ANNALS of Faculty Engineering Hunedoara – International Journal of Engineering
Tome XX [2022] | Fascicule 2 [May]

[1.]Dimitar LOLOV, [2.]Svetlana LILKOVA–MARKOVA

DYNAMIC STABILITY OF A FLUID–IMMERSED PIPE CONVEYING FLUID AND RESTING ON A DAMPED WINKLER ELASTIC FOUNDATION

[1,2.]University of Architecture, Civil Engineering and Geodesy, Faculty of Hydraulic Engineering, Sofia, BULGARIA

Abstract: The present article examines the dynamics of a pipelining on a damped Winkler elastic foundation. The dependence of the critical velocity of the fluid in the pipe on the critical velocity of the external fluid for different parameters of the damped Winkler elastic foundation is investigated. The dynamic stability of a pipeline resting on a damped Winkler elastic foundation and immersed in fluid that is moving with a particular velocity is investigated. The Galerkin method is employed to approach numerically the problem. Conclusions are drawn on the influence of the damped Winkler elastic foundation on the critical flow velocity of the pipeline.

Keywords: stability, fluid, critical flow velocity, immersed pipe, damped Winkler elastic foundation

1. INTRODUCTION

Pipes with flowing fluid and immersed in fluid are used in many areas of the industry. The flow of the fluid in the tube as well as the flow of the external fluid causes oscillations in it. A number of scientists are conducting research in the field of fluid–structure interaction.

Gregory and Païdoussis [4] were the first to present a solution to the equations describing the dynamics of the pipe. They define the velocity of the fluid at which the pipeline loses stability as the critical velocity of the fluid.

Deng and Yang [2] studied the dynamics of pipes with different types of flowing fluid. The tubes are immersed in fluid. The pipe is considered as a cylindrical shell. Numerical surveys have been performed. The fluid–filled tubes examined in [3] are buried in an elastic solid or immersed in fluid. A semi–analytic finite element method is applied. The results are compared with those obtained by the scaled boundary finite element method. Great coincidence of the results is observed.

In [5] is presented an investigation about the dynamic stability of a pipe with a flowing fluid immersed in a non–viscous fluid moving at a constant speed. It is also given an analytical solution for the same type of pipe with a rigid body attached at one of its ends.

Lin and Qiao [6] examined an axially moving pipe immersed in a fluid. The Differential quadrature method is applied. Pipes with three types of supports at both ends were studied: fixed–fixed, pinned–pinned, pinned + torsion spring – pinned + torsion spring. Parametric studies have been performed.

In study [7] is considered a moving tube immersed in fluid. An analytical solution for non–viscous and viscous fluid is presented. Brennan [1] presents research on the inertial forces with which a fluid acts on a body immersed in it. Examples of analytical research and experiments are presented.

Wu and Shin [8] performed a dynamic study of a continuous tube conducting fluid. Transfer matrix method is used. Lolov [9] investigates fluid conveying pipe immersed in moving fluid and lying on Winkler elastic foundation. Numerical studies have been performed to determine the critical velocities of the two fluids.

The present article examines the dynamics of a pipelining on a damped Winkler elastic foundation. The dependence of the critical velocity of the fluid in the pipe on the critical velocity of the external fluid for different parameters of the damped Winkler elastic foundation is investigated.

2. VIBRATION OF A FLUID–IMMERSED STRAIGHT PIPE CONVEYING FLUID

The transverse vibration of a fluid–immersed straight pipe conveying inviscid fluid and lying on a damped Winkler elastic foundation is governed by the following differential equation [5]:

$$\mathrm{EI}\frac{\partial^4 \mathrm{w}}{\partial \mathrm{x}^4}+\left(\mathrm{m_f V^2}+\mathrm{m_e V_e^2}\right)\frac{\partial^2 \mathrm{w}}{\partial \mathrm{x}^2}+2\left(\mathrm{m_f V}+\mathrm{m_e V_e}\right)\frac{\partial^2 \mathrm{w}}{\partial \mathrm{x}\,\partial \mathrm{t}}+\left(\mathrm{m_f}+\mathrm{m_p}+\mathrm{m_e}\right)\frac{\partial^2 \mathrm{w}}{\partial \mathrm{t}^2}+\mathrm{d_w}\frac{\partial \mathrm{w}}{\partial \mathrm{t}}+\mathrm{k_w w}=0 \qquad (1)$$

where t is the time, $w(x,t)$ is the lateral displacement of the pipe axis, x is the coordinate along the axis, EI is the rigidity of the pipe. The mass of the pipe per unit length is denoted by $\mathrm{m_p}$ and the mass of the fluid per unit length of the pipe by $\mathrm{m_f}$. $\mathrm{m_e}$ is the added mass of the external fluid. V is the flow velocity of the fluid in the pipe and $\mathrm{V_e}$ is the velocity of the external fluid. $\mathrm{k_w}$ and $\mathrm{d_w}$ are respectively the rigidity and the damping coefficient of the foundation.

The added mass of the external fluid per unit length of the pipe $\mathrm{m_e}$ in the case when the pipe is close to a horizontal plane (Figure 1) is calculated by the following formula, given in [1]:

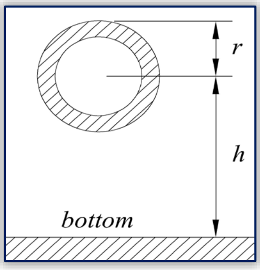

Figure 1. A scheme for obtaining the added mass of the external fluid

$$m_e = \pi \rho_e r^2 \left(1 + \frac{r^2}{2h^2}\right) \tag{2}$$

where ρ_e is the density of the external fluid.

The spectral Galerkin method is applied to approximate the solution of differential equation (1). The solution is sought in the following form:

$$w(x,t) = \sum_{i=1}^{n} y_i(x) z_i(t) \tag{3}$$

where: $z_i(t)$ – are unknown functions; $y_i(x)$ – are basic functions that satisfy the boundary conditions of the pipe. Such functions are the functions describing the *i*–th mode of vibration of a beam with the same static scheme as the immersed pipe. On the basis of the differential equation, describing the lateral vibrations of an immersed tubular beam, filled with stationary fluid ($V = 0$) is obtained [8]:

$$y_i^{IV}(x) = \gamma_i^4 y_i(x) \tag{4}$$

where:

$$\gamma_i = \sqrt[4]{\frac{(m_f + m_p + m_e)\omega_i^2}{EI}} \tag{5}$$

where ω_i is the circular frequency of the beam.

Substituting equation (3) into equation (1), one obtains the residual function:

$$R(x,t) = \sum_{i=1}^{n} \left\{ (m_f + m_p + m_e) y_i \ddot{z}_i + [d_w y_i + 2(m_f V + m_e V_e) y_i'] \dot{z}_i + [(EI\gamma^4 + k_w) y_i + + (m_f V^2 + m_e V_e^2) y_i''] z_i \right\} \tag{6}$$

In (6) and in the sequel, primes denote derivatives with respect to x and dots with respect to the time t.

The Galerkin method requires the residual function $R(x,t)$ to be orthogonal to the basic functions in the interval $x \in [0;1]$:

$$\int_0^1 R(x,t) y_k(x) dx = 0, \text{ for } k = 1,\dots,n \tag{7}$$

Equation (7) is rewritten in the following form:

$$\sum_{i=1}^{n} \int_0^1 \{ (m_f + m_p + m_e) y_i \ddot{z}_i + [d_w y_i + 2(m_f V + m_e V_e) y_i'] \dot{z}_i + [(EI\gamma^4 + k_w) y_i + \\ + (m_f V^2 + m_e V_e^2) y_i''] z_i \} y_k \, dx = 0 \text{ for } k = 1,\dots,n \tag{8}$$

Equation (8) represents a system of n differential equations with n unknown functions $z_i(t)$. In order to solve the system, the described in [8] method is applied. According to it the pipe is divided to sections with length Δx. The following relationships are taken into account:

$$\int_0^1 y_i y_k dx = \{y_i\}^T \{y_k\} \Delta x \tag{9}$$

$$\int_0^1 y_i' y_k dx = \{y_i'\}^T \{y_k\} \Delta x \tag{10}$$

$$\int_0^1 y_i'' y_k dx = \frac{1}{EI} \{M_i\}^T \{y_k\} \Delta x \tag{11}$$

where in (9),(10) and (11):

$\{y_i\}$ – is a column vector consisting of the lateral displacements of the stations on the axis of the pipe, corresponding to the i–th eigen form in the case of stationary fluid ($V = 0$);

$\{y_i'\}$– is a column vector consisting of the rotations of the cross–sections in the stations on the axis of the pipe, corresponding to the i–th eigen form in the case of stationary fluid ($V = 0$);

$\{M_i\}$ – is a column vector consisting of the bending moments in the stations on the axis of the pipe, corresponding to the i–th eigen form in the case of stationary fluid ($V = 0$).

Substituting (9),(10) and (11) in (8) the following system of n differential equations with n unknown functions $z_i(t)$ is obtained:

$$\sum_{i=1}^{n}\Big\{(m_f+m_p+m_e)\{y_i\}^T\{y_k\}\ddot{z}_i+\left[d_w\{y_i\}^T\{y_k\}+2(m_f V+m_e V_e)\{y_i'\}^T\{y_k\}_i\right]\dot{z}_i+$$
$$+\left[(EI\gamma^4+k_w)\{y_i\}^T\{y_k\}+(m_f V^2+m_e V_e^2)\frac{1}{EI}\{M_i\}^T\{y_k\}\right]z_i\Big\}\Delta x=0 \qquad (12)$$

The system (12) could be rewritten in matrix form:

$$M\ddot{z}+C\dot{z}+Kz=0 \qquad (13)$$

The general solution of the system (12) is expressed through the roots ($\lambda_1,\ldots,\lambda_{2n}$) of the equation:

$$\det X=0 \qquad (14)$$

The elements of the matrix X are given by:

$$X_{ik}=\lambda^2 M_{ik}+\lambda C_{ik}+K_{ik} \qquad (15)$$

$$M_{ik}=(m_f+m_p+m_e)\{y_i\}^T\{y_k\}\Delta x,\quad M_{ik}=0\ (\text{when } i\neq k) \qquad (16)$$

$$C_{ik}=\left[d_w\{y_i\}^T\{y_k\}+2(m_f V+m_e V_e)\{y_i'\}^T\{y_k\}_i\right]\Delta x \qquad (17)$$

$$K_{ik}=\left[k_w\{y_i\}^T\{y_k\}+(m_f V^2+m_e V_e^2)\frac{1}{EI}\{M_i\}^T\{y_k\}\right]\Delta x+E_{ik} \qquad (18)$$

$$E_{ik}=EI\gamma^4\Delta x,\quad E_{ik}=0\ (\text{when } i\neq k) \qquad (19)$$

On the basis of obtained roots ($\lambda_1,\ldots,\lambda_{2n}$) could be drawn conclusions about the stability of the system. The system is stable if the real part of all the roots of the characteristic equation (14) is negative.

The roots ($\lambda_1,\ldots,\lambda_{2n}$) depend on all the parameters of the system. If all of them are fixed except the velocity of the conveyed fluid V or the velocity of the external fluid V_e, one could obtain the corresponding critical velocities.

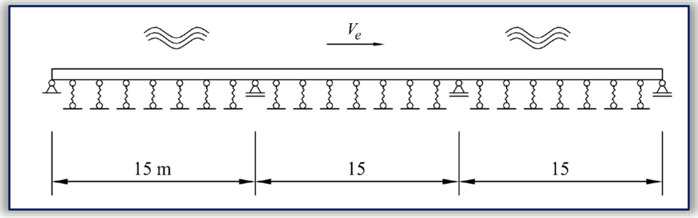

Figure 2. Static scheme of the investigated pipe conveying fluid

3. RESULTS AND DISCUSSION

Numerical studies have been carried out for the fluid conveying pipe in Figure 2.

The geometric and the material characteristics of the pipes are: rigidity $EI=771.26\,kNm^2$; $m_p=10.80\,kg/m$; $m_e=18.02\,kg/m$. The density of the external fluid is $\rho_e=1\,t/m$ and the density of the internal fluid is $\rho_f=1.2\,t/m$.

In the Figures below is shown the dependence of the critical velocity of the fluid in the pipe V_{cr} on the critical velocity of the external fluid $V_{e,cr}$ for different parameters of the of the damped Winkler elastic foundation. The sign 'minus' on the graphics corresponds to a velocity of the extremal fluid V_e that is in opposite direction of the velocity of the internal fluid V.

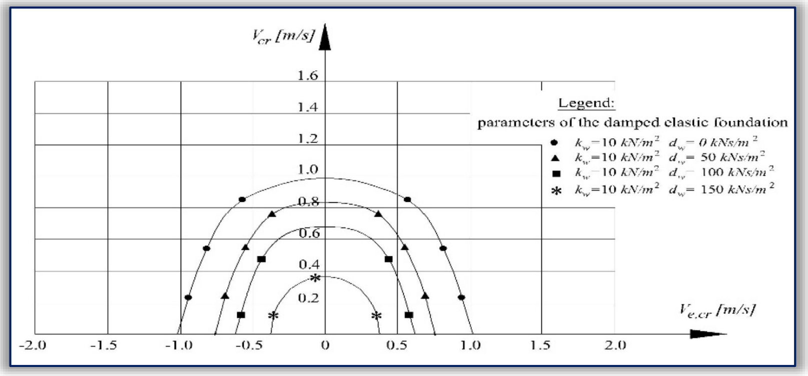

Figure 3. Dependence of the critical velocity of the fluid in the pipe V_{cr} on the critical velocity of the external fluid $V_{e,cr}$ for $k_w=10\,kN/m^2$

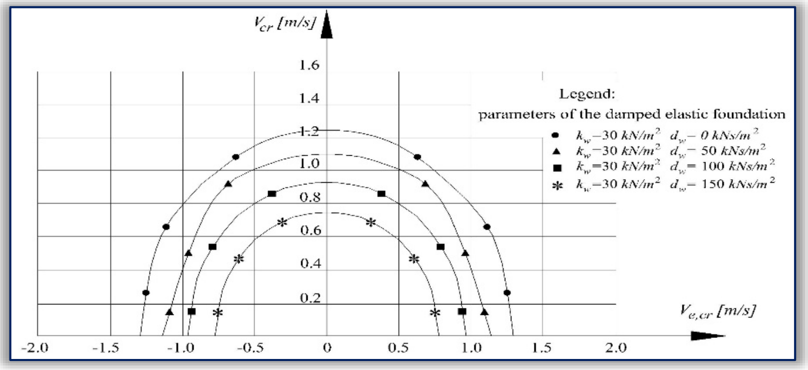

Figure 4. Dependence of the critical velocity of the fluid in the pipe V_{cr} on the critical velocity of the external fluid $V_{e,cr}$ for $k_w = 30\,kN/m^2$

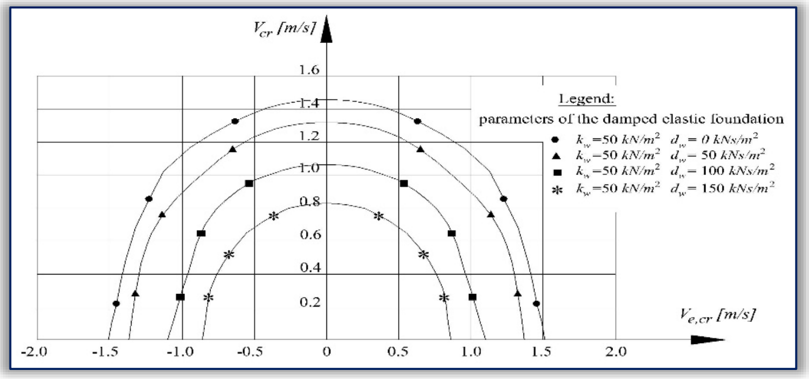

Figure 5. Dependence of the critical velocity of the fluid in the pipe V_{cr} on the critical velocity of the external fluid $V_{e,cr}$ for $k_w = 50\,kN/m^2$

4. CONCLUSION

The results in Figure3, Figure4 and Figure5 show that for the investigated system the damping of the Winkler elastic foundation has a destabilizing effect. The bigger the damping parameter d_w the smaller is the stability area depicted in the Figures.

Note: This paper was presented at IIZS 2021 – The XI International Conference on Industrial Engineering and Environmental Protection, organized by Technical Faculty "Mihajlo Pupin" Zrenjanin, University of Novi Sad, in Zrenjanin, SERBIA, in 07–08 October, 2021.

References

[1] Brennan, C., A review of added mass and fluid internal forces, Technical Report, Naval Civil Engineering Laboratory, 1982.
[2] Deng, Q., Yang Z., Wave propagation in Submerged Pipe Conveying Fluid, Acta Mechanica Solida Sinica, Vol. 32, No.4, pp. 483 – 498, 2019.
[3] Duan, W., Kirby, R., Guided wave propagation in buried and immersed fluid–filled pipes: Application of the semi analytical finite element method, Computers & Structures, Vol. 212, pp. 236–247, 2019.
[4] Gregory, R., Païdoussis, M., Unstable oscillations of tubular cantilevers conveying fluid, Proceedings of the Royal Society of London, Vol. A, pp.512–523 and pp.528–542, 1966.
[5] Hellum, A., Mukherjee, R., Hull, A., Flutter instability of a fluid–conveying fluid–immersed pipe affixed to a rigid body, Journal of Fluids and Structures, Vol. 27 , pp. 1086–1096, 2011.
[6] Lin, W., Qiao, N., Vibration and stability of an axially moving beam immersed in fluid, International Journal of Solids and Structures, Vol. 45, pp. 1445–1457, 2008.
[7] Taleb, I., Misra, A., Dynamics of an axially moving beam submerged in a fluid, Journal of hydronautics, Vol. 15, Issue 1–4, pp. 62–66, 1981.
[8] Wu, J., Shih, P., The dynamic analysis of a multispan fluid–conveying pipe subjected to external load, Journal of Sound and Vibration, Vol. 239, pp.201–215, 2001.
[9] Лолов, Д., Динамична устойчивост на потопен тръбопровод, провеждащ флуид и лежащ върху Винклерова еластична основа (Dynamic stability of fluid–immersed pipe conveying fluid and lying on Winkler elastic foundation), XVI международна научна конференция ВСУ'2016, Structural Mechanics, Structural Engineering, Earthquake Engineering, pp. 85–90, 2016.