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KINEMATIC MODEL OF AN ANTHROPOMORPHIC ROBOTIC ARM

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Abstract: The kinematic model of an anthropomorphic robotic arm was created. The phalanges were considered rigid rods. The dimensions, the initial position and orientation, as well as the relative rotation angles of each element with respect to the previous one were considered. An analysis was performed through the direct kinematics method, using the rotation matrices corresponding to each hand element. Based on the theoretical considerations, the final positions of the components have been determined, as well as the trajectories of the finger–endpoints. The results are intended for the control of a hand–arm system built by the authors.

Keywords: anthropomorphic, direct kinematics, robotic arm, rotation matrices

1. INTRODUCTION

The anthropomorphic grippers have recently became an increasingly interesting research subject for many researchers due to the versatility of the grapping method and because they confer the possibility of manipulating an already grasped object, in contrast with the capabilities of the grippers currently used in the industry (Murray, R.M. et al, 2006; Touvet, F. et al, 2012).

The human hand is a complex biological mechanism composed of 27 bones (14 phalanges, 5 metacarpals and 8 carpals) which can be mechanically modelled as a system of rods connected by single and double joints with 25 degrees of freedom (Abdel–Malek, K. et al, 2006; Săvescu, A.V. et al, 2004).

The human arm, excluding the hand, is a redundant mechanism because the number of controllable degrees of freedom, 3 in the shoulder joint, 1 in the elbow joint and 3 in the wrist joint, is higher than the total number of degrees of freedom for the arm and forearm, which is equal to 6 (Murray, R.M. et al, 2006).

For an adult person, there can be found in the literature (Adewusi, S. et al, 2014) the following mean values for the length of the upper arm and for the forearm: 285mm and 260mm, respectively. Mean lengths for the phalanges and metacarpals can be found in the literature, as well (Buryanov, A., Kotiuk, V., 2010). These dimensions are presented in Table 1, where the finger number is defined in Figure 1.

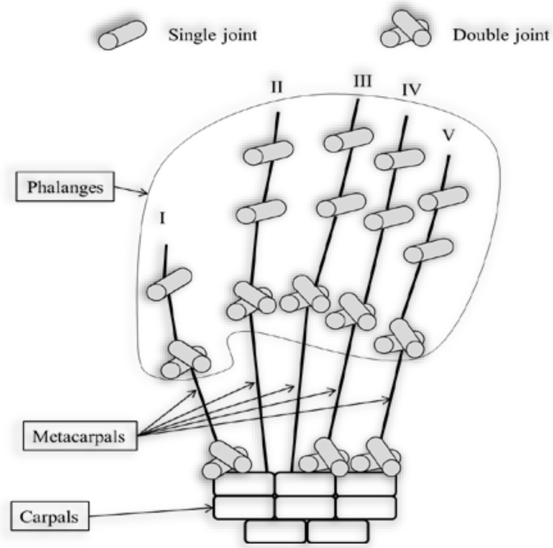


Figure 1 - The human hand in the model proposed by Abdel-Malek (Abdel-Malek, K. et al, 2006)

There are two main methods of analysing the kinematics of the hand–arm system: direct kinematics and inverse kinematics (Kurfess, T.R. (Editor), 2005). In this paper, the final position of every component of the hand–arm system will be determined using the direct kinematics method, for given rotation angles about the joints and known mean anthropomorphic dimensions of the human arm and hand.

2. MATHEMATICAL MODEL

The bones are considered rectilinear rigid rods with the dimensions given in Table 1. Although the human hand has only simple and double joints, the mechanical modelling becomes considerably easier by using the general mathematical transformations, valid for spherical joints. Unlike the phalanges, the carpals do not have a relative motion with respect to each other, therefore they make up a rigid subsystem.

Table 1. The mean lengths of the metacarpals and phalanges (Buryanov, A., Kotiuk, V., 2010)

Finger	Length of the metacarpal [mm]	Length of the proximal phalanx [mm]	Length of the middle phalanx [mm]	Length of the distal phalanx [mm]
I	46.2	31.6	–	21.7
II	68.1	39.8	22.4	15.8
III	64.6	44.6	26.3	17.4
IV	58.0	41.4	25.7	17.3
V	53.7	32.7	18.1	16.0

The nineteen bones in the human hand which move relatively with respect to each other (14 phalanges and the 5 metacarpals) create a mechanical system with 57 degrees of freedom (DOF). By adding the degrees of freedom conferred by the other three spherical joints of the shoulder, elbow and wrist, a 66 DOF mechanical system can be obtained, as seen in Figure 2.

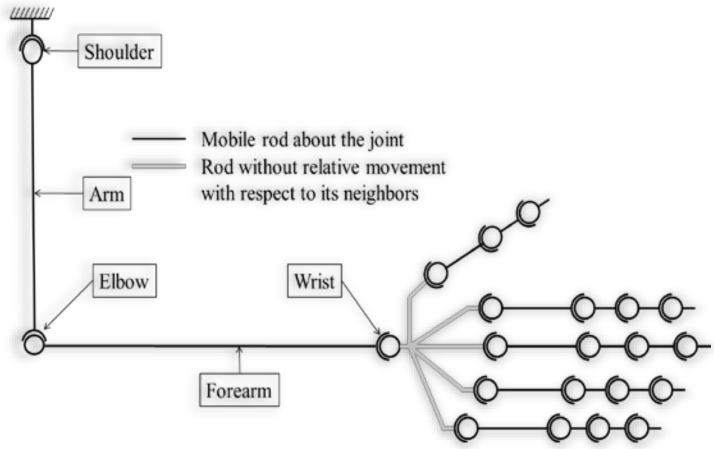


Figure 2 - The model of the human hand used for the study

Considering that the time variations of the rotation angles are known, it follows that the transformation between two positions of an arbitrary point is represented as a function of the initial position and time. Therefore, in the case of the rotation, this transformation has the form (Murray, R.M. et al, 2006)

$$f(X_1,t)=[R](t)\cdot\{X_1\}, \tag{1}$$

where $[R](t)$ is a 3 by 3, time dependent rotation matrix.

The matrix of rotation about an arbitrary axis is given by Rodrigues' formula (Bishop, R.H. (Editor), 2002)

$$[R](\{u\},\alpha)=\{u\}(u)+([I]-\{u\}(u))\cdot\cos\alpha+[AS](\{u\})\sin\alpha\,, \tag{2}$$

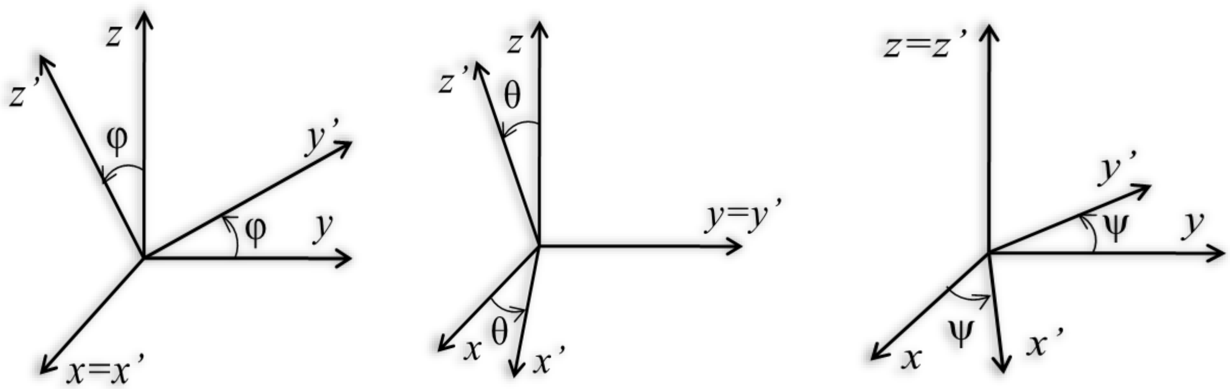
where:

- $\{u\},(u)$ represents the column and the row matrix, respectively, of the components of the rotation axis unit vector;
- $[I]$ represents the identity matrix in $R^{3\times3}$;
- α is the rotation angle;
- $[AS](\{u\})$ is a skew–symmetric matrix

$$[AS](\{u\})=\begin{bmatrix}0 & -u_z & u_y\\ u_z & 0 & -u_x\\ -u_y & u_x & 0\end{bmatrix}. \tag{3}$$

Using the notations c=cos(α) and s=sin(α), the rotation matrix can be written as:

$$[R]=\begin{bmatrix}u_x^{\,2}(1-c)+c & u_xu_y(1-c)-u_zs & u_xu_z(1-c)+u_ys\\ u_xu_y(1-c)+u_zs & u_y^{\,2}(1-c)+c & u_yu_z(1-c)-u_xs\\ u_xu_z(1-c)-u_ys & u_yu_z(1-c)+u_xs & u_z^{\,2}(1-c)+c\end{bmatrix}. \tag{4}$$



a) Rotation about the Ox axis with the angle φ   b) Rotation about the Oy axis with the angle θ   c) Rotation about the Oz axis with the angle ψ

Figure 3 – The rotations of the reference system with respect to the three coordinate axes

If the rotation takes place with the angle φ, about the Ox axis (Figure 3a), the unit vector is $(u)=(1\quad 0\quad 0)$, and the rotation matrix becomes (Murray, R.M. et al, 2006; Angeles, J., 2006):

$$[R_x](\varphi)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\varphi & -\sin\varphi\\ 0 & \sin\varphi & \cos\varphi\end{bmatrix}. \tag{5}$$

If the rotation takes place with the angle θ, about the Oy axis (Figure 3b), the unit vector is $(u)=(0 \quad 1 \quad 0)$, and the rotation matrix becomes (Murray, R.M. et al, 2006; Angeles, J., 2006):

$$[R_y](\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}. \tag{6}$$

If the rotation takes place with the angle ψ, about the Oz axis (Figure 3c), the unit vector is $(u)=(0 \quad 0 \quad 1)$, and the rotation matrix becomes (Murray, R.M. et al, 2006; Angeles, J., 2006):

$$[R_z](\psi)=\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}. \tag{7}$$

If the reference system is rotated first about Ox, next about Oy and finally about Oz (Figure 4), the composed rotation matrix is obtained by multiplying the rotation matrices (5)–(7) (Angeles, J., 2006; Spong, M.W. et al, 2002):

$$[R_{xyz}](\varphi,\theta,\psi)=[R_x](\varphi)[R_y](\theta)[R_z](\psi). \tag{8}$$

It results

$$[R_{xyz}](\varphi,\theta,\psi)=\begin{bmatrix}c\psi\cdot c\theta & -c\theta\cdot s\psi & s\theta\\ c\varphi\cdot s\psi + c\psi\cdot s\varphi\cdot s\theta & c\varphi\cdot c\psi - s\varphi\cdot s\theta\cdot s\psi & -c\theta\cdot s\varphi\\ s\varphi\cdot s\psi - c\varphi\cdot c\psi\cdot s\theta & c\psi\cdot s\varphi + s\theta\cdot s\psi & c\varphi\cdot c\theta\end{bmatrix}, \tag{9}$$

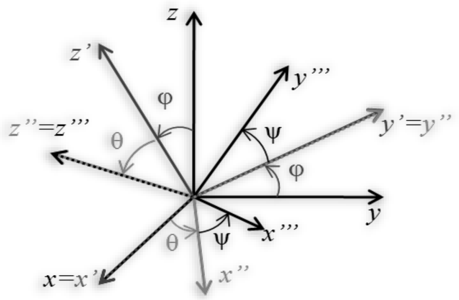


Figure 4 - Succession of rotations

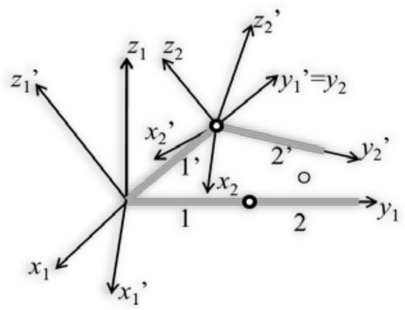


Figure 5 - Composing the rotations of a system with 2 rods

Since the mathematical model of the arm consists of rods and spherical joints, the problem of determining the configuration of the system can be reduced to one of finding the positions of the endpoints of the rods for any given angle. For a simpler system consisting of only two rods, as seen in Figure 5, in order to determine the final position of the second rod, the final position of the endpoint of the first rod must be determined first.

The initial coordinate system Ox2y2z2 of the rod 2 can be obtained by translating the rotated coordinate system Ox1'y1'z1' of the rod 1' along the Oy1' direction.

By introducing the column matrix of the components of the unit vector of the direction of rod "i"

≡ before the rotation, {X}i,

≡ after the rotation, {X}i',

the following relations can be written:

$$\{X\}_1=[R_{xyz}](\varphi_1,\theta_1,\psi_1)\{X\}_{1'}, \tag{10}$$

$$\{X\}_2=[R_{xyz}](\varphi_2,\theta_2,\psi_2)\{X\}_{2'}, \tag{11}$$

As stated before, systems $Ox_2y_2z_2$ and $Ox_1'y_1'z_1'$ are parallel, meaning that the directions of their respective rods are identical:

$$\{X\}_2=\{X\}_{1'}, \tag{12}$$

By substituting relations (10) and (11) in relation (12), the final direction of the second rod can be determined without previously determining the final direction of the first rod:

$$\{X\}_1=[R_{xyz}](\varphi_1,\theta_1,\psi_1)[R_{xyz}](\varphi_2,\theta_2,\psi_2)\{X\}_{2'}, \tag{13}$$

By extrapolating relation (13), a formula can be established for a chain of n rods, in order to find the direction of the n–th rod using the rotation matrices:

$$\{X\}_1=[R_{xyz}](\varphi_1,\theta_1,\psi_1)\cdots\cdots[R_{xyz}](\varphi_n,\theta_n,\psi_n)\{X\}_{n'}, \tag{14}$$

This rotation matrices are orthogonal, i.e. their transposes are equal to their inverses.

3. INPUT DATA

The model of the hand–arm system has been analysed in a right–handed orthogonal coordinate system, with axis Oz initially in the vertical direction and Oy along the corresponding segment.

The origin of the system (the shoulder) is located at a height H=600mm above a fixed 300x300mm platform.

Both the arm and the forearm are initially parallel, with the previously given dimensions.

The initial directions of the metacarpals and the phalanges are identical and given through their rotation angles around the Oz axis in Table 2, with the lengths defined in Table 1.

The variation of the angles has been chosen linear between 0 and their maximum value. The maximal rotation angles chosen for the arm, forearm and wrist joints can be found in Table 3.

Table 2. The rotation angles of the metacarpals and phalanges of each finger around the Oz axis

Finger	Ψ_0[rad]
I	$\pi/4$
II	$\pi/30$
III	0
IV	$-\pi/30$
V	$-\pi/15$

The carpals are represented as rods, connecting the endpoints of the metacarpals with the wrist joint and have similar motions. Therefore, there is no need to define the rotation angles of the carpals, since they are identical to those of the wrist.

Table 3. The rotation angles of the arm, forearm and wrist

Angle	Arm	Forearm	Wrist
φ	$-\pi/4$	$\pi/4$	0
θ	$\pi/4$	0	$\pi/2$
ψ	$-\pi/4$	$\pi/2$	0

Around the Oy axis, neither the metacarpals, nor the phalanges are rotated. The rotation angles chosen for the metacarpals and phalanges about the Ox and Oz axes of the reference system are defined in Tables 4–5.

Table 4. The rotation angles of the metacarpals and phalanges about the Ox axis

Finger	Metacarpal	Proximal phalanx	Median phalanx	Distal phalanx
I	$-\pi/2$	$\pi/4$	–	$\pi/5$
II	0	$-\pi/4$	$-\pi/4$	$-\pi/4$
III	0	$-\pi/4$	$-\pi/4$	$-\pi/4$
IV	0	$-\pi/4$	$-\pi/4$	$-\pi/4$
V	0	$-\pi/4$	$-\pi/4$	$-\pi/4$

Table 5. The rotation angles of the metacarpals and phalanges about the Oz axis

Finger	Metacarpal	Proximal phalanx	Median phalanx	Distal phalanx
I	$-\pi/8$	$-\pi/6$	–	$-\pi/6$
II	0	0	0	0
III	0	0	0	0
IV	0	0	0	0
V	0	0	0	0

4. RESULTS

Taking into consideration that the initial directions of every rod, as well as the rotation angles about each of the main axes are given, a MATLAB program has been written in order to represent the arm and hand, as well as the variation of the elements of the rotation matrices during the entire motion.

The trajectories of the tip of each finger regarding the 3 fixed reference system axes are given in Figures 6–8. Using the data given in Tables 2–5, the initial and the final position of the hand–arm system were determined and were illustrated in Figure 9.

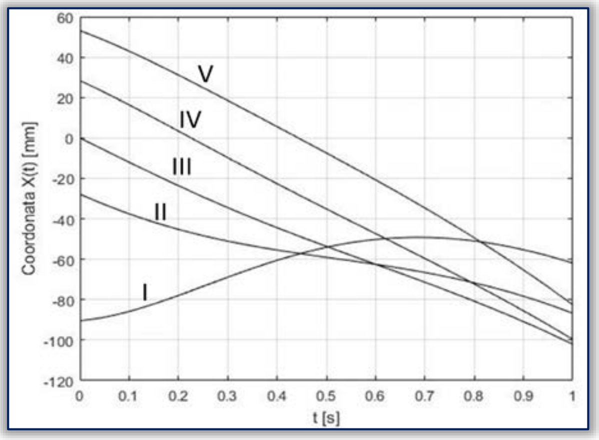


Figure 6 – Trajectory of the fingertips with respect to the Ox axis

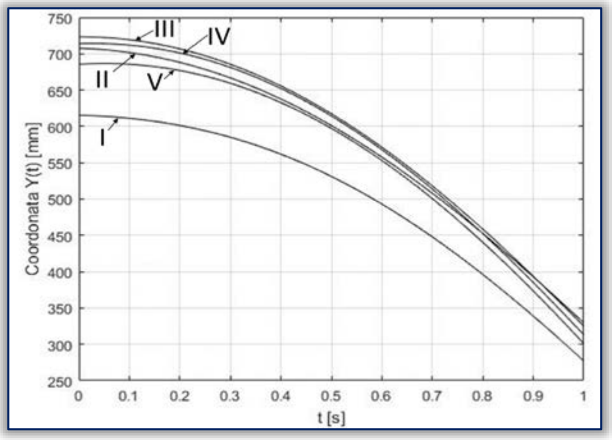


Figure 7 – Trajectory of the fingertips with respect to the Oy axis

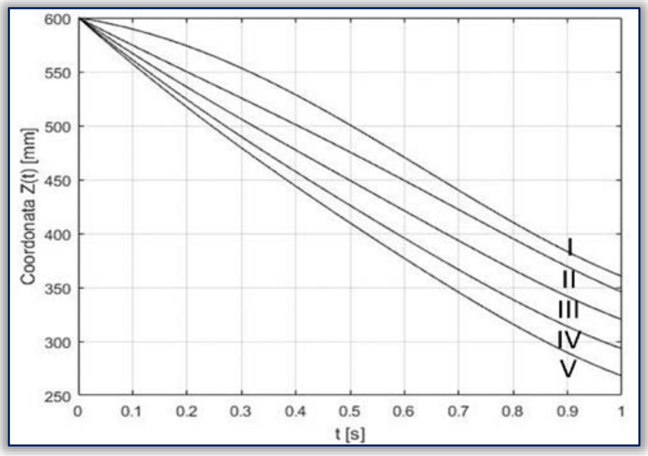


Figure 8 – Trajectory of the fingertips with respect to the Oz axis

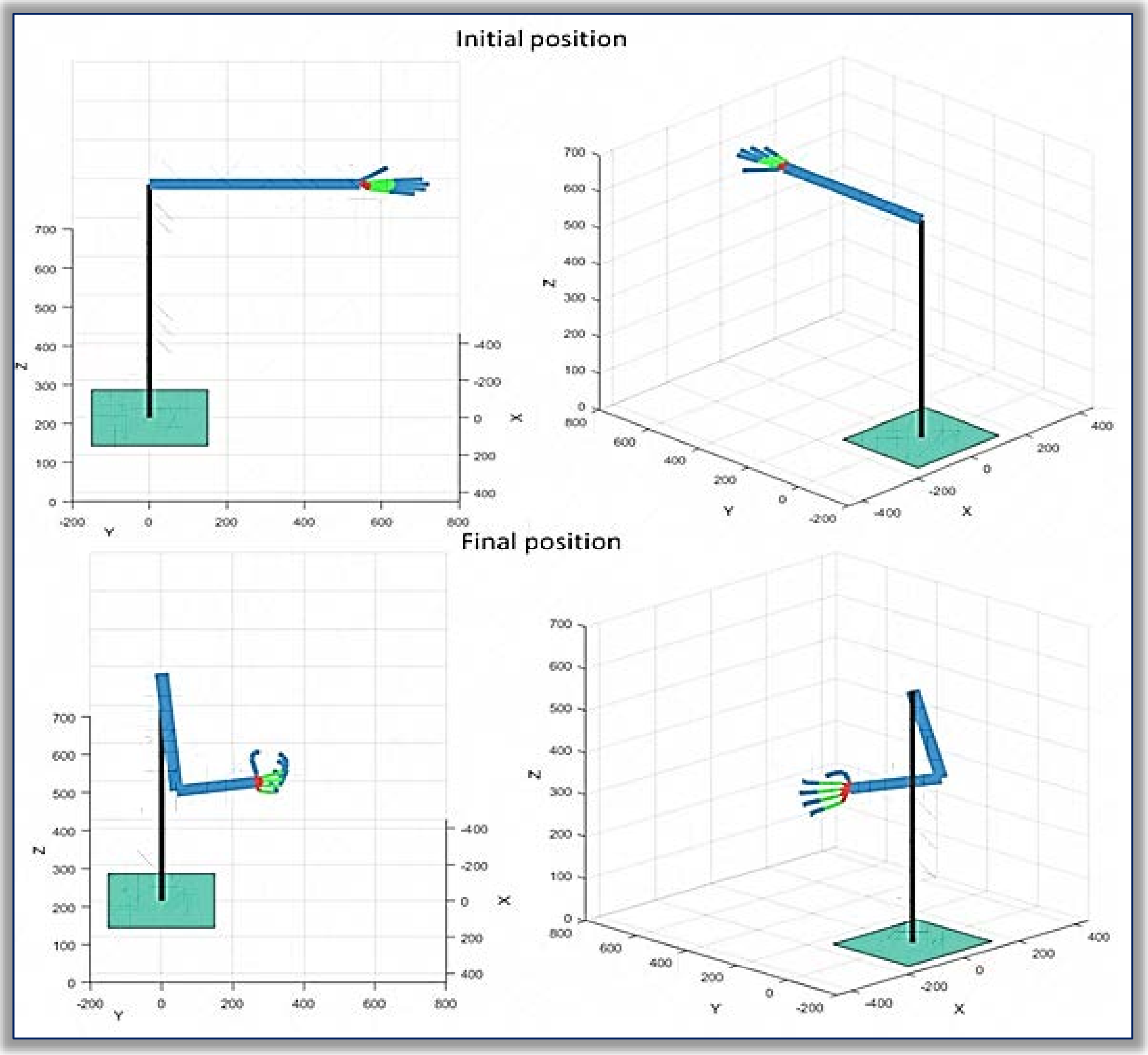


Figure 9 – Initial and final position of the hand–arm system

5. CONCLUSIONS

The full rotation matrices of a rod can be determined through successively multiplying the independent rotation matrices of the intermediate rods. The calculus of the matrices allows the determination of the direction of each element.

In order to define the position of a rod, it is necessary to know its origin point, the dimension and direction of the respective element. The direction can be obtained by means of the rotation matrices and the dimension of the rod is already known and remains constant, due to its rigidity. The origin point of the rod coincides with the end point of the previous rod in the kinematic chain.

Using the direct kinematic method, the final position of the hand–arm system can be determined by knowing the dimensions and initial orientation of each segment of the system, as well as the rotation angles about the 3 principal directions of the system.

The considerations presented in this paper are intended for the construction and control of a hand–arm system.

Note: This paper was presented at ISB–INMA TEH' 2021 – International Symposium, organized by University "POLITEHNICA" of Bucuresti, Faculty of Biotechnical Systems Engineering, National Institute for Research-Development of Machines and Installations designed for Agriculture and Food Industry (INMA Bucuresti), National Research & Development Institute for Food Bioresources (IBA Bucuresti), University of Agronomic Sciences and Veterinary Medicine of Bucuresti (UASVMB), Research-Development Institute for Plant Protection – (ICDPP Bucuresti), Research and Development Institute for Processing and Marketing of the Horticultural Products (HORTING), Hydraulics and Pneumatics Research Institute (INOE 2000 IHP) and Romanian Agricultural Mechanical Engineers Society (SIMAR), in Bucuresti, ROMANIA, in 29 October, 2021.

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