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THE INFLUENCE OF CARBON FIBER ORIENTATION ANGLE ON BUCKLING PROPERTIES OF FOUR–LAYER SYMMETRIC LAMINATE UNDER BIAXIAL COMPRESSION

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Abstract: The present work focused on the buckling of four–layer symmetric laminate subjected to biaxial compression. Governing equations are derived based on Classical Laminated Plate Theory (CLPT). The composite plates are bonded by an internal elastic medium and surrounded by external elastic foundation. The influences of carbon fiber orientation angle and aspect ratio on critical buckling load are demonstrated for symmetric laminate. We analyzed four lamination scheme which fiber angle orientation is equal to 0°, 30°, 45° and 90°.
Keywords: analytical modelling, buckling, composite plates, carbon fiber angle

1. INTRODUCTION

A composite material is composed of reinforcement (fibres, particles, flakes, and/or fillers) embedded in a matrix (polymers, metals, or ceramics). The matrix holds the reinforcement to form the desired shape while the reinforcement improves the overall mechanical properties of the matrix. A laminate is called symmetric if the material angle, and the thickness of plies are the same above and below the midplane.

To use the laminated composite plates efficiently, it is necessary to develop appropriate analysis theories to predict accurately their structural and dynamical behavior. Currently, the analysis of the behavior of the laminated plates is an active research area because of their complex behavior [1]. The structural instability becomes an important concern in a reliable design of composite plates. Several studies on laminated plates stability were concentrated on rectangular plates [2–5]. It is known that buckling strength of the rectangular plates depends on the boundary conditions [3], plies orientation [3,4,6] and geometrical ratio [3,5–7]. The thin composites structures which are largely used become unstable when they are subjected to mechanical or thermal loadings which leads to buckling .

To predict buckling load and deformation mode of a structure, the linear analysis can be used as an evaluation technique [8]. The analysis of the laminated plates is more complicated than the analysis of an isotropic and homogeneous material [9]. Finite element method is used for the analysis of the buckling behavior of the notched antisymetrical fibers plates under compression [10]. The majority of the investigations on laminated plates utilize either the classical lamination theory (CLT), or the first–order shear deformation theory (FSDT).

The main contribution of this work is to perform a composite laminated plates analysis by using the Classical Laminated Plate Theory (CLPT) is described in [11–13]. Various geometries of the plates subjected to compressive load are studied.

2. THEORETICAL FORMULATION

Let us consider composite plate the length of *a* , width *b* and height *h*, as shown in figure 1. The composite plates are surrounded by external elastic medium and subjected to biaxially compression. The external medium is modeled as Pasternak–type foundation which is equivalent to Winkler modulus parameter k_w and shear modulus parameter k_G of polymer matrix Figure 1.

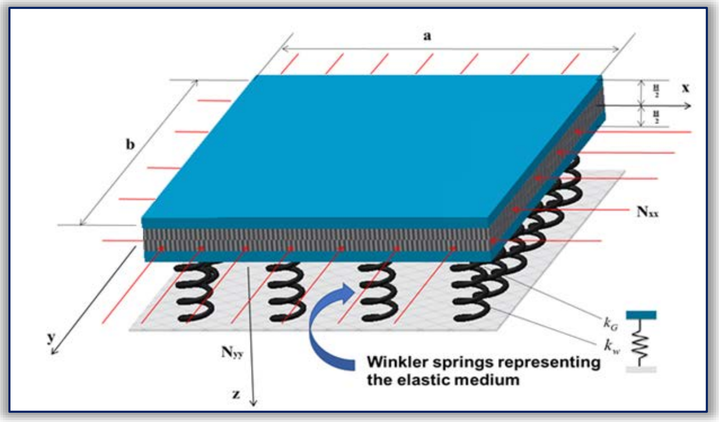


Figure 1. Schematic diagram of symmetric laminate subjected to biaxial compression

Bending displacements of the plate–1 and plate–2 are $w_1(x, y, t)$ and $w_2(x, y, t)$, respectively. It was assumed that each composite plate had the length, a and width, b. We assume that composite plates are biaxially compressed by forces N_{xx} and N_{yy} in the directions of x and y axes.

— Governing equations of biaxially compressed composites plates

The governing equation for biaxially compressed orthotropic composite plate embedded in an elastic medium [14], which is based on Classical Laminated Plate Theory CLPT, have following form.

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+k_w w-k_G\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=0 \quad (1)$$

We assume that composite plate is biaxially compressed in the directions of x and y axes, $N_x=N_y$. Now we can define compression ratio which equals the ratio between the forces acting in y and x directions

$$\delta=\frac{N_{yy}}{N_{xx}}\rightarrow N_{yy}=\delta N_{xx} \quad (2)$$

Substitution of equation (2) in equation (1) we derive the general form of governing equation

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+N_x\left(\frac{\partial^2 w}{\partial x^2}+\delta\frac{\partial^2 w}{\partial y^2}\right)+k_w w-k_G\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=0 \quad (3)$$

Before solving constituent equation (3), boundary conditions should be defined. In this study it is assumed that all edges on both nanoplates are simply supported. This means that both the displacements and moments at the composite plate edges are zero. This can be expressed by following equations

$$w_i(0,y,t)=0,\quad w_i(a,y,t)=0,\quad w_i(x,0,t)=0,\quad w_i(x,b,t)=0 \qquad i=1,2 \quad (4)$$

$$M_i(0,y,t)=0,\quad M_i(a,y,t)=0,\quad M_i(x,0,t)=0,\quad M_i(x,b,t)=0 \quad (5)$$

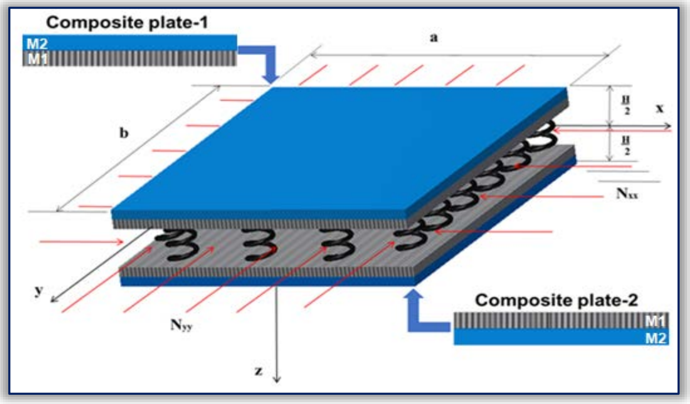


Figure 2. Symmetric laminate coupled by an elastic medium

If in the middle of the four–layer laminate sistem we insert an elastic medium that separate the laminate into two symmetric parts, we will have two composite plates with two laminae (Figure 2.) whose main equations are:

≡ Plate 1:

$$D_{11}\frac{\partial^4 w_1}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w_1}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w_1}{\partial y^4}+N_x\frac{\partial^2 w_1}{\partial x^2}+N_y\frac{\partial^2 w_1}{\partial y^2}+k_w\left(w_1-w_2\right)-k_G\nabla^2\left(w_1-w_2\right)=0 \quad (6)$$

≡ Plate 2:

$$D_{11}\frac{\partial^4 w_2}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w_2}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w_2}{\partial y^4}+N_x\frac{\partial^2 w_2}{\partial x^2}+N_y\frac{\partial^2 w_2}{\partial y^2}+k_w\left(w_2-w_1\right)-k_G\nabla^2\left(w_2-w_1\right)=0 \quad (7)$$

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \quad (8)$$

— Buckling loads of biaxially compressed composite plates

In this section different explicit cases of biaxial buckling will be considered. The symmetric laminate is subjected to both biaxial as well as biaxial compressive forces. The cases studied will be composite plates buckling with out–of–phase (asynchronous); in–phase (synchronous); and when one of the composite plates is considered to be fixed.

— Asynchronous–type buckling (out–of–phase)

Composite plates system is assumed to be bi–axially buckled. Figure 3 shows the three–dimensional configuration of double composite plates system with the asynchronous sequence of buckling:

$$w_1(x,y,t)-w_2(x,y,t)\neq 0$$

In out–of–phase, sequence of buckling the nanoplates is buckled in opposite directions. We evaluate the buckling load for asynchronous–type buckling and use equations (6,7) for the biaxial buckling solution of double composite plates system.

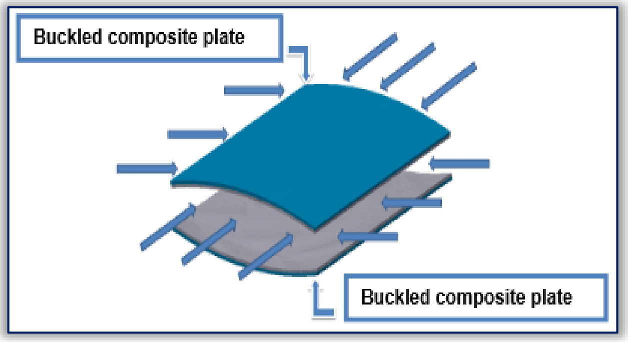


Figure 3. Asynchronous–type buckling

Subtracting equation (6) from equation (7) we get:

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+2k_w w-2k_G\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)=0 \tag{9}$$

$$w=w_1-w_2-\ w_2+w_1=2w_1-2w_2=2\left(w_1-w_2\right)=2w \tag{10}$$

We assume that the buckling mode of the double–nanoplate system as

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin\left(\alpha x\right)\sin\left(\beta y\right) \tag{11}$$

In the upper equation:

$$\alpha=\frac{m\pi}{a},\quad \beta=\frac{n\pi}{b} \tag{12}$$

where m and n are the half wave numbers.

Substituting equation (11) into equation (9), we get critical buckling load for asynchronous type of buckling

$$N_{cr}=\frac{D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4+2k_w+2k_G(\alpha^2+\beta^2)}{(\alpha^2+\delta\beta^2)} \tag{13}$$

— Synchronous–type buckling (in–phase)

The schematic illustration buckling of the orthotropic composite plates in–phase is shown in Figure 4, which is the first mode synchronous type buckling. For the present system, the relative displacements between the two composite plates are

$$w_1(x,y,t)-w_2(x,y,t)=0$$

In synchronous buckling state, the double composite plates system can be considered to be as one of the composite plates.

We apply the same procedure as earlier for solving equations

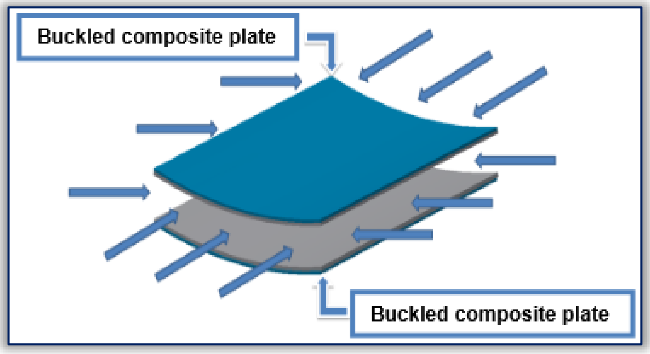


Figure 4. Synchronous–type buckling

Following procedure similar to that of out–of–phase buckling, critical buckling load for synchronous type of buckling can be written as

$$N_{cr}=\frac{D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4}{(\alpha^2+\delta\beta^2)} \tag{14}$$

Biaxial compression of double composite plates sistem can be effectively treated as a single composite plate because for this case the critical buckling load is independent of the stiffness of the coupling springs.

— Buckling with one composite plate fixed

Consider the case of composite plates sistem when one composite plate is stationary w2 = 0 which is shown in Figure 5. Critical buckling load for this type of buckling can be written as

$$N_{cr}=\frac{D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4+k_w+k_G\left(\alpha^2+\beta^2\right)}{(\alpha^2+\delta\beta^2)} \tag{15}$$

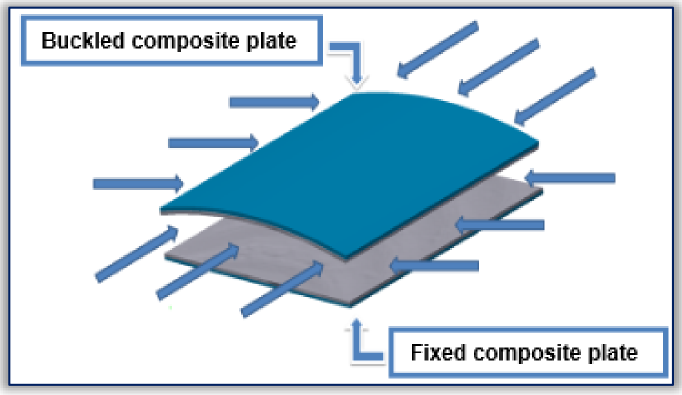


Figure 5. Buckling with one composite plate fixed

In fact, when one of the composite plates in composite sistem is fixed (w2 = 0), the composite system behaves as composite plate on an elastic medium.

3. NUMERICAL RESULTS

This section shows analysis of four–layer symmetric laminate made of two types of materials:

≡ Kevlar 49/CE 3305 (material M1)

≡ Graphite–Epoxy AS–1/3501–5A (material M2)

For laminates of total thickness of 1mm with four sheets of individual thickness of 0.25mm, bending stiffness matrix D has the following form [14]:

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{N}\left(\overline{Q}_{ij}\right)^{k}\left(h_k^3-h_{k-1}^3\right)$$

$$=\frac{1}{3}\left(\overline{Q}_{ij}\right)_{\theta}\left(h_1^3-h_0^3\right)+\frac{1}{3}\left(\overline{Q}_{ij}\right)_{\theta}\left(h_2^3-h_1^3\right)+\frac{1}{3}\left(\overline{Q}_{ij}\right)_{\theta}\left(h_3^3-h_2^3\right)+\frac{1}{3}\left(\overline{Q}_{ij}\right)_{\theta}\left(h_4^3-h_3^3\right)$$

$$=\frac{1}{3}\left(\overline{Q}_{ij}\right)_{\theta}\left[(-0,25)^3-(-0,5)^3\right]+\frac{1}{3}\left(\overline{Q}_{ij}\right)_{\theta}\left[(0)^3-(-0,25)^3\right]+\frac{1}{3}\left(\overline{Q}_{ij}\right)_{\theta}\left[(0,25)^3-(0)^3\right]+$$

$$\frac{1}{3}\left(\overline{Q}_{ij}\right)_{\theta}\left[(0,5)^3-(0,25)^3\right]$$

Based on the above expression and using the MATLAB software package, bending stiffness matrix for selected laminate schemes $\theta=0°,30°,45°,90°$ are obtained.

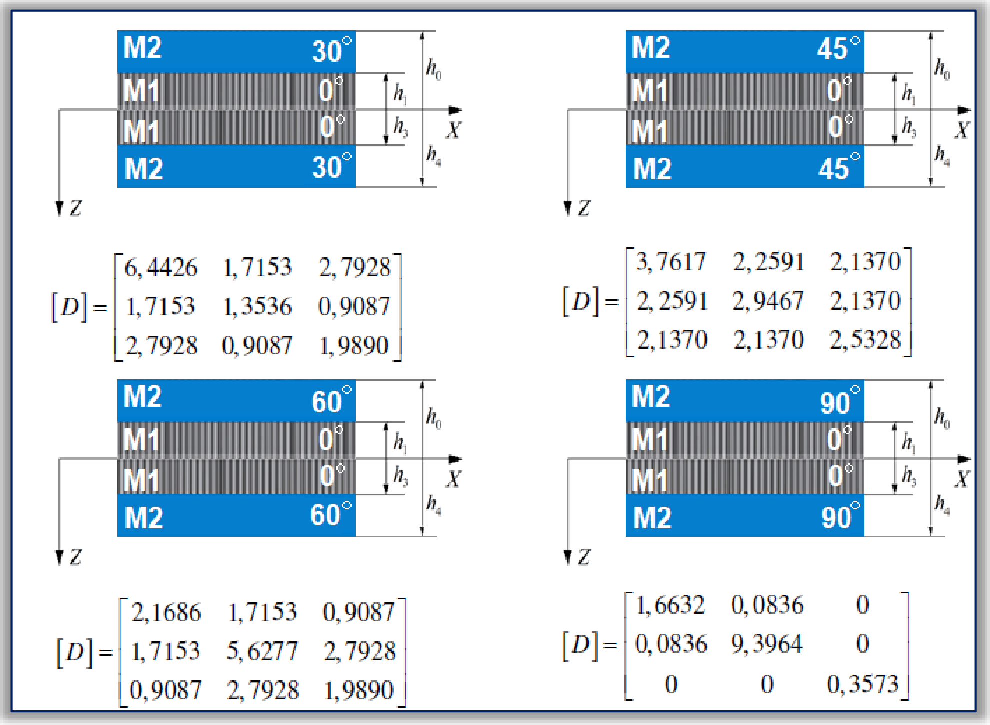
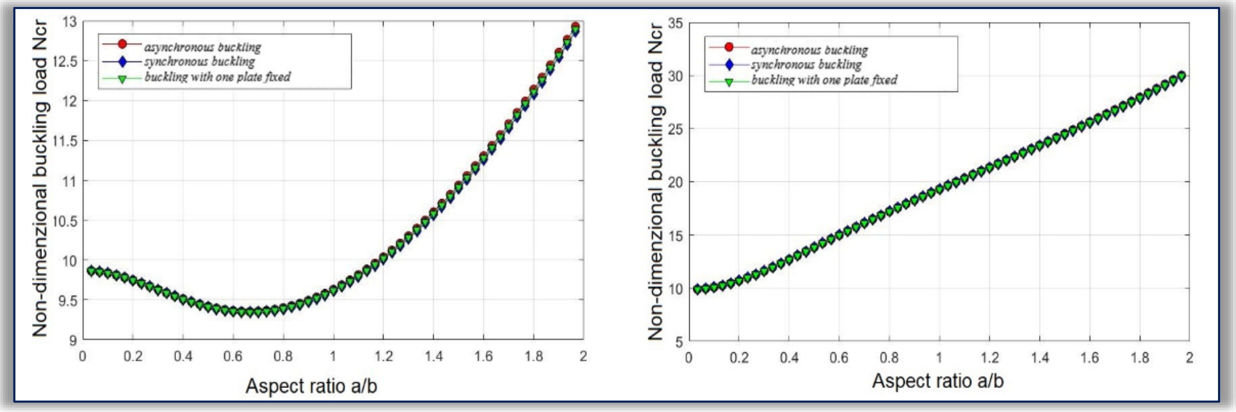


Figure 6. Schematic layout of symmetric laminate with bending stiffness matrix

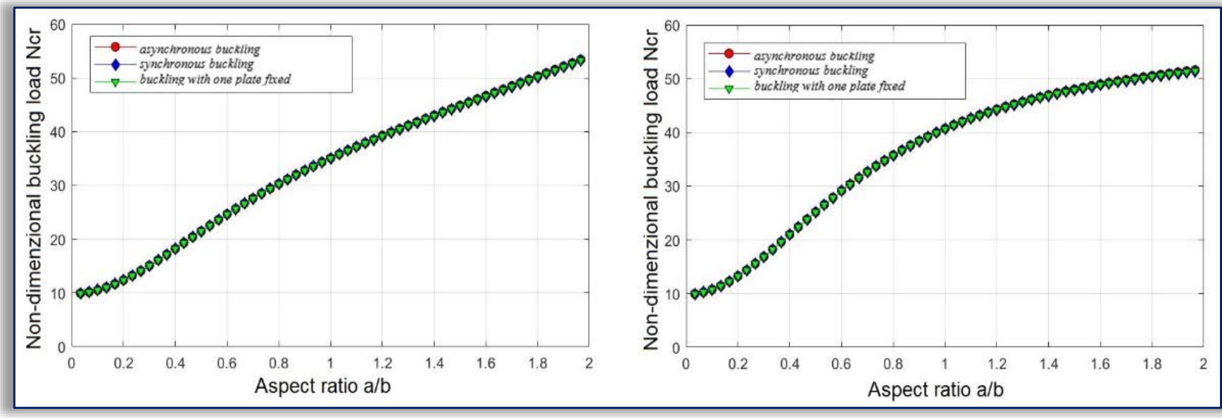
Substituting the values of bending stiffness matrix in the previously set equations we obtain values of non–dimensional critical force for three types of buckling.

Based on equations (13), (14) and (15), in this section follows analysis of carbon fiber orientation angle on the non–dimensional buckling load. Nondimensional buckling load is calculated for the value of Winkler modulus $k_w = 10N/m^3$ while the shear modulus parameter $k_G = 1N/m$. The number of half waves was m = 1, n = 1, while the compression ratio was δ =2. The thickness of one composite plate is h = 0,25 mm, while the length takes values within a=0–0,6m range and width takes value b=0,3.



$a) M 230° M 10° M 10° M 230°$ $b) M 245° M 10° M 10° M 245°$

Figure 7. Effect of carbon fiber orientation angle on non–dimensional buckling load



$a) M 260° M 10° M 10° M 260°$ $b) M 290° M 10° M 10° M 290°$

Figure 8. Effect of carbon fiber orientation angle on non–dimensional buckling load

It can be concluded that for different values of fiber orientation angle, the curves are very close together for all three buckling types. For value of fiber orientation angle $45°, 60°, 90°$ (Figure 7–b and Figure 8–a, 8–b) the non–dimensional critical force value is constantly increasing. The critical force has a minimum only for fiber orientation angle $30°$ and aspect ratio a/b=0,7 (Figure 7–a). A very small value of the non–dimensional critical force leads to the deformation of the composite plates and the occurrence of instability of the system.

CONCLUSION

In this paper, there are analytical expressions for non–dimensional buckling load for three characteristic cases of buckling of simply supported composite plates. Based on CLPT, in this paper was analyzed influence of fiber orientation angle and aspect ratio a/b on the non–dimensional buckling load on biaxial compressed composite plates embedded in elastic medium.

It has been shown that with the change of fiber orientation angle, the value of the non–dimensioning critical load is changed for all three characteristic buckling cases. Laminate have different minimum and maximum values of non–dimensional critical force at the same value of aspect ratio. For laminate with fiber orientation angle $M 230° M 10° M 10° M 230°$ non–dimensional critical buckling has minimum.

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