control. Depending on the different types of damping devices their implementation can take different forms. Depending on the frequency band you want to treat the damping devices can be damping pulls [2,3] or sticked panels to the structure[4]. In the past few years, many damping devices have been developed and applied in civil engineering structures, especially in earthquake-prone regions[5 – 7]. Viscous dampers generally represent a broad class of passive energy dissipation devices and have emerged as one of the most popular ones. By providing additional damping and energy dissipation, they can efficiently suppress earthquake-induced vibrations and, therefore, limit the damage to structural and non-structural components [8 – 10]. To better understand the damping properties of such dampers, many experimental studies have been conducted, including damper element tests [11,12] and shaking table tests of small-scale and full-scale structures with viscous dampers[13,14]. However, studies focusing on in-service structures with such dampers when subjected to large earthquakes are relatively rare[15,16]. This practical issue requires accurate state quantification for dampers and structure itself, which holds a key position in structural health, safety, and risk assessments[17,18]. Recent applications involving dampening treatments show a tendency to incorporate treatments into vibrating structures. These new methods, some of which are presented by[19], most often result in choosing a "smart glue" in order to reduce the vibration amplitudes of the structure. The design of an efficient damping system is a challenge that engineers are addressing with the demand for increasingly stressed and lighter structures. To do this, the most well-known and most widely used means is the use of viscoelastic materials in various forms. Thus, these can come in the form of uniform layers inserted between two elastic layers (viscoelastic heart sandwiches) [20 – 23] in the form of a honeycomb filled with viscoelastic material[24,25]. There are also soul-material systems made of tangled fibers[26].

For folded shells, the finite element modelling of the dynamic response of structures consisting of quadrangle panel assembly is a well-known technique. When these structures are subject to dynamic actions significant dynamic shear stresses occur at shell junctions posing a dangerous threat to structural rigidity and stability. To overcome this disadvantage one of the techniques used is the incorporation of damping elastoplastic inserts at all junctions in order to reduce the vibratory effects that are detrimental to the mechanical performance of folded shells. This leads to often very high implementation costs.

The aim of this work is to propose a method of identification the optimal location of elastoplastic damping joints in low folded shells under dynamic action in order to reduce their number and implementation costs.

## 2. MATERIALS AND METHODS

Studied structure is a low folded square-plane shell with a mathematical model presented in figure 1.

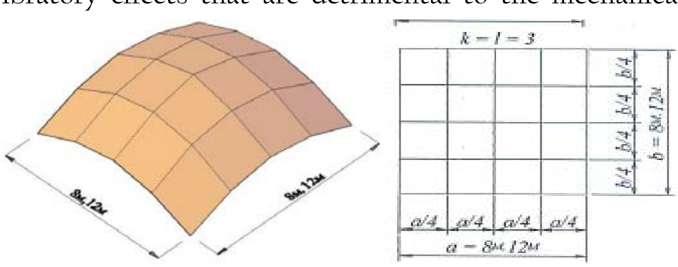


Figure 1. Mathematical model of square-plan folded shell

Determination of the elastoplastic damper joint location in the shell is based on the conditions of appearance the minimum frequencies and associated vibratory forms.

The system of governing motion equations for low shell is given by [27]:

$$\begin{cases} D\Delta^2 w + \Delta_k \varphi + \gamma h \ddot{w} = 0 \\ \frac{1}{Eh}\Delta^2\varphi - \Delta_k w = 0 \end{cases} \quad (1)$$

D – Low shell cylindrical stiffness; $\varphi$, w - stress and displacement functions ; h - shell thickness; E - Yong module; $\gamma$ – material density; $\Delta$ - Laplace's differential operator, $\Delta_k$ - Vlasov's differential operator

$$\Delta_k = k_x \frac{\partial^2}{\partial y^2} + k_y \frac{\partial^2}{\partial x^2} \quad (2)$$

Generalized functions are widely used for analysis of folded middle-plane shells, particularly the $\delta$-Dirac functions and Heaviside unit function.

In expression (2) for folded shell with double curvature, $k_x$. and, $k_y$ are given by [28]:

$$\begin{cases} k_x = \sum_{i=1}^{k} \theta_i \delta(x - x_i); \\ k_y = \sum_{j=1}^{l} \theta_j \delta(y - y_i), \end{cases} \quad (3)$$

$\theta_i$, et $\theta_j$ are the angles formed between the plane elements of the average shell surface.

$\delta(x - x_i)$ , $\delta(y - y_j)$ – the Dirac's delta functions, i =1,2...k ; j=1,2...l ;

k , l - the number of surface fold edges in directions x and y axes respectively.

By substituting (2) and (3) in equation system (1) one obtains equation system of folded shell motion in following form:

$$\begin{cases} D\Delta^2 w + \frac{\partial^2\varphi}{\partial y^2}\sum_{i=1}^{k}\theta_i\delta(x - x_i) + \frac{\partial^2\varphi}{\partial x^2}\sum_{j=1}^{l}\theta_j\delta(y - y_i) + \gamma h\ddot{w} = 0; \\ \frac{1}{Eh}\Delta^2\varphi - \frac{\partial^2\varphi}{\partial y^2}\sum_{i=1}^{k}\theta_i\delta(x - x_i) - \frac{\partial^2 w}{\partial x^2}\sum_{j=1}^{l}\theta_j\delta(y - y_i) = 0. \end{cases} \quad (4)$$

By deriving the motion equations (4) we get:

$$D\sum_{m}^{\infty}\sum_{n}^{\infty} w_{mn}V^2_{mn}\sin\alpha_m xy - \sum_{i=1}^{k}\theta_i\delta(x - x_i)\sum_{m}^{\infty}\sum_{n}^{\infty}\varphi_{mn}\beta^2_n \sin\alpha_m x\sin\beta_n y -$$
$$\sum_{j=1}^{N}\theta_j\delta(y - y_i),\sum_{m}^{\infty}\sum_{n}^{\infty}\varphi_{mn}a^2_n \sin\alpha_m x\sin\beta_n y - yh\sum_{m}^{\infty}\sum_{n}^{\infty} w_{mn}\omega^2_{mn}\sin\alpha_m x\sin\beta_n y = 0;$$
$$\frac{1}{Eh}\sum_{m}^{\infty}\sum_{n}^{\infty}\varphi_{mn}V^2_{mn}\sin\alpha_m x\sin\beta_n y + \sum_{i=1}^{k}\theta_i\delta(x - x_i)\sum_{m}^{\infty}\sum_{n}^{\infty}\varphi_{mn}\beta^2_n\sin\alpha_m x\sin\beta_n y +$$
$$\sum_{j=1}^{N}\theta_j\delta(y - y_i)\sum_{m}^{\infty}\sum_{n}^{\infty} w_{mn}\alpha^2_m \sin\alpha_m x\sin\beta_n y = 0. \quad (5)$$

In this form the Boubnov-Galerkin method can be used to solve these equations. After mathematical transformations one obtains the quadratic expression of free vibration frequencies of symmetrically folded surface of any configuration m and n, hypotheses for thin-walled low shells:

$$\omega^2_{mn} = \frac{1}{yh}\left( DV^2_{mn} + \frac{4Eh\left(\frac{\beta^2_n}{a}\vartheta_x + \frac{\alpha^2_m}{b}\vartheta_y\right)^2}{V^2_{mn}} \right) \quad (6)$$

Here $\vartheta_x = \sum_{i=1}^{k}\theta_i \sin^2\alpha_m x$ et $\vartheta_y = \sum_{j=1}^{l}\theta_j\sin^2\beta_n y$, $V_{mn} = \alpha^2_m + \beta^2_n$ ; $D = \frac{Eh^2}{12(1-\mu^2)}$ – the shell cylindrical rigidity.

$\propto_m = \frac{m\pi}{a}$, $\beta_m = \frac{n\pi}{b}$ ; m and n the wave number following x and y axes respectively ; a and b the shell dimensions following x and y axes respectively.

Expression (6) determines the free vibration frequencies of low folded shell with any fold number in both directions.

## 3. RESULTS AND DISCUSSION

### — Determining the minimum frequencies of square-plan folded shells

The structure under consideration is a square plane folded shell of 12m x12m.

In the figure 2 and figure 3 are shown the frequency variation of the square-plane low shell depending on the wave number m and n and the fold ridge number k and l in both directions.

Analysis of the curves of figure 2 and figure 3 shows that:

≡ When the edge number *k = l* is between 1 and 3, there is a minimum frequency for *m=k+1* and *n=l+1*. As a result the dangerous sections will be the sections at the junctions from which occur rotations and shears of plane elements one in relation to the other.

≡ When *k=l*> 3 we have a minimum frequency for *m=1* and *n=1*. In this case no shear in the central section.

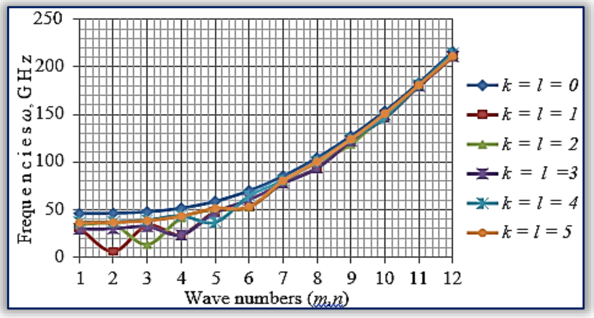



Figure 2. Frequency variation curves based on the wave numbers (m, n) for 12x12m shell

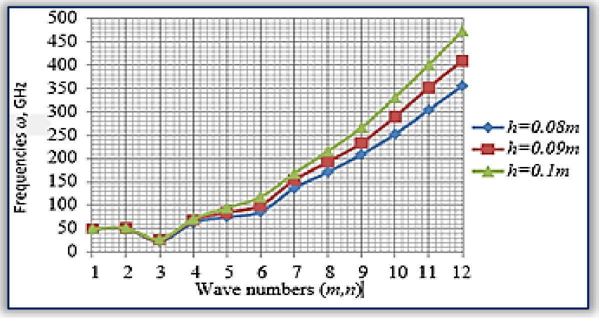



Figure 3. Frequency variation curves based on 12x12m shell thickness

There is a decrease in the minimum frequency for decrease in the edge number and thickness but there is decrease in the minimum frequency for increase in the shell dimension on the plane. Hence the need to provide damping devices in the structure.

Table 1 shows that the free vibration frequency values of studied low-plane folded shell increase significantly when the wave number in both directions m and n is greater or equal to 4. On the other hand, when the shell's dimensions on the plane are larger there is a decrease in frequency values.

Table1. Free vibration frequency values of 12m x12m square plane shells

| 12m x 12m shells | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| m = n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| k = l = 0 | 46.0 | 46.3 | 47.8 | 51.6 | 58.8 | 69.9 | 85.2 | 104.4 | 127.3 | 153.5 | 183.1 | 215.9 |
| k = l = 1 | 30.8 | 5.9 | 33.4 | 23.4 | 47.8 | 52.7 | 78.1 | 93.8 | 122.6 | 146.5 | 179.9 | 211.0 |
| k = l = 2 | 34.1 | 34.6 | 13.2 | 41.4 | 50.0 | 52.7 | 79.5 | 99.8 | 118.7 | 150.4 | 180.5 | 211.0 |
| k = l = 3 | 29.4 | 30.0 | 32.2 | 23.4 | 47.0 | 60.4 | 77.6 | 93.8 | 122.3 | 149.4 | 179.7 | 211.0 |
| k = l = 4 | 36.8 | 37.2 | 39.0 | 43.6 | 36.6 | 64.3 | 80.6 | 100.7 | 124.2 | 146.5 | 181.0 | 214.1 |

Table 2 Minimum frequencies, wave numbers and edge numbers values for 12x12m square plane shells

| Square plane folded shells | | | |
|---|---|---|---|
| Wave numbers | Fold edge numbers | Minimum frequency $\omega_{min}$ (GHz) | Wave numbers for which $\omega_{min}$ |
| m = n = 1...12 | k = l = 1 | 5.86 | m = 2; n = 2 |
| | k = l = 2 | 13.18 | m = 3; n = 3 |
| | k = l = 3 | 23.44 | m = 4; n = 4 |
| | k = l = 4 | 36.78 | m = 1; n = 1 |
| m = 1<br>n = 1...12 | k = l = 1 | 24.86 | m = 1; n = 2 |
| | k = l = 2 | 31.53 | m = 1; n = 3 |
| | k = l = 3 | 29.43 | m = 1; n = 1 |
| | k = l = 4 | 36.78 | m = 1; n = 1 |
| m = 2<br>n = 1...12 | k = l = 1 | 5.86 | m = 2; n = 2 |
| | k = l = 2 | 25.44 | m = 2; n = 3 |
| | k = l = 3 | 27.71 | m = 2; n = 4 |
| | k = l = 4 | 36.93 | m = 2; n = 1 |
| m = 3<br>n = 1...12 | k = l = 1 | 13.42 | m = 3; n = 2 |
| | k = l = 2 | 13.18 | m = 3; n = 3 |
| | k = l = 3 | 26.25 | m = 3; n = 4 |
| | k = l = 4 | 37.47 | m = 3; n = 1 |

The fold edge numbers k and l as well as the wave numbers m and n are reported in Table 2. In this same table are reported the calculation results of the minimum frequencies as function of different fold edge numbers and different wave numbers.

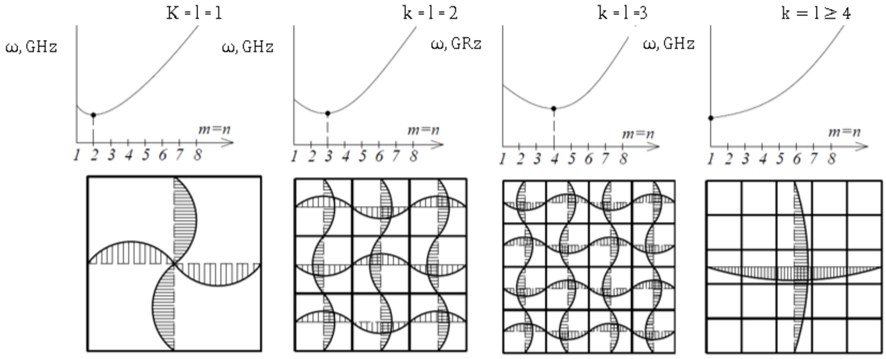
— **Determining of optimal location of elastoplastic damping joints in square-plane folded low shells**

In Figure 4 are shown the curve appearances of the free frequencies and vibratory form variation of 12m x 12m square plan folded shell as a function of different wave numbers and different fold edge numbers. In this same figure are shown the vibratory forms corresponding to given minimum frequency.

Curve analysis shows that for shells with a fold edge in each direction (k=l=1) the minimum frequencies occur when m=n=2; for shells with three edges (k=l=3) the minimum frequency occurs when m=n=4. So, the joints will be arranged in the central sections to the right of the fold edges (figure 4-c).

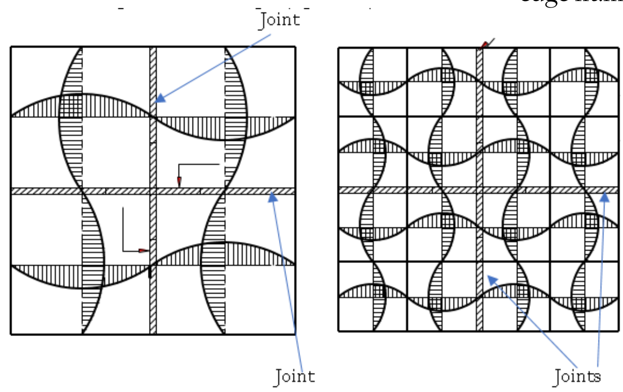
For shells with two edges (k = l = 2) a minimum frequency is obtained when m = n = 3, corresponding to a symmetrical vibratory form. In this case if the joint was laid out in the center section it would not work at shear this is why it is more rational to arrange it on the edge lines (figure 5).

For k=l≥4 we have minimum frequency when m=n=1. So, for even fold edge number and odd wave number, the joints arranged on the edge lines of the symmetrical sections relative to the central axes will be subjected to shear forces (figures 6-7) while for odd fold edge number and even wave number damping joints arranged on central section edge lines and sections symmetrical to the central lines will be subjected to shear forces (figure 5).




a) $\omega_{min}$ for m=n=2 b) $\omega_{min}$ for m=n=3 c) $\omega_{min}$ for m=n=4 d) $\omega_{min}$ for m=n=1

Figure 4. Free frequency and vibratory form variation of square plane folded shell as function of wave numbers and fold edge numbers




a) b)

Figure 5. Arrangement of joints (inserts) of square-plane shells: a) k=l=1; b) k=l=3

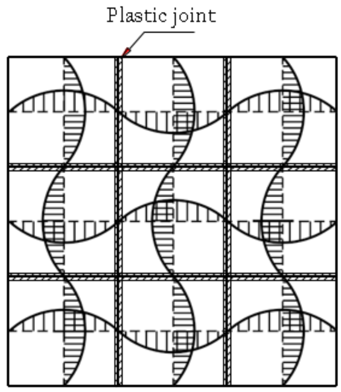



Figure 6. Arrangement of plastic joints in square-plane folded shells for k=l=2

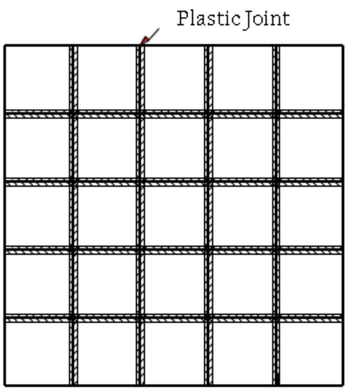



Figure 7. Arrangement of plastic damping joints of square-plane folded shell for k=l=4

## CONCLUSION

In this work we can retain the following:

— After having built a mathematical model of square plane folded shell it is proposed a method of determining the optimal location of the elastoplastic damping joints at the junctions of the folded shell flat elements.

— The minimum frequencies in free vibration of 12m x 12m square plane folded shell shall be determined.

— From obtained minimum frequencies it is constructed the associated vibratory forms for different wave number values m and n, different fold edge numbers k and l.

— Finally, it is proceeded to the determination of the optimal location of the elastoplastic damping joints in low square plane folded shells according to the wave numbers and the fold edge numbers.

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